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The Stanford Relativity Gyroscope Experiment (A): History and Overview

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1. BACKGROUND

Kip Thorne in the preceding paper has expounded the reasons for seeking to measure the relativistic precessions of gyroscopes in earth orbit. This and the next six papers describe the experiment we are developing at Stanford under NASA support, usually referred to by its NASA denomination Gravity Probe B. Nowhere in all the work inspired by William Fairbank is the "near zero" principle so broadly exemplified as here.

The idea of testing general relativity by means of gyroscopes was separately discussed by Schouten [1], Fokker [2] and Eddington [3] soon after Einstein had advanced the theory in 1915. De Sitter [4] in 1916 had calculated that the earth-moon system would undergo a relativistic rotation in the plane of the ecliptic of about 19 m arc-sec/yr due to its motion around the sun. Fokker proposed searching for a corresponding precession of the earth's axis, while Eddington observed that "if the earth's rotation could be accurately measured by Foucault's pendulum or by gyrostatic experiments, the result would differ from the rotation relative to the fixed stars by this amount." This prediction, which Eddington credited to Schouten, omits the effects of the earth's rotation on the gyroscope, investigated many years later by L. I. Schiff [5].

Eddington's discussion stimulated Blackett [6] in the 1930's to examine the prospect for building a laboratory gyroscope to measure the 19 m arc-sec/yr precession. He concluded that with then-existing technology the task was hopeless. There the matter rested until 1959, when, two years after the launch of Sputnik, and following also upon the improvements in gyroscope technology since World War II, Schiff and G. E. Pugh [7] independently proposed to test Einstein's theory by observing the precessions with respect to a distant star of one or more gyroscopes placed in an earth orbiting satellite. According to Schiff's calculations such a gyroscope may be expected to undergo relativistic precessions given by

$$\Omega = \frac{3}{2}\frac{GM}{c^2R^3}(\mathbf{R}\times\mathbf{v}) + \frac{GI}{c^2R^3}\left[\frac{3\mathbf{R}}{R^2}(\boldsymbol{\omega}\cdot\mathbf{R}) - \boldsymbol{\omega}\right] \quad , \tag{1}$$

where $\mathbf{R}$ and $\mathbf{v}$ are the instantaneous position and velocity of the gyroscope, and M, I and $\boldsymbol{\omega}$ are the mass, moment of inertia and angular velocity of the rotating central body, in this instance the earth.

The first term in equation (1) is the *geodetic* precession Ω_G resulting from the motion of the gyroscope through the curved space-time around the earth, the counterpart of de Sitter's effect from motion about the sun. In a 650 km near-circular orbit around an ideal spherical earth, it amounts to 6.6 arc-sec/yr in the plane of the orbit. The second term in the equation is the *motional* (now sometimes called *gravitomagnetic*) precession due to the rotation of the central body. The integrated value in a 650 km polar orbit is 42 m arc-sec/yr in the plane of the earth's equator. The precessions are measured with respect to the line of sight to a suitable guide star and must, as Schiff pointed out, be corrected for the aberration in the apparent position of the star. As will be explained in section 5.2, three other effects of general relativity appear in the data, of which the largest is the 19.0 m arc-sec/yr solar geodetic precession.

The experiment, as now conceived, places four gyroscopes and a reference telescope, all at liquid helium temperature, in a polar orbiting statellite. The duration of the mission is between one and two years, and the aim is to measure the two principal relativity effects, plus other effects to be discussed in sections 5.2 and 5.6, to rather better than 1 m arc-sec/yr. Thus the geodetic precession will be determined to about 1 part in 10^4 (the most precise test yet attempted of any effect of general relativity) and the motional precession to between 1 and 2 percent. Figure 1 illustrates the orientation of the gyroscopes. The spin vectors of all four lie approximately in the plane of the orbit and along the line of sight to the guide star, Rigel, with one pair spinning clockwise and the other counterclockwise. This configuration makes the two effects Ω_G and Ω_M predicted by Schiff appear at

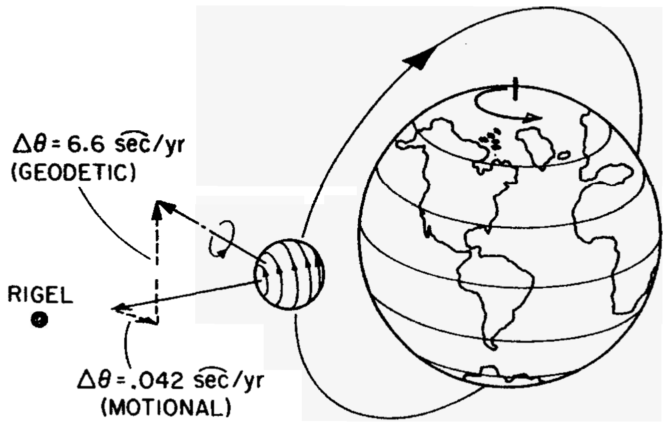

FIGURE 1. Relativity effects as seen in gyroscopes with spin vector lying parallel to line of sight to star.

right angles in the output of each gyroscope, giving fourfold redundancy in the measurement of each effect. The spacecraft rolls slowly with 10 minute period about the line of sight to the star. The technique for reducing the data is indicated in section 4.2 and explained in detail in section 5.

The program is being developed, under NASA support, jointly by the Stanford Physics and Aero-Astro Departments with Lockheed Missiles and Space Corporation as aerospace subcontractor. The first formal contact with NASA was a two-page letter from Fairbank and Schiff to Dr. Abe Silberstein, dated January 27, 1961, describing an instrument to be mounted on the Orbiting Astronomical Observatory that would measure the geodetic precession to a few percent. This early proposal, short as it was, embodied several ideas that were to prove crucial in the experiment as we now conceive it.

