Polhode Motion of the Gravity Probe-B Gyroscopes

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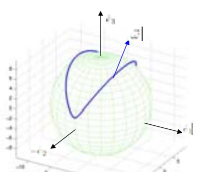
Euler's Equations

The Gravity Probe B gyroscopes are the roundest objects ever manufactured. They operate in a near torque-free environment. As round as they are, they still have asymmetry, and exhibit polhode motion as they spin.

Angular Momentum $\vec{L} = \vec{I} \cdot \vec{\omega}$

Kinetic Energy: $E = \frac{1}{2}\vec{\omega} \cdot \vec{L}$

General Euler Equations (no external torque): $\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = 0$



Polhode Motion
Spin axis moves along polhode path, which is determined by the intersection of two ellipsoids: constant angular momentum (L) and constant kinetic energy (E).

Conservation of energy and angular momentu whose expressions are:

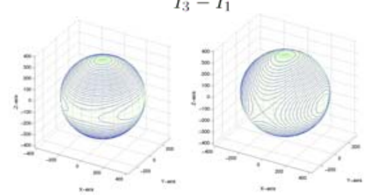
$$2E = I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2$$

$$L^2 = (I_1\omega_1)^2 + (I_2\omega_2)^2 + (I_3\omega_3)^2$$

Rotor Asymmetry

Given E and L, the shape of the polhode path is determined by a single asymmetry parameter Q^2 (I_i is the moment of inertia about i^{th} principal axis)

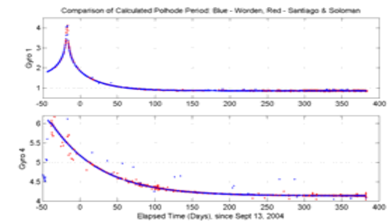
$$Q^2 = \frac{I_2 - I_1}{I_3 - I_1}$$



$Q^2 = .10$ $Q^2 = .50$

Even if there is no energy dissipation, knowledge of Q^2 is needed to accurately determine the polhode path.

Observations & Explanation



The polhode period of each gyro was observed to be changing over time. Each of the four exhibited similar asymptotic behavior. The only explanation that agrees with all the observation was that of energy dissipation in the rotor body. Dissipation moves the spin axis towards the maximum inertia axis where energy is minimum.

$$\frac{E_{max} - E_{min}}{E_{min}} = \frac{I_3 - I_1}{I_1} \sim 10^{-6}$$

With kinetic energy on the order of 1 J, the dissipated power needed to move the spin axis all the way from min to max inertia axis over one year is ~10^{-13} W.

While we can assume constant energy over one polhode period, we need to account for the dissipation of energy over the course of the mission.

Dissipation Model (1)

To account for energy dissipation, a term is added to the Euler equations which satisfies 3 fundamental assumptions:

1) Angular Momentum is conserved. $\dot{L} = 0$
2) Energy is never added. $\dot{E} \leq 0$
3) Energy dissipation goes to zero ONLY when spin axis aligned with principal axes of gyro. $\dot{E} = 0$

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \mu \frac{(\vec{L} \times \vec{\omega}) \times \vec{L}}{L} \;*$$

$$\frac{dE}{dt} = -\mu[\omega^2 L^2 - (2E)^2] \leq 0 \quad : \quad \mu \geq 0$$

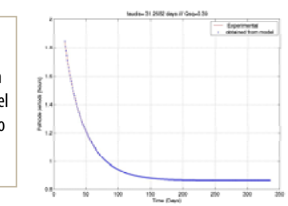
* Modified Euler Equations introduced by Alex Silbergleit.

Algorithm to find Q^2:

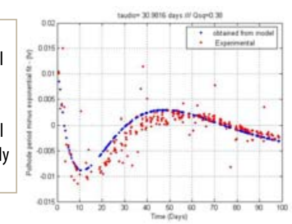
- Chose a long time stretch of data and set Q^2 (and some other parameters).
- Compute initial energy (E_i) from polhode period ($T_p(t_i)$).
- Integrate energy equation, and find predicted $E_{pr}(t)$.
- Convert $E_{pr}(t)$ to polhode period ($T_{ppr}(t)$).
- Fit to polhode period data using quadratic cost function.
- Repeat process to find minimum cost. Vary Q^2 over desired range, and find best Q^2 by analyzing the cost function.

Dissipation Model (2)

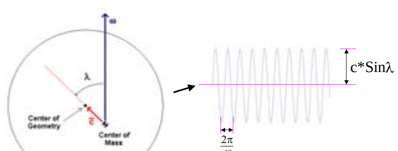
Model Fit
The predicted polhode period from the dissipation model fits extremely well to the measured polhode period.



