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Gravity Probe B Relativity Mission

VERIFICATION OF KNOWLEDGE OF GUIDE STAR PROPER MOTION
from VLBI MEASUREMENTS

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CHANGE RECORD

LIST OF ACRONYMS, ABBREVIATIONS AND SYMBOLS

TPM	Technical Performance Metric
VLBI	Very Long Baseline Interferometry
VLBA	Very Long Baseline Array
mas	milliarcseconds
marsec	milliarcseconds
GHz	gigahertz
NRAO	National Radio Astronomy Observatory
FWHM	Full Width at Half Maximum
RSS	Root Sum of Squares

TABLE OF CONTENTS

APPROVALS	II
CHANGE RECORD.....	III
LIST OF ACRONYMS, ABBREVIATIONS AND SYMBOLS.....	IV
LIST OF TABLES	VI
1. SUMMARY.....	1
2. INTRODUCTION.....	1
2.1. T003 REQUIREMENT.....	2
3. VERIFICATION OF KNOWLEDGE OF GUIDE STAR PROPER MOTION FROM VLBI REQUIREMENTS.....	2
3.1 STATISTICAL UNCERTAINTY	2
3.2 UNKNOWN BIAS.....	3
3.3 COMBINED UNCERTAINTY.....	4
APPENDIX. PROPER MOTION UNCERTAINTY FROM VLBI OBSERVATIONS OF IM PEG	5
<i>Statistical Uncertainty</i>	5
<i>Unknown Bias</i>	6
REFERENCES.....	11

LIST OF TABLES

Table 1-1. T003 Requirement 25.4.1

Table 1-2. Mitigations for low-probability potential unknowns

Table 3-1. Summary of proper motion uncertainty

1. SUMMARY

This document verifies T003 requirement 25.4.1, which is the requirement on knowledge of the proper motion of the radio centroid of the guide star. This proper motion is a programmatic technical performance measure (TPM), which is tracked regularly in monthly reviews. Recently, we identified potential low-probability error terms, which, if unmitigated, would lead to larger-than-expected high-confidence ($>3\sigma$) intervals for the uncertainty of the proper motion. The result of this analysis is included in the appendix. The verification is based on known error terms and mitigation plans for addressing the identified potential low-probability unknowns. Table 1-1 summarizes the uncertainty based on known error terms. Table 1-2 summarizes the mitigations for low-probability potential unknowns.

Table 1-1. T003 Requirement 25.4.1

Requirement	Specification	Value	Pass
T003 Requirement 25.4.1	< 0.14 mas/yr	0.10 mas/yr	Yes

Table 1-2. Mitigations for low-probability potential unknowns

Improbable Potential Unknowns	Mitigations
Future drop in source radio flux density may lead to reduced number of future position measurements.	Continue to make VLBI measurements until required accuracy is achieved and maintain the quarterly schedule. (Note: Extending the observing schedule is likely to be necessary to prevent proper motion from being the largest contributor to the overall GP-B experiment error.)
Significant structure-related proper motion common to the two primary extragalactic reference sources	Addition of a third reference source to the observing program. This addition will likely reveal any such common proper motion.
Errors in the proper motion of the guide star proportional to the separation on the sky of that star from the line between the two primary extragalactic reference sources.	Addition of a third reference source to the observing program. This addition provides sensitivity to position-dependent errors with an arbitrary gradient on the sky, and should greatly reduce the probability of a significant hidden error resulting from a chance alignment of that error gradient perpendicular to the line between the first two reference sources.

2. INTRODUCTION

This analysis document provides verification of the T003 requirement “25.4.1 Proper Motion of Radio Centroid” which is stated in section 2.1. Section 3 summarizes the more detailed discussion contained in the appendix, providing values for statistical uncertainty and unknown bias in the proper motion measurements. For reference, (Schultz, 1998) provides the rationale for selection of IM Peg as the GP-B guide star. The uncertainty in

6/10/03

the rate of change of the spacecraft orientation due to the (small) offset on the sky between the radio centroid of IM Peg and the effective guide point of the telescope are considered elsewhere.