Section 2 below describes the origin of the idea in the work of Schiff and Pugh. Section 3 briefly summarizes the history of the program. Sections 4 through 8 describe the experiment, further details being given in succeeding papers.

2. ORIGIN OF THE IDEA

Historians of science have often descanted on the mysterious phenomenon of simultaneous discovery. Pugh and Schiff illustrate the theme. The two men came upon the idea of the orbiting gyroscope experiment within a few weeks by different paths, knowing nothing about each other's work, and each contributed significantly to the scheme of the experiment as it is now conceived. Schiff's investigation, begun in November 1959 after an earlier broad study of experimental tests of general relativity, first appeared in an article submitted to *Physical Review Letters* on February 11, 1960. Pugh's took the form of a proposal, dated 12 November 1959, in the unlikely sounding Weapons System Evaluation Group Research Memorandum Number 11, from the Pentagon. The two men learned about each other toward the end of February 1960, and exchanged manuscripts and pleasingly cordial letters shortly thereafter. For clarity I take Schiff's contribution first despite Pugh's priority. In retrospect Pugh must be seen as unlucky in not having chosen a more traditional vehicle for publication. His work, though lacking the theoretical depth of Schiff's, was of exceptional quality and deserves wider recognition than it has had.

2.1 Schiff's Investigation

The path by which Schiff arrived at the idea of the gyroscope experiment was oddly circuitous. His interest in general relativity went back a long way, as far as 1939, when under Oppenheimer's influence he wrote a paper on the application of Mach's principle to rotating electric charges. Considerations about Mach's principle were, as we shall see, crucial to Schiff's work on the gyroscope experiment; appropriately in 1964 he contributed a paper "On the Observational Basis of Mach's Principle" to the issue of *Reviews of Modern Physics* in honor of Oppenheimer's sixtieth birthday.

Another of Schiff's interests was in the equivalence principle, which he applied in 1958 in a very ingenious way to a topic that happened also to attract Bill Fairbank's interest at the same time, the question whether there is a gravitational repulsion between particles and antiparticles which might account for the separation of matter and antimatter in the universe. By considering the gravitational status of virtual particles, Schiff deduced that positrons like electrons should fall downwards. This unexpected insight, right or wrong, seems to have led Schiff to look for other novel applications of the equivalence principle, and these came to a focus in an article "On Experimental Tests of the General Theory of Relativity" [8] submitted to the *American Journal of Physics* on October 6, 1959. There Schiff demonstrated just how tenuous the evidence for Einstein's theory was.

Einstein in 1915 had identified three observational tests of general relativity. the gravitational redshift, the deflection of starlight by the sun, and the precession of the perihelion of the planet Mercury. Forty-four years later, in 1959, these remained the only feasible observations, regarded by most people as "crucial tests" of Einstein's theory. Schiff set out to "examine to what extent the full formalism of general relativity is called upon in the calculation of these three effects, and to what extent they may be correctly inferred from weaker assumptions that are well established by other experimental evidence" [8]. The redshift certainly does not require the full formalism. Einstein in 1911, before general relativity existed, had computed it on the basis of special relativity and the equivalence principle, results which are established by the Eötvös experiment and observations with high energy particles. In formulating his critique in this way Schiff, while right on the main point, was tacitly ignoring the conceptual extension involved in applying equivalence to calculate the redshift, an omission which led R. H. Dicke, who refereed Schiff's paper for the *American Journal of Physics*, to write a forceful accompanying paper urging the importance of directly testing Einstein's redshift formula. The exchange between Schiff and Dicke bore fruit later in Dicke's work on the experimenal basis of gravitational theories and in the "Schiff conjecture" about the relationship between weak and strong equivalence [9].

What of the deflection of starlight? Conventional wisdom had it that this does require the full formalism of general relativity, because when Einstein, in the same paper of 1911, had applied special relativity and the equivalence principle to calculate a deflection, he obtained a formula identical with the one given by a classical ballistic theory of light, but only half of what he was to get four years later from general relativity. It was just this doubling of the classical (and the special relativistic) deflection that Eddington made such play of in 1919 when he claimed that the solar eclipse observations by Dyson, Davidson and himself supplied a decisive proof of general relativity.

Schiff thought otherwise. Applying a line of reasoning previously sketch ed out in a different context by W. Lenz [10], he claimed an oversight in Einstein's 1911 calculation. In addition to the deflection from time dilation, worked out by Einstein, there is a second special relativistic effect from the "FitzGerald contraction" of space radially in toward the sun. This term (analogous to the space-curvature contribution to gyroscope precession discussed by Kip Thorne) Schiff found to be equal in magnitude to the Einstein term and in the same direction. The sum of the two just equals the observed deflection.

Schiff's argument on light deflection has been sharply criticized, a point to which I shall return in section 2.3. Meanwhile for Schiff himself in 1959, two of the three "crucial tests" of general relativity failed of being crucial. The third, the precession of the perihelion of Mercury, was a real test, first because it could not be calculated without introducing an equation of motion (the geodetic equation), and second because in applying that equation changes in clock rate of order $(GM/c^2r)^2$ must be taken into account, and these "cannot be found by the methods of this paper " [11]. Schiff's prognosis was gloomy. Improved redshift or light deflection experiments offered little hope of anything new, and

> By the same token, it will be extremely difficult to design a terrestrial or *satellite* [my italics] experiment that really tests general relativity,

> and does not merely supply corroborative evidence for the equivalence principle and special relativity. To accomplish this it will be necessary either to use particles of finite rest mass so that the geodesic equation may be confirmed beyond the Newtonian approximation, or to verify the extremely small time or distance changes of order $(GM/c^2r)^2$. For the latter the required accuracy of a clock is somewhat better than one part in 10^{18}.

Within three months Schiff had conceived in the gyroscope experiment not one but two new tests of general relativity, to be done in a satellite, both checking the equations of motion beyond the Newtonian approximation, and one (the measurement of the motional precession) checking a wholly new aspect of the theory.