Even when the baseline exponential is removed, the dissipation model captures the residual behavior impressively well.

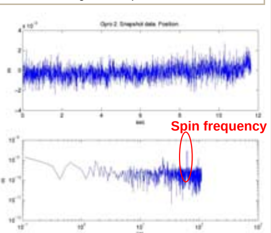


GSS Position Model (1)

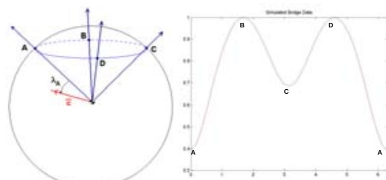


Mass unbalance creates a signal at spin frequency in the position as measured by the Gyro Suspension System (GSS). The amplitude of the signal is a function of both the length of the mass offset, and its angle to the spin axis.

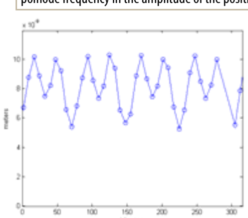
The GSS position is measured with RMS error of 0.45 nm. The signal due to mass unbalance is of order 1 to 10 nm, and is clearly visible in the frequency analysis of the signal.



GSS Position Model (2)



Even if the asymmetry parameter (Q^2) was equal to zero, and thus the polhode paths were circular, we would expect "M" shaped variations at polhode frequency in the amplitude of the position signal at spin frequency.

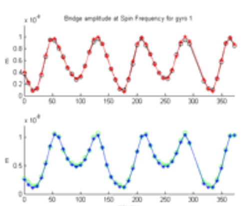


The "M" shape curves indeed are visible for some times of the mission. This example is for gyro 1, relatively early in the mission (October, 2004).

The period of this signal matches known polhode frequency.

This also provides an estimate of the mass unbalance vector.

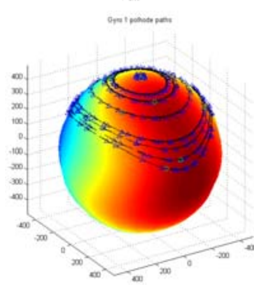
GSS Position Model (3)



To match the signal, a relatively simple model of the gyro surface in 1st and 3rd spherical harmonics was employed to account for slight differences between two sets of orthogonal measurements.

The resulting model matches the data throughout the entire mission.

The best fit model was found for a range of Q^2 values. Each best fit was evaluated by a least squares cost function.

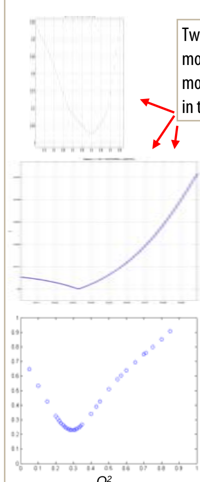


Gyro 1 is shown here with its surface model as found using this method.

Also, some of the polhode paths used in the fitting process are shown.

Triangles denote locations of the spin axis at each data point.

Cost Analysis of Q^2



Two versions of the dissipation model and the GSS position model for gyro 1. Results shown in terms of fit cost function vs Q^2.

The alternate dissipation model method is based on a 3rd order exponential interpolation of the measured polhode period data points:

$$T_p(t) = T_{pa} + \sum_{n=1}^{3} C_n e^{-n(t-t_0)/\tau_{diss}}$$

The above dissipation model allows for theoretical expressions of the fit coefficients C_n as functions of Q^2. By matching those theoretical expressions with the found values provides the sought number for the asymmetry parameter (the only approach to accommodate gyros 3 and 4).

All three methods suggested Q^2 for gyro 1 is in the region of .29 to .39

Results And Applications

By combining all our results, we get an estimate for Q^2 for each of the four gyros.

	Gyro 1	Gyro 2	Gyro 3	Gyro 4
Q^2	0.33 (0.29 – 0.38)	0.36 (0.14 - 0.43)	0.72 (0.56 – 0.75)	0.32 (0.30 – 0.40)

The importance of knowing Q^2 is that it is used in computation of polhode phase and polhode angle for each gyroscope for the entire mission, which is needed for science data analysis.

Accurate knowledge of the polhode enables us to account for the polhode frequencies in a the signals in which it appears. Some of the measurements which include polhode frequencies are: GSS position, GSS suspension force, vehicle attitude and control system, vehicle orbit, LF science SQUID signal, HF SQUID trapped flux signal, measured spin rate and spin phase. The knowledge of polhode phase is critical to the Trapped Flux Mapping efforts (see Trapped Flux Mapping poster).

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