2.1. T003 Requirement

25.4.1 *Proper Motion of Radio Centroid*

The proper motion of the radio centroid of the guide star relative to extragalactic sources shall be determined via VLBI astrometry with an error ≤ 0.14 marcsec/year both in declination and in right ascension times the cosine of the declination.

Mission-specific proper motion is defined in T002 #3 and listed below for reference:

The proper motion value critical to the mission is the best estimate that will be available, about 1 year after the end-of-mission, of the proper motion at the midpoint of the span of science data. The projected uncertainty in proper motion is the uncertainty of that estimate, derived prior to launch and based on conservative assumptions regarding the success of all existing or planned observations whose analysis is not yet complete.

3. VERIFICATION OF KNOWLEDGE OF GUIDE STAR PROPER MOTION FROM VLBI REQUIREMENTS

3.1 Statistical Uncertainty

The statistical uncertainty in the proper motion of IM Peg's radio centroid is projected to be 0.075 mas/yr under the plausible (but optimistic) assumption that 8 additional position values are obtained between January 2003 and December 2004. This projection is based on the analysis of all currently available VLBI position estimates. The latter are obtained via established analytical procedures which are described in papers by Bartel et al., (1986), Herring, Davis, and Shapiro (1990), Lestrade et al. (1995), Lebach et. al. (1999), and references therein. The above uncertainty value includes the benefits of Kalman-filter analysis, which reduced the uncertainty by ~10% below that obtained based upon fits to IM Peg's positions estimated without the Kalman-filter software. The results of the proper-motion analysis have been reported regularly at GP-B Monthly Reviews. The review process for this analysis has included:

- Peer review for the published papers cited above
- Guide Star Review meeting of May 2001
- Review by GP-B Science Advisory Committee (December 2002)

Even if low flux density were to cause the loss of the last two projected position estimates, which are the most valuable ones, the projected statistical uncertainty would increase only to 0.086 mas/yr. We take this value as a reasonably conservative estimate of the statistical uncertainty that will be obtained using all successful VLBI observations completed by the end of the mission. The above values allow for the conservative assumption that IM Peg could be accelerating. The existing Kalman-filter position estimates are, however, consistent with zero acceleration, and if acceleration were not

included, the standard error of the proper motion would be smaller by a factor greater than 3.

A low-probability potential unknown associated with the statistical uncertainty is whether a large fraction or even all of the remaining pre-December 2004 measurements result in non-detections. If this were the case, then the mitigation would be to extend the observing plan for a longer period until sufficient statistics were achieved. Since the current error tree indicates the GP-B hardware performance will exceed requirements by a wide margin, extension of the observing program is likely to be desirable in any case.

3.2 Unknown Bias

One estimate of the unknown bias error (at the end of the mission) in the guide star proper motion is the magnitude of the currently estimated apparent relative proper motion of the two radio sources that have been the primary VLBI position references. (As explained in the appendix, for this purpose we allow no acceleration in the estimated relative proper motion.) This estimated proper motion is -0.02 ± 0.02 mas/yr in right ascension and -0.01 ± 0.02 mas/yr in declination. However, the magnitude of this result is largely determined by the positions estimated for the four most recent VLBI sessions analyzed to date with the Kalman filter. These positions exhibit relatively large postfit position residuals. Exclusion of any one of these positions yields a large fractional change in at least one proper-motion component. Thus, rather than accepting 0.01 or 0.02 mas/yr in each component as our estimated standard error due to unknown bias, we instead adopt the value 0.05 mas/yr (see appendix).

This is a conservative upper bound, because it is highly probable that the extended reference source 3C 454.3 introduces an error that is at least a few times larger than the more compact B2250+194, and therefore that the addition of a third reference source will reduce the uncertainty to a significantly smaller value. (A third reference source has been included in the VLBI observations since November 2002.) Furthermore, other observations by NASA/GSFC, which employ data reduction methods that tend to produce larger relative motion shifts than the CfA methods, are consistent with our choice of 0.05 mas/yr as a reasonable upper-bound on this type of unknown bias.

Two improbable sources of unknown bias will be mitigated by the addition of the third reference source. The first is the possibility that the two original reference sources share a common dominant component of proper motion. If this were the case, then the common component would not contribute to the differential motion between the two reference sources, and it would therefore lead to the underestimation of the unknown bias associated with reference source proper motion. Observations of the third source will greatly reduce the risk of such an underestimate, and will likely provide a means to reduce the bias.