Evidence for the development of Schiff's ideas comes from 68 pages of handwritten notes, dated 12/20/59 with the parenthetic comment "work started about a month earlier," plus the recollections of Mrs. Schiff, Bill Fairbank, Bill Little and Bob Cannon. The notes are in fair copy, Schiff's habit being to destroy his first rough calculations, but internal evidence suggests that only the first 11 or so pages were about work done before December 20. This reconstruction differs from the recollections of Fairbank and Cannon in that it puts the critical events after the start of Christmas vacation, but is, I believe, necessitated by the evidence of the notes.

In the December 1959 issue of *Physics Today* there appeared an advertisement for the Jet Propulsion Laboratory [12] trumpeting "important developments at JPL" through an account of "The Cryogenic Gyro. A fundamentally new type of gyroscope with the possibility of exceptionally low drift rates," accompanied by an artist's rendition of a spinning metal sphere afloat on the distorted lines of magnetic force above a horizontal loop of wire. Schiff's notes start with a description of such a gyroscope as he envisioned it, followed by 11 pages about two scientific experiments that might be done with it: a test of Mach's principle and a relativistic clock experiment. Marginally inserted next to the JPL reference is "See also GE article in *Wall St Journal* 1/21/60," an account of the parallel program "Project Spin" at General Electric [13]. The insert clearly followed the bulk of Schiff's work; Bill Fairbank's recollection, on the other hand, is that even before the JPL advertisement he and Schiff had talked about an article from MIT on uses of magnetically levitated superconductors. Knowing how good Bill's memory is for details of this kind, I have little doubt that such an article will turn up; however, with *Physics Today* coming out in the middle of the preceding month as it then did, Schiff's statement "work begun about a month earlier" is consistent with the JPL advertisement's being the trigger for his thinking.

Neither of the experiments analyzed in the first 11 pages of Schiff's notes is the gyroscope experiment as we now know it. His test of Mach's principle had not yet gone beyond Oppenheimer. Relativistic predictions did not enter; the idea was to check agreement between the local and general frames by comparing the direction of a gyroscope or rather "several gyroscopes pointing in different directions, and started at different times of the day and the year" with the line of sight to some suitable heavenly object. As Schiff put it,

> This experiment is much in the spirit of the Eötvös experiment, a negative result is expected, and if established with precision and reliability is useful. A positive result, definitely established, would be the scientific event of the century! (Non-visible matter in the universe may be determining.) [14]

The gyroscopes were to be ground-based at the earth's equator; the aim was for "an accuracy comparable with the best astronomical observations, something of the order of 0.01″ to 1″ of arc"; and there was a long and technically intriguing analysis of disturbances acting on the gyroscope and of two methods of reading out the direction of spin, one due to Bill Little, the other to Schiff himself.

If Schiff did not then know that such a gyroscope would undergo a relativistic precession, what led him to the idea? The answer lies in the second experiment where the gyroscope is treated as a clock. Schiff, of course, was hoping for a clock good enough to measure the second order redshift. A spinning superconductor, being free of eddy-current disturbances and temperature-dependent changes in dimension, holds promise of being an exceptionally good clock. Schiff's notes and Bill Fairbank's recollections trace the course of this idea as applied in measuring frequency differences between clocks at the top and bottom of Hoover tower on the Stanford campus (300 ft), between a balloon at 100,000 ft and the ground, and finally (p. 9 of notes): "For a satellite in an eccentric orbit (as proposed by Zacharias), Δh might be 600 miles or 10^8 cm, an effect of fractional amount 10^{-10}."

At page 10 Schiff lists six points to be investigated. The first is "Does the gyro act as a clock in the relativistic sense?" The others are technical questions on clock performance. The remaining 58 pages of notes never take him beyond point one.

If a gyroscope is a clock in the relativistic sense, its spin rate will change with changes in the gravitational potential $\phi = gh/c^2$. Applying a technique from the paper on gravitational properties of antimatter [15], Schiff examined the effects of ϕ on the Hamiltonians of oscillator clocks and gyroscopes. For an oscillator, the redshift formula $\omega = \omega_0(1 + \phi)$ came out easily; for a rotator the same holds provided the angular momentum J does not change from motion through the field. Schiff added "as expected" to the sentence stating this result; after which, heavily lined out in characteristic

Schiff fashion, comes (p. 11, my italics): "*Now J is not changed as the rotator is moved in the field since no torques are exerted. Motion (polar vector) cannot produce torque (axial vector).*"

This, I conjecture, is the point Schiff had reached by December 20. Abruptly the next paragraph reverses direction with "We must now see how J changes with the motion in the field," followed by reference to a paper of 1951 "Spinning test-particles in general relativity" by E. Corinaldesi and A. Papapetrou [16]. Schiff at this point evidently suspected that a rotational clock would not obey the Einstein redshift formula, and indeed one of his first calculations (p. 14) gave an expression for the spin rate of a gyroscope with axis normal to the orbit plane of a satellite, equivalent to $\omega = \omega_0(1 + 2\phi)$, twice the Einstein value. Long before reading Schiff's notes I recall Bill Fairbank's telling me how Schiff, while working on the clock problem, telephoned him to say that there was a mistake in Papapetrou's paper. The issue, as Schiff makes clear in his final published result [17], is the choice of supplementary conditions on the equation of motion. In the notes, Schiff reached the proper answer after 20 pages. Changes in gravitational potential do indeed alter the gyro spin speed in accordance with the standard redshift formula. Motion along Newtonian equipotentials leaves the spin speed unaffected but causes a precession: the geodetic precession of the first term in equation (1) above.

