

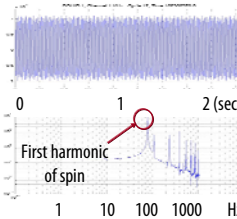
TRAPPED FLUX MAPPING FOR THE GRAVITY PROBE B GYROSCOPES



Michael Salomon, John W. Conklin, Michael Dolphin G. Mac Keiser, Alex Silbergleit, Paul Worden

High Frequency Signal and Trapped Flux Mapping

High Frequency snapshot and FFT



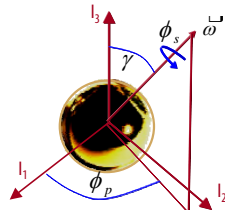
Why a High frequency signal ?

- Below critical temperature: magnetic anomalies (fluxons) freeze on gyro surface
- Fluxons spin with gyro
- SQUID signal modulated at harmonics of spin

Principle of Trapped Flux Mapping (TFM)

- Fluxons responsible for HF signal
- Use spin harmonics from HF signal to map the magnetic potential distribution created by the fluxons around the gyroscope
- Trapped flux through the pick-up loop = 1% of Low Frequency signal
- **Trapped flux mapping provides info on main science signal**

Trapped Flux Model



Expansion of magnetic potential in body-fixed frame

- Use spherical harmonics basis in body frame
 - Rotate to the pick-up loop frame
 - Compute flux through SQUID pick-up loop
- Each harmonic H_n of spin is modeled and linearly related to magnetic potential coefficients**

$$H_n(t) = \frac{\phi_0}{2} \sum_{\substack{\text{odd} \\ l \geq |n|}} \left(\frac{r_g}{b} \right)^l \sum_{m=-l}^l I_l e^{im\phi_p} d_{nm}^l(-\gamma) d_{0n}^l\left(-\frac{\pi}{2} + \beta\right) A_{lm}$$

n^{th} harmonic of spin: read from HF signal

Euler rotation from body to pick-up loop frame

Magnetic Potential Coefficients

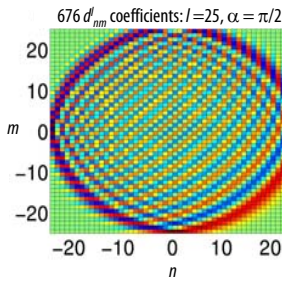
Linear Model in A_{lm} !

Rotation of spherical harmonics

$$d_{nm}^l(\alpha)$$

Issue with rotation of spherical harmonics

- Over 400,000 coefficients for each fit to data
- Computation involves sum of factorials: numerical errors, time-consuming, ...



Algorithm developed for this study

- Use hypergeometric polynomials: pre-computing coefficients
- Up to **1 million** d_{nm} coefficients in 1 second on 800 Mhz Sun Blade workstation
- 13 digits accuracy up to $l=51$

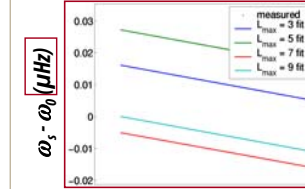
Finding nanoHertz level spin speed

$$H_n(t)$$

Absolute spin phase is unknown

- Harmonics of spin measured from FFT of 2-second long snapshots: => spin phase only known w.r.t. snapshot time origin
- But **true orientation** of fluxons w.r.t. pick-up loop needed for model of SQUID signal: **absolute spin phase needed**
- Estimate spin speed and spin-down rate with high accuracy

Gyro Spin Speed for 4 potential distributions



Algorithm developed for this study

- Modification of Nelder-Mead non linear search
- Find spin phase which yields H_n best fit by model
- **10 nanoHertz precision in spin speed obtained**

Trapped Flux Truth Model

Description

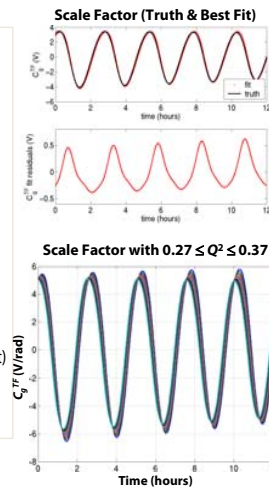
- Use known set of parameters to generate HF SQUID signal

Purpose

- Validate fitting algorithms
- Find bugs in code
- Assess sensitivity of fits to input parameters
- Gain insight into information contained in HF SQUID signal

Some Results

- TFM algorithm determines scale factor to <10% (top right)
- Scale factor found to be insensitive to asymmetry parameter Q^2 (bottom right)

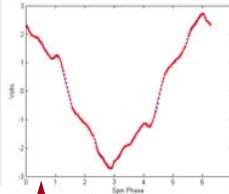
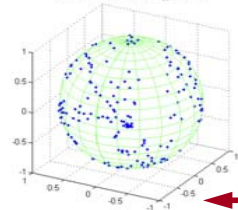


Estimation of Fluxon Locations

Alternate Method of TFM

- Start with random distribution of fluxons trapped on rotor surface
- Compute magnetic flux as superposition of flux generated by each fluxon
- Stochastically vary fluxon positions until computed flux signal coincides with the measured one

Locations of found fluxons on gyro surface



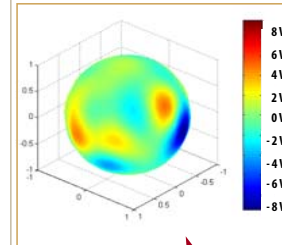
Results for Gyro 1

- Measured and best fit magnetic flux through SQUID pick-up loop (above, in red and blue)
- Map of fluxon locations on surface of rotor (best fit)

Magnetic Potential Determination

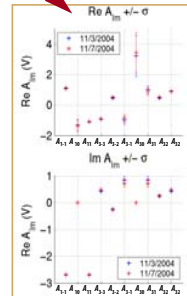
Gyro 1 Magnetic Potential Coefficients

Best fit A_{lm} coefficients of trapped magnetic potential consistent from day to day (~5%)



Gyro 1 Magnetic Potential Map

Best fit map of scalar magnetic potential on surface of gyro 1

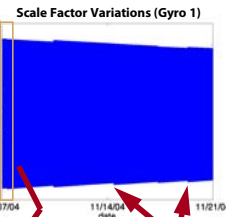
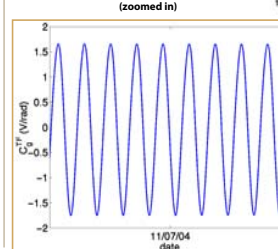


Gyro Scale Factor Determination

Trapped Flux Mapping Estimates LF Gyro Scale Factor

Variations in gyroscope LF scale factor (~1% of total scale factor) are proportional to 0th harmonic of spin (H_0)

Scale Factor Variations (Gyro 1) (zoomed in)



With a map of the trapped magnetic potential, scale factor variations can be reconstructed for primary science data analysis