The second improbable source of unknown bias stems from the possibility that our estimate of the guide-star proper motion contains an unexpectedly large error whose magnitude is proportional to the star's separation on the sky from the line connecting the two primary extragalactic reference sources. Such an error would not be revealed as an

apparent relative proper motion of those reference sources. A large error of this type could occur only if the gradient on the sky of the separation-dependent proper motion errors were oriented with a dominant component perpendicular to the line connecting the two reference sources. This orientation would allow the magnitude of the differential motion between the two reference sources to greatly underestimate that of the bias in the guide-star proper motion. Again, data from the third reference source greatly reduce the risk of such an underestimate, and will likely also provide a means to reduce the unknown bias .

3.3 Combined Uncertainty

The combined uncertainty is the RSS of the statistical uncertainty and the estimate of unknown bias as summarized in Table 3-1.

Table 3-1. Summary of proper motion uncertainty

Source	Value
Statistical Uncertainty	0.086 marcsec/yr
Unknown Bias	0.05 marcsec/yr
Combined Uncertainty	0.10 marcsec/yr

APPENDIX. Proper motion uncertainty from VLBI observations of IM Peg

Statistical Uncertainty

The plausible statistical error is the currently projected 1 sigma value (0.075 mas/yr, if the Kalman filter is used). This value is obtained from a covariance study of a data set consisting of all existing positions for the radio emission from the guide star, plus eight projected additional results from observations from January 2003 through December 2004. The proper motion is estimated at the epoch 2004.5 (the approximate midpoint of the science mission for a launch at the end of October 2003) with an astrometric model that allows for an unknown proper acceleration. The statistical standard error allowed for each individual measured position is taken to be 0.49 mas in each coordinate. This value makes the chi-square per degree-of-freedom exactly 1.0 in the latest astrometric solution for the motion of the radio emission based on Kalman-filter-derived positions through April 2002. (This solution includes all the Kalman-filter-derived VLBI positions observations available as of May 2003. Two additional VLBI sessions in 2002 that yielded preliminary position estimates have yet to be processed with the Kalman filter.) Alternatively, it could be argued that the existence of a significantly nonzero proper acceleration is implausible, since there is no evidence of any kind for a sufficiently massive body near the guide star to accelerate it. If this argument were accepted, then the (constant) proper motion could be estimated with severalfold less statistical uncertainty than is claimed here.

The worst case statistical error expected to be present at the end of the mission depends significantly on the number of VLBI positions that will be obtained by that time. In particular, this error depends upon the probability that, due to repeated instances of low flux density, very few or none will be obtained. Since positions have been obtained for 27 of the 28 epochs observed and analyzed since January 1997, one might be tempted to conclude that this probability is too low to significantly influence the plausible error considered above. However, a detectable downward trend in the guide star's radio flux density in the interval from 1997 through 2002 implies that the probability is not so small. Before considering the worst case, we first discuss a reasonably conservative estimate of the error that will be obtained from the analysis of all VLBI positions available within several months of the end of the mission. Here we allow for the possibility that two of the expected eight additional positions either not yet scheduled or analyzed from sessions during 2003 and 2004 yield no position estimate. To be conservative, we calculate the statistical error assuming that this failure occurs for the last two sessions during the mission, which are potentially the most valuable. The resulting projected error is 0.086 mas/yr. Nevertheless, for the purpose of a worst case analysis, it would be imprudent to adopt a value less than 3% for the probability that we obtain no new positions after 2002, or less than 10% for the probability that we obtain only one such position. (These minimum prudent values are based upon the history of the guide star's radio flux density and its optical brightness, and upon the slight but theoretically expected anticorrelation of the two. These values likely overstate the actual probabilities.)

The definition of the worst case error is potentially ambiguous when the probability distribution of the error value depends on the occurrence of discrete events, such as obtaining a particular number of positions. For this box, a "case" is interpreted as the occurrence that particular number of VLBI positions are obtained and that the resulting statistical error in proper motion exceeds some value, v . Only cases with a probability of at least 0.27% are considered. (The probability 0.27% is simply 100% minus the 99.73% double-sided 3 sigma confidence level that applies in the case of a normal distribution of errors.) The worst case error is then the maximum value of v for all cases considered.