When and why did Schiff start thinking about a space experiment? From one side he was preconditioned to it through thinking about the orbital clock experiment "as proposed by Zacharias," as well by his own earlier reflections on satellite experiments. From another side he must in all probability have seen, through analyzing the mass unbalance torques on a ground-based gyroscope (pp. 3–8 of the notes), that gyro performance would be enormously improved by operating in zero-g. But what really put him into space was—*mathematical convenience!* As Schiff noted (pp. 11–12) the Corinaldesi-Papapetrou equations "only apply to free motion in the Schwarzchild field & it seems very difficult to work out a constrained motion." Hence (new paragraph) "We first consider the motion of the gyroscope in a free satellite."

The next 30 pages of notes, with one short exception, all relate to the satellite experiment. The exception "suggested by Fairbank" (p. 18) is a preliminary calculation for a ground-based gyroscope yielding a precession that is "of the order of the Mach effect & must therefore be considered there as a correction, along with the Thirring-Lense effect," together (at this stage) with a 12 hour periodic variation in clock rate. All this "assumes the C-P equations are valid for constrained motion." Later (p. 40) Schiff derived proper constrained equations, eliminated the periodic effect, and showed that the precession for an earth-based gyroscope, allowing for the reduction in translational velocity and presence of a Thomas precession term is about 0.4 arc-sec/yr instead of 6.6 arc-sec/yr.

After this elaborate investigation Schiff quickly worked out the seemingly more difficult motional effect. The metric was known from the work of de Sitter (1916) [18] and Lense and Thirring (1918) [19]. Lense and Thirring had deduced that a moon orbiting a rotating planet undergoes a relativistic advance of its ascending node. Schiff's formula for gyroscope precession (the second term in equation (1)), was more complex, giving a precession which, unlike the Lense-Thirring drag, depends on orbit inclination and reverses sign in equatorial orbits. Schiff concluded the notes with investigations of sundry other effects, the aberration of starlight, gravity gradient disturbances on the gyroscope, and the relativistic effects of the sun, the moon and the galaxy. The article in *Physical Review Letters* was published on March 1, 1960, followed later in the year by the definitive paper in the *Proceedings of the National Academy*.

Just as Pugh and Schiff lighted on the experiment almost together, so in the fall of 1959 came an unforeseen confluence of interests at Stanford. Schiff, through writing his *American Journal of Physics* article, had fixed on the need for new tests of general relativity. Bill Fairbank, new to Stanford from Duke, was focusing on experiments that would go beyond traditional low temperature phenomena into wider applications. No topic could be more apt than the superconducting gyroscope. Meanwhile Bill had met Bob Cannon, also new to Stanford (from MIT), who had been thinking about the large improvements that could be gained in the performance of air bearing or electrically suspended gyroscopes by operating them in space. Cannon has often told the story of Fairbank introducing him to Schiff at the Stanford swimming pool, and of three "dripping men" deciding there and then to pursue the experiment, with Cannon saying to his wife that evening, "I have met a man who needs a gyroscope even better than the ones we have been talking about." Bill Fairbank's fertile imagination and powers of intellectual catalysis drew this diverse group together and started the adventure we are now on.

2.2 Pugh's Proposal

After Schiff's elaborately roundabout, though scientifically profound, route to the gyroscope experiment, Pugh's approach seems almost embarrassingly simple. In 1958 H. Yilmaz [20] had advanced as an alternative to general relativity a new generally covariant theory of gravitation, which yielded identical predictions to Einstein's for the three "crucial tests," though it has since been shown to be nonviable on other grounds. Originally Yilmaz did not discuss frame-dragging, but at the January 1959 New York meeting of the American Physical Society he gave a 10 minute talk on experimental consequences of generally covariant scalar, vector and tensor theories of gravitation. All would yield identical results for the three "crucial tests" but a possible experiment to distinguish the nature of the interaction would be "to launch an artificial satellite whose plane contains the axis of rotation of the earth" [21]. According to Yilmaz, a vector or tensor interaction would cause a Lense-Thirring drag of about 0.56 arc-sec/yr, a scalar interaction would cause no drag [22].

Pugh's "Proposal for a Satellite Test of the Coriolis Prediction of General Relativity" started where Yilmaz had left off. Yilmaz's suggestion, argued Pugh,

> does not appear feasible as a definitive test, because of the extremely large magnitude of perturbations that are not accurately known, such as the quadrupole and higher order moments of the earth's gravitational field. However the spin of a satellite is much less influenced by such effects and may provide a feasible technique for the experiment. [23]

Pugh's initial idea, then, was to place a large spinning body in orbit around the earth and compare its direction of spin with the line of sight to a guide star. The reference telescope would be mounted on the spinning body, aligned closely enough with its maximum axis of inertia to keep the star image within the field of view. The output would consist in observing gradual changes in diameter of the circles traced by the star image in the focal plane of the telescope. To accommodate a 10 inch aperture telescope and still provide sufficient gryoscopic action, Pugh proposed using a 1 m diameter satellite weighing about half a ton. Variants on Pugh's scheme were afterwards studied by groups led by Howard Knoebel at the University of Illinois [24] and David Frisch at MIT [25].

Pugh assumed that the satellite axis would precess through the same angle as the orbit-plane, and since the nodal drag calculated for him by Yilmaz was 0.36 arc-sec/yr (twice the correct value) the precession he gave was eight times what it should have been. From Peter Bergmann, Pugh learned of the de Sitter-Fokker precession, for which he gave, with acknowledgement to F. Pirani, the correct value of 6.6 arc-sec/yr. With these figures as goals for measurement, Pugh proceeded to analyze nonrelativistic disturbances on the satellite's spin axis from atmospheric drag, gravity gradients and magnetic effects.

Atmospheric drag was the Achilles' heel of the experiment. There seemed no way of making the disturbances from it small enough with a bare satellite. Pugh reacted with the brilliant suggestion that the primary satellite should be

> encased in a larger hollow sphere or "tender" satellite having the same center of mass. The tender satellite could be equipped with

> light beams or other sensing mechanisms to monitor the position of the primary satellite *without* exerting significant forces or torques on the primary satellite. The use of *external* vernier rocket jets would allow repositioning of the tender with respect to the primary satellite so that the two would not collide during the course of the experiment. [26]

Thus was born the concept of the drag-free satellite. Pugh at once pointed out almost all the applications of drag-free technology that have since been pursued by others: aeronomy, geodesy, gravitational modeling of the earth, navigational satellites, the correction of "classical perturbations in orbital-type relativity" (*i.e.*, perihelion tests), even (though neither Paul Worden nor I knew of this when we began) a test of the equivalence principle.

It was Bergmann who put Schiff and Pugh in touch with each other. Pugh in a letter to Schiff (March 2, 1960) noted regretfully that "I must multiply my estimated effect by .25 for a polar orbit and −.5 for an equatorial orbit," and observed that "You could hardly pick a worse location than Stanford to measure the Lense-Thirring effect." Schiff replied (March 14), "Your double satellite experiment and my gyro within a satellite are really the same thing. I would certainly like to see this done, although it is possible that the earth-based gyro will not be as difficult at this stage of satellite development." Concerning their views of theory, Schiff continued,

> We differ slightly in motivation in that you regard an experiment that would distinguish between Einstein's and other theories of gravitation as being of primary importance, whereas I would regard an experiment that tests any of the theories, Einstein's in particular, beyond the equivalence principle as being of primary importance. Hence you tend to stress the Lense-Thirring effect, whereas I tend to regard it and the non-rotating earth effect as of equal importance. [27]

2.3 Significance of the Effects

Even today, 25 years after Schiff, general relativity lacks a secure experimental foundation. Einstein advanced a theory of great conceptual elegance, radically different from Newtonian theory, with few testable consequences. One result over a long period was a mad proliferation of rival theories all claiming to account for the three "crucial tests."

A partial answer came in the late 1960's through the formulation of the PPN (parametrized post-Newtonian) framework for classifying gravitational theories and comparing their experimental consequences. The idea goes back to a discussion in Eddington's *Mathematical Theory of Relativity* [28] revived by Schiff in 1960 [29]. In the hands of Nordvedt and Will, PPN analysis has eliminated a large number of theories previously thought to be viable, either by disclosing internal inconsistencies or by showing that the theories lead to peculiar effects not present in general relativity or nature.

Will [30] concludes that of some fifty to eighty theories of gravitation once held, only about half a dozen survive, all of which yield identical or nearly identical results with general relativity for solar system tests.

Invaluable as this demolition work is, it does not establish Einstein. Ultimately perhaps the most interesting consequence of the PPN investigation is the discovery of how many potentially occurring effects vanish in general relativity. Einstein's is a minimalist theory. Apart from some rather marginal deductions about gravitational radiation in the Taylor-Hulse binary pulsar, the one truly new positive discovery since 1960 has been the Shapiro time delay effect [31], the relativistic increase in transit times of radar signals reflected from bodies (planets or spacecraft) as they pass behind the sun. Time delay is closely related to light deflection. Measurements of it, light deflection and redshift in recent years all support the Einstein predictions within experimental limits (2 parts in 10^4 for redshift in the Vessot-Levine experiment, 1 part in 10^2 for light deflection in VLBI measurements on radio stars in the ecliptic plane, and 1 part in 10^3 for Viking Mars lander time delay measurements). Each result has been analyzed with unusual care, but some reserve is appropriate since each depends on elaborate data modeling. Also there is the peculiar status of the precession of the perihelion of Mercury, which has oscillated between agreement and disagreement with the Einstein prediction with changing data on the sun's oblateness. The latest results suggest a 1% discrepancy with general relativity [32].

If Schiff's 1959 argument were correct that redshift and light deflection (and by the same token radar time delay) only test special relativity and the equivalence principle, this would be a meager crop. Actually the argument has been disputed, and is now usually thought to have been disproved, though Schiff himself continued to grant it heuristic value. To follow the issues, certain distinctions have to be made.

First, in the Schwarzchild solution of Einstein's field equations around a static spherically symmetric body, the departure from flat space-time, when written in standard rather than isotropic coordinates, does indeed involve a modification of the time coordinate and a modification of the radial component of the space coordinates. The modification to time is just the Einstein clock shift; the modification to space is such that the radial distance from the center to a sphere of surface area A, as measured by observers at rest around the central body, is somewhat greater than the square root of $A/4\pi$. In stating that Einstein had omitted space curvature from his 1911 argument, we are on safe ground.

Schiff (and several other writers) sought to repair that omission. Their line of reasoning is roughly as follows [33]. Assume that in weak fields, such as exist around the earth and the sun, the gravitational potential is given to first approximation by the Newtonian formula GM/r, so that a reference frame R falling inwards freely from infinity will at distance r have velocity $v = \sqrt{2GM/r}$ with respect to the central body. According to the equivalence principle, the frame R is inertial so the laws of special relativity apply to it. Suppose an observer on R possesses clocks and measuring rods which he compares with clocks and rods attached to a static spherically symmetric coordinate frame at distance r from the origin. The time dilation and space contraction of special relativity will affect the comparisons. If the inertial observer measures the interval between ticks of the clock in the static frame S as δt, its proper time in S is actually the shorter interval $\delta t\sqrt{1 - v^2/c^2}$. If he measures the radial distance interval as δr, its proper value in S is $\delta r/\sqrt{1 - v^2/c^2}$. Comparisons of transverse distances remain unaffected. If then one can use these intervals δr and δt as differentials for the radial and time coordinates in the stationary coordinate system S, one arrives at the curved three-dimensional space and modified time of the Schwarzchild metric (and hence at the general relativistic light deflection) without invoking the elaborate machinery of general relativity. The seductiveness of the argument is enhanced when one learns that it can indeed be made rigorous for the falling clock, where of course it agrees with Einstein's 1911 result and with other simple arguments that yield a gravitational redshift without invoking general relativity.