In this scheme, the probability of a "case" is the product of the probability of obtaining a given number of positions and the conditional probability of the statistical error exceeding v . For any given number of positions, the largest value of v that allows for a case probability of 0.27% depends on the probability of obtaining the given number of positions. For cases in which no new positions are obtained, v must be exceeded with a conditional probability of $0.0027/0.03 = 0.09 = 9\%$, i.e., with a 9% probability given that no new positions are obtained. Since the statistical error of a least-squares estimate, such as our proper-motion estimate, has an approximately normal distribution, the bound v that is exceeded with 9% (double-sided) probability can be found from tables of the normal distribution. The required bound is the 1.7 sigma level. Since the standard error of the estimate derived from the current data set is 0.17 mas/yr, the 1.7 sigma value is 0.29 mas/yr. Similarly, for cases in which just one new position is obtained, v must be exceeded with a conditional probability of $0.0027/0.1 = 0.027 = 2.7\%$, which for a normal distribution corresponds to the 2.2 sigma level. The earlier the epoch for which the new position is obtained, the larger the expected experimental standard error of proper motion. Hence, a relatively conservative error value can be obtained under the assumption that the one new position is during September 2003. The standard error with this new position is 0.15 mas/yr, and the 2.2 sigma value 0.33 mas/yr. This last value is adopted as the worst case statistical error. (For comparison, if our current projection of eight additional positions held true, resulting in a 0.075 mas/yr statistical standard error, the 3 sigma confidence level would be $3 \times 0.075 \text{ mas/yr} = 0.23 \text{ mas/yr}$.)

Unknown Bias

The VLBI observations of the guide star are interleaved with observations of two extragalactic radio reference sources. Thus we can determine its astrometric motions with respect to one or both of these reference sources. The reference source lying closer on the sky to the guide star (3C 454.3, 0.7 deg) has a redshift $z = 0.859$ (Hewitt and Burbidge 1989, as quoted in on-line NASA/IPAC NED database), and the one lying farther from the guide star (B2250+194, 3.0 deg) has a redshift $z = 0.284$ (Engels et al. 1998, also in NED). Based on a wide variety of evidence, it is generally understood that each such compact radio source is closely tied to the center of mass of its host galaxy. At the distances of these sources, the proper motions of these centers of mass are all expected to be very small ($\ll 0.01 \text{ mas/yr}$), based on the observed range of radial velocities of nearby galaxies. Nevertheless, angular velocities of order 0.1 mas/yr are seen in the relative positions of certain compact components in the radio brightness maps computed for some radio sources. These (often unresolved) components are identified as local brightness maxima in the maps, and they typically are observable for between one

year and several years. In most such cases it is possible to identify one peak that is more permanent and apparently more stationary than the others, and this peak is called the core component. In one case observations confirmed that the relative angular velocity of the core components of two galaxies are mutually stationary to within ~ 0.01 mas/yr (Rioja et al. 1997, A&A, 325, 383). On the other hand, for radio sources in which the core component is blended with, rather than well resolved from, neighboring components, no identifiable feature is likely to be so stationary. The closer VLBI reference source, 3C 454.3, is one such case. The resolution of the VLBI images produced for GP-B is characterized by an elongated synthesized beam ~ 2 mas by ~ 1 mas in size (FWHM). But higher resolution observations of this source at 43 GHz during the period 1995-1997 revealed that there were two additional variable features within 0.7 mas of the apparent core (Jorstad et al. 2001, ApJS, 134, 181). Thus, the peak of the blended component that serves as the astrometric position reference may move (with respect to the host galaxy's center of mass) by a few tenths of a milliarcsecond on a time scale of a few years.

Most of the systematic errors in our astrometry depend on the relative separation on the sky of the star and the reference sources, but not on the nature of the individual sources. Such errors include those in our models for the ionosphere, troposphere, earth orientation, continental drift, and antenna structure, as well as processing blunders and programming bugs. The key points concerning these errors are that they must vanish in the limit of zero separation; and, for angles much less than 1 radian, they will likely be linear in the (vector) separation on the sky. To minimize these separation-dependent errors, we selected reference sources close on the sky to the guide star. (Admittedly, a blunder or bug could affect only the guide star. On the other hand, based on the sample consisting of recently fixed errors other than those producing easily caught astrometric outliers, a single-source problem is at least tenfold less likely than others.)