An obvious objection, first advanced by Schild [34], is that in the PPN framework the value for the light deflection (and likewise for the time delay) is proportional to $(1+\gamma)GM/c^2r$, where the 1 represents the contribution from the Einstein clock shift and the γ the contribution from space curvature, equal to 1 in general relativity but not in all metric theories of gravity. In the Brans-Dicke scalar-tensor theory, for example, with Dicke's original value of 6 for the scalar parameter ω, γ is 0.92. The Brans-Dicke theory is certainly consistent with special relativity and the equivalence principle, yet matter in it produces less curvature of space than in Einstein's theory. It does so because of the admixture in Brans-Dicke of a scalar potential along with Einstein's tensor potential. Tucked away in Schiff's argument seems to be a hidden assumption that gravitation depends solely on a tensor potential.

But Schiff's argument can be objected to on other grounds. A certain intellectual legerdemain appears in the transfer from contraction of a falling body to Schwarzchild curvature. In 1968 Sacks and Ball [35], and independently Rindler [36], exposed various fallacies in Schiff's and other simple derivations of space curvature. Rindler indeed claimed to have found a decisive counterexample to all such derivations. My own impression is that Rindler proved too much and that one version put forward by Tangherlini [37] in 1962 might with proper modification be made rigorous. However, pursuit beyond a certain point is futile. The true value of such arguments is pedagogical. If they remove some of the mystique of incomprehensibility from general relativity and give some hint as to what aspect of the theory is under test, they have served their turn.

In treating the gyroscope experiment, Schiff remarked that calculation of *either* the motional or the geodetic effect involves the larger structure of general relativity since each requires an equation of motion beyond the Newtonian approximation. Here Schiff followed the thought of his *American Journal of Physics* article where the precession of the perihelion of Mercury could not be calculated "by an extension of the methods of this paper [since] we require in addition an equation of motion for a particle of finite rest mass, to replace the argument used above that the speed of light measured by a [falling clock] B is c" [8]. This check of the equation of motion was what made Schiff tend to regard the nonrotating earth effect as equal in importance with the motional or Lense-Thirring effect.

Most physicists, including Schiff himself in other contexts, have viewed the motional effect as the more important because the dragging of the inertial frame by rotating matter is an aspect of general relativity different in kind from anything seen in earlier tests. The difference may be put in various ways: measuring off-diagonal terms in the metric, searching (as in Kip Thorne's elegant discussion) for a gravitomagnetic field, determining a particular combination of PPN parameters. The most searching discussion is C. N. Yang's. After pointing out that general relativity "though profoundly beautiful, is likely to be amended," and that the amendment is likely to involve a new, beautiful and symmetrical geometrical concept, he asks,

> What is this new geometrical symmetry? We do not know ... However, many of us believe that whatever this new geometrical symmetry will be, it is likely to entangle with spin and rotation, which are related to a deep geometrical concept called torsion. But, no one has figured out what precise new concept related to rotation is the relevant one. From the viewpoint of gauge theory, I had pointed out [*Phys. Rev. Lett.* **33**, 445 (1974)] that the natural amalgamation of Einstein's theory with gauge theory is to involve the derivatives of R_{ik}, hence to involve spin.
>
> That the amendment of Einstein's theory may not disturb the usual tests is easy to imagine, since the usual tests do not relate to spin. The proposed Stanford experiment is especially intesting since it *focusses on the spin*. I would not be surprised at all if it gives a result in disagreement with Einstein's theory. [38]

What then of the geodetic effect, and equations of motion? Certainly the calculations involve, as Schiff said, terms in an equation of motion beyond the Newtonian approximation, as may be seen from Schiff's paper on the gyroscope and more simply from de Sitter's investigation of the earth-moon system, where the precession comes from a relativistic perturbation on the

Newtonian equations for lunar motion [39]. Two statements of clarification are needed, however. First, in general relativity the equations of motion are not independent postulates. They follow with almost sinister inevitability either from an extension of a variational principle known to apply in special relativity or, more unexpectedly and unlike any other theory in physics, by direct deduction from the field equations. Schiff's easy separation of different aspects of the theory does not quite work. Second, the rotation of direction is not restricted to a gyroscope. Eddington made the point when he said that the de Sitter effect "does not have exclusive reference to the moon: in fact the elements of the moon's orbit do not appear in [the final equation]. It represents a property of the space around the earth—a precession of the inertial frame in this region relative to the general inertial frame of the sidereal system." [40]

The weak-field result can be understood by referring to Kip Thorne's discussion. There the geodetic precession comes in two parts: two-thirds from lateral translation of the gyroscope through the curved three-dimensional space around the earth, one-third from spin-orbit coupling in the gravitoelectric field. In this treatment the experimentally observed decrease in clock rate with distance from the center of the earth is replaced by an artificial universal time. Sticking to the empirical (or at least notionally empirical) one may continue to attribute two-thirds of the geodetic precession to space-curvature, as defined earlier, while reinterpreting the other one-third as an effect of the lateral motion of the gyroscope through the radial gradient in the time dimension of the Schwarzchild metric. For theories within the PPN framework the total precession is $(1+2\gamma)GM/2c^2r$, with γ being the measure of space curvature. A measure of the geodetic precession effects two things: (a) assuming a precision of 1 part in 10^4 on Ω_G, it fixes γ to a part in 7000, rather more than a factor of ten better than the best radar ranging determination; (b) it tests to a part in 3000 the precession from lateral motion of the gyroscope through a time gradient. An interesting investigation not yet attempted would be to see whether the time gradient part of the gyro precession can be derived from the equivalence principle by an argument analogous to Einstein's argument for light deflection. Schiff would have said no. The significance of the geodetic measurement of course depends on whether the result agrees or disagrees with general relativity. Barring conpensating deviations, agreement would confirm both the time and the space aspects of the theory. Disagreement would on first presumption be attributed to γ's being different from 1, but only because one is thinking within a PPN framework.