The current VLBI analysis strategy is to determine the position of the guide star with respect to the closer reference source. The coordinates for this source are fixed at the values determined by other (geodetic) VLBI observations, while those of the guide star and the other reference source are estimated for each of our VLBI sessions. Treating the reference sources this way yields a smaller expected separation-dependent error in the derived guide-star positions than fixing the position for the other reference source and determining the guide star's position with respect to that source. Alternatively, in our Kalman-filter analysis of the VLBI data the positions of both reference sources can be held fixed, in order to better account for the effects of the ionosphere and troposphere on the VLBI results. More complex schemes may further reduce the separation-dependent error, but no such scheme has yet been implemented or planned.

For want of a better procedure, the plausible unknown bias error is inferred from the currently estimated relative proper motion of the reference sources. Since the Kalman-filter analysis yields positions with less scatter than those derived from phase-reference mapping using NRAO software alone, we used the Kalman-filter results. The current set of VLBI observations with the two reference sources, 21 epochs from January 1997 to April 2002, spans only a little more than half of the length of time that such observations should span by the end of the mission. Consequently, one estimate of the unknown bias

error (at the end of the mission) in the guide star proper motion is the magnitude of the currently estimated apparent reference source proper motion, obtained with no proper acceleration estimated. This estimated proper motion is -0.02 ± 0.02 mas/yr in right ascension and -0.01 ± 0.02 mas/yr in declination. However, the magnitude of this result is largely determined by the positions estimated for the four most recent VLBI sessions, which exhibit relatively large postfit position residuals. Exclusion of any one of these positions yields a large fractional change in at least one proper-motion component. Thus, rather than accepting 0.01 or 0.02 mas/yr in each component as our estimated standard error due to unknown bias, we instead adopt the value 0.05 mas/yr, consistent with other evidence discussed below.

It is highly probable that the source-specific error for the closer reference source is several times larger than for the farther one and systematically time-dependent, too. The closer reference source (3C 454.3) is known to have extensive structure, including the multiple compact components resolved at 43 GHz. In the 8 GHz GP-B VLBI data, the so-called closure-phase observable that is a measure of the astrometric importance of the structure is also large for this source, especially after December 1999. (Unfortunately, the pre-2000 data alone yield a relative proper motion estimate with a standard error ~ 0.05 mas/yr in each coordinate, and hence they do not yield any better limit on the bias error.) In comparison to the closer reference source, the farther one (B2250+194) appears largely unresolved by these observations. To obtain additional information about the positional stability of our two reference sources, starting in November 2002 we include a third reference source. We hope that by the end of the mission the time series of relative positions among these sources will clarify which of the other two sources is the more stationary. In addition, B2250+194 is now regularly included in the NASA/GSFC “Research and Development VLBI” series of geodetic VLBI observations. This series consists of wide-bandwidth observations with NRAO’s VLBA several times per year. So far, comparison of just one such high-precision position in December 2002 with another from late 1997 reveals an apparent change of ~ 0.25 mas in each coordinate of effective position. This change corresponds to a mean rate of ~ 0.05 mas/yr. However, for resolved radio sources the data reduction method employed for geodetic observations tends to yield larger apparent position shifts than does ours. We therefore regard 0.05 mas/yr as a plausible preliminary upper bound on the standard error in IM Peg proper motion due to changes in this source.

The estimated relative proper motion of the reference sources includes the contribution from the separation-dependent error corresponding to a 3.5 deg separation. Since the guide star is closer than this to both reference stars, the separation-dependent error in its proper motion will likely be smaller. The estimated relative proper motion of the reference sources also includes contributions from the errors specific to both reference sources. In this regard, too, this estimated motion provides a conservative estimate of the bias error in the proper motion of the guide star due to the reference source structures. This bias error should be no larger than the error due to structure in one or another of the reference sources. (There can be no correlation between these errors for any pair of the three sources, due to the absence of any physical association among them.) The less than 30% difference of the rms scatter in the relative right ascension and the relative

declination of the two reference sources suggests that at each epoch similar accuracy is obtained in the two coordinates. In light of this similarity, we adopt 0.05 mas/yr as the plausible bias error in both components of the guide star proper motion.