Having studied weak field effects, it is instructive to see what happens to a gyroscope orbiting a black hole. Consider a nonrotating black hole of Schwarzchild radius r_0. The extreme of light deflection and also of radar time delay occurs when the light or radar pulse goes into orbit; this occurs at radius $3r_0$. For a gyroscope, the parallel question is at what distance does the spin axis become locked in through geodetic precession radially towards the center of the black hole (or tangential to a circular orbit around it)? Since the lowest stable captive orbit for massive objects is $6r_0$, one may conjecture that the locking radius also is $6r_0$, a result which if true neatly illustrates the difference between experiments with gyroscopes and experiments with electromagnetic signals. Mark Jacobs and I are investigating this problem. For rotating black holes, the story is more complicated. With an equatorial orbit the Lense-Thirring-Wilkins [41] drag on the orbiting body alters the radius of the lowest stable orbit, decreasing it for corotation and increasing it for counterrotation, and the motional precession of the gyroscope has the opposite sense from the Wilkins effect. For counterrotating orbits, locking becomes impossible. For corotating orbits, its occurrence would depend on whether the reverse effect of the motional precession is outweighed by the enhanced geodetic precession for the closer-in orbit. These and the more complicated effects in inclined orbits offer an intriguing field of study.

3. EARLY DAYS AND THE CONCEPT OF THE LONDON MOMENT READOUT

I have described in the introduction to this volume my own first encounter with Bill Fairbank and the gyroscope experiment. Upon arriving at Stanford in 1962 I found that, even though NASA funding had yet to commence, significant work was already going on in both physics and aero-astro departments. Morris Bol, with Bill, had demonstrated the principle of a new kind of gyroscope readout based on the Mössbauer effect and was beginning his doctoral research to detect the London moment in a spinning superconductor; Roger Bourke and Benjamin Lange, with Bob Cannon, were respectively studying the dynamics of a magnetically supported superconducting rotor [42] and the design and performance of the drag-free satellite [43].

As originally conceived, the experiment used a spherical superconducting gyroscope, magnetically levitated, with a Mössbauer readout. This readout, a typically ingenious Fairbank idea, was based on having a small ^{57}Fe source on the gyro rotor, and a detector mounted on a corotating cylinder interposed between the gyroscope and the reference telescope. Any misalignment between the axes of the gyroscope and the cylinder would result in a periodic linear displacement between the source and detector, and the instantaneous velocity associated with this displacement could be measured by the Mössbauer effect. A separate measurement would then be made of the orientation of the cylinder with respect to the telescope.

Relativity data would come from differencing three signals: telescope-to-star, cylinder-to-telescope, gyroscope-to-cylinder.

One concern with a magnetically suspended superconducting gyroscope is the effect of the London moment in the spinning superconductor. Fritz London, in his book on *Superconductivity* [44], extending an earlier investigation of Becker, Sauter and Heller [45], had shown that a spinning superconductor develops a magnetic moment proportional to spin speed aligned with the instantaneous axis of rotation. A simple calculation reveals that the torque from the magnetic support field may easily cause gyro drift several orders of magnitude larger than the 10 m arc-sec/yr limit aimed at by Fairbank and Schiff. It was partly from this concern as well as from interest in the phenomenon itself, that Bill and Morris Bol decided in 1961 to set about measuring the magnitude and properties of the London moment. Similar experiments were started independently about the same time by Hildebrandt [46] and by King, Hendricks and Rorschach [47], but Bol and Fairbank's [48] work was especially interesting because in addition to detecting the London moment in a superconductor spun up below its transition temperature, they demonstrated that a solid strain-free superconductor, spun in the normal state, would generate a London moment spontaneously on cooling through the transition. This effect, the analog of the Meissner field exclusion effect, is one of the many intriguing consequences of superconductivity's being an equilibrium state.

A cardinal principle of physical experimentation is that one should try to convert obstacles into advantages. A few months after my arrival at Stanford I struck upon two possible ways of exploiting this bothersome London moment. One was to apply the magnetic torque on the gyroscope in aligning the spin axis with the reference direction. The other (very tentative) was that since the London moment is tied to the direction of spin, it might supply the basis for a gyro readout. My original notion here was to adapt one of the Blackett astatic magnetometers we had used in paleomagnetic measurements at Imperial College. This, as now appears, would have been problematical. The Blackett magnetometer [49], though a brilliantly conceived instrument deserving of far more acclaim than it has received, would not have fitted well. The delicate mechanical suspension would have made it hard to adapt to space operations. The reaction torque on the gyroscope from the detecting magnets would have caused difficulty. The sensitivity, which was at best 5×10^{-11} G/cm in a 0.03 Hz bandwidth, would have yielded a readout resolution of 1 arc-sec in 100 sec of time, marginal for an experiment at 10 m arc-sec/yr and unacceptable for the 1 m arc-sec/yr we now seek.