A worst case error would occur if both current reference sources are subject to persistent position shifts due to structure changes during the last few years before the end of the mission. What reasonable bound can be placed on this error? Geodetic VLBI observations (Ma et al. 1998, AJ, 116, 516) yielded the apparent position of 3C 454.3 for 15 years, and at no time did the position estimate differ from its mean value by more than 1.2 mas, with the exception of one highly uncertain estimate. More typical examples shown by Ma et al. exhibit peak residuals under 0.5 mas. None of the observed sources is known to exhibit apparent motions much larger than those seen in 3C 454.3. Moreover, it is likely that the relative positions determined from the GP-B VLBI measurements (by means of an analysis of the observed interferometer phases) contain significantly smaller errors due to structure than do the positions derived by Ma et al. (who must rely on the intrinsically less precise group delays). There is no rigorous procedure to estimate the 99.73% (3 sigma) confidence limit for this contribution to the bias in proper motion. Somewhat arbitrarily, we adopt 0.5 mas/yr. This value allows, for example, for a steady drift of 2 mas in 4 yrs. (Shorter lived but still larger shifts, if any, would likely be detected as relative position outliers, and therefore be eliminated from the final IM Peg data set.)

Can the possibility of obtaining few or no new guide star positions lead to a higher worst case bias error? Consider the possibility that none is obtained after July 2002, an outcome for which the assumed probability is up to 3%. For such a data set, the uncertainty of the guide-star proper motion will be smallest for an epoch near the 1997 mean epoch of the VLBI observations. The required proper motion at the ~July 2004 mean epoch of the GP-B mission will be dominated by the uncertainty in the proper acceleration. The worst case bias error in the proper acceleration will be dominated, in turn, by the error due to the structure of the closer reference source, which was the only reference source in the pre-1997 observations. According to the reasoning used to derive the worst case statistical error, the worst case proper acceleration error with at least 0.27% probability (3 sigma case) due to an unknown bias will be the error magnitude which is exceeded with a probability of 9%. Once again, the best data from which to infer this value are the positions determined by Ma et al. Somewhat arbitrarily, again, to allow for a change of 0.3 mas/yr in the mean rate of motion of the peak radio emission from this source between the periods 1991-1997 and 1997-2002 (representing the two halves of our existing data set), we take 0.06 mas/yr^2 as the worst case mean proper acceleration during 1991-2002. When this value is integrated from the 1997 mean VLBI epoch to the 2004 mission-midpoint epoch, an error of 0.4 mas/yr results. This case is not worse than the 0.5 mas/yr worst case bias error identified under the assumption that we successfully obtain a guide-star position from each projected VLBI session. Based upon these two examples, it seems unlikely that intermediate cases accounting for partial success of future VLBI observations would yield an error larger than 0.5 mas/yr, but the appropriate calculations have not been made.

6/10/03

Could separation-dependent errors add significantly to the above value? There is no indication from the GP-B VLBI data analysis that there are significant correlations among the separation-dependent errors in the position results for different observing epochs. If these errors are uncorrelated from epoch to epoch, their contribution to the experimental error will be accounted for in the statistical standard error computed from the observed scatter. Could the fact that the guide star is displaced about 0.2 deg from the line connecting the two reference sources allow for a larger worst case error? If the gradient on the sky of these errors is by chance very nearly perpendicular to the line connecting the two reference sources, the separation-dependent error of the guide star proper motion with respect to either reference source could exceed the error in the relative proper motion of the two reference sources, even though they lie ~ 3.5 deg apart. If the orientation of the gradient is assumed to be completely random, the 99.73% (3 sigma) confidence level on the error contributed by the 0.2 deg offset is ~ 13 times the separation-dependent error in the reference source relative motion. Nevertheless, since the latter error is likely smaller than the 0.05 mas/yr adopted bound on the relative proper motion, the worst case position error due to separation-dependent errors is probably not as large as 0.5 mas/yr. Consequently the worst case error due to unknown bias remains 0.5 mas/yr.

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