Bill Fairbank and I discussed these ideas, but did nothing with them. Then a month later Bill came rushing up to me bursting with excitement about the London moment readout. Apparently he had forgotten our

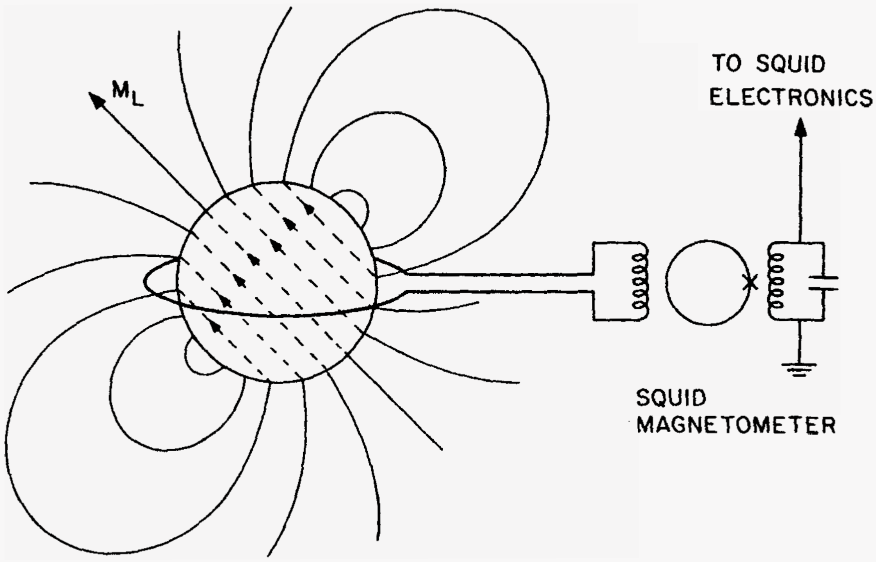

FIGURE 2. London moment readout of the gyro spin axis.

earlier conversations and had come upon the same idea independently. Be that as it may, he brought a new and crucial factor into the discussion: the modulated inductance magnetometer that he, Bascom Deaver and John Pierce [50] were just then beginning to work on. If this could reach the hoped-for precision, it would be the key to a successful gyro readout. We quickly saw that the right design was to have a readout loop centered on the gyroscope, and wrapped closely around it in order to make use of the self-shielding action of the superconducting rotor in rejecting changes in the external field from the readout (figure 2). Later we abandoned the modulated inductance magnetometer for the more subtle and sophisticated SQUID (Superconducting QUantum Interference Device), whose invention by others still lay ahead, but that was a detail, albeit an important one. The London moment readout unlocked the riddle, first by allowing us to eliminate the awkward intermediate rotating cylinder between the telescope and gyroscope, and second by making us realize that with a magnetic readout one should substitute for the magnetic suspension an electrical suspension system of the kind invented by the late Arnold Nordsieck [51] of the University of Illinois and developed commercially by Honeywell and Rockwell.

At this point a conversation with Howard Knoebel, Nordsieck's successor at Illinois, is worth recalling. Knoebel visited Stanford shortly after I arrived and before we had thought of the London moment readout. Rather cryptically even to myself, I expressed concern about gyro readout, and he immediately responded, "Well, you are worrying about the right problem; the readout is the difficult part." Correctly so. Paradoxical as it may sound, development of a gyroscope with drift rate at the milliarc-sec/yr level is not the hard problem. The real difficulties are two: reading out the direction of spin and spinning up the gyroscope.

The London moment readout has four advantages: (a) since the London moment is tied to the gyroscope's instantaneous axis of spin rather than its body axes, the readout can be applied to an ideally round, ideally homogeneous rotor; (b) it has adequate angular resolution; (c) it is sensitive only in second order to the centering of the gyro rotor in the readout loop; (d) the reaction torque of the readout current on the London moment is negligible.

More detailed information on all four points is given in the paper by Anderson (paper (C) of this series).

3.1 Building the Research Group

NASA funding commenced in March 1964 retroactive to November 1963, with initally a supplement from the U.S. Air Force. The original proposal was "To Develop a Zero-*G* Drag-Free Satellite and Perform a Gyro Test of General Relativity in a Satellite," with Cannon and Fairbank as co-principal investigators. The place of "drag-free satellite" in the title was significant. From the beginning, our intention was to develop the gyroscope experiment jointly between the Stanford Physics and Aero-Astro Departments, but Bob Cannon, unlike Bill and myself, foresaw that this would be a long process. Perhaps even Cannon did not realize just how long it would be, but in any event he wisely set for the Aero-Astro Department an independent goal of gaining early flight experience through development of the drag-free satellite. Cannon's hope was to apply the drag-free satellite in aeronomy and geodesy, as Pugh had suggested. As it turned out, the first application was to the U.S. Navy's TRIAD Transit Navigation Satellite, build by Johns Hopkins Applied Physics Laboratory and launched in July 1972, which carried the DISCOS (DISturbance COmpensation System) drag-free controller developed by the Stanford Guidance and Control Group under Daniel B. DeBra [52].

In parallel with the main program Lange [53] explored a variant on Pugh's idea of making the gyroscope itself a drag-free proof mass. He proposed using a carefully mass-balanced silicon rotor a few cm in diameter, for which the direction of spin would be read out from an optical flat accurately positioned on the pole of the rotor's maximum axis of inertia. The telescope was to be mounted on the outer satellite, not on the spinning proof mass as in Pugh's proposal. Although this "unsupported gyroscope" was never reduced to practice, it led to important research by several graduate students, not least among them Bradford Parkinson, who has now returned to Stanford in another role as program manager for the relativity gyroscope experiment.

Once NASA-Air Force funding was available, the aero-astro side of the enterprise rapidly expanded. Dan DeBra, James Mathiesen and Richard Van Patten all joined the professional staff in 1964 (at first principally to work on the drag-free satellite), and several graduate students followed. Within physics we did not expand so fast, but a close working relationship established with DeBra and Van Patten from 1965 onwards led to many of the basic ideas for the flight mission, especially in attitude and translational control of the spacecraft, in work on the reference telescope, and in the conception of a "science data instrumentation system" to subtract and process the gyroscope and telescope signals [54].

Meanwhile, in cooperation with Honeywell Incorporated we started the long and arduous process of developing the gyroscope, both in its mechanical design and in adapting the electrical suspension system to our application. Daniel Bracken, then a physics graduate student, played a critical role through his ingenious work on the gas spin up system [55] (see paper (B) of this series). Another important event in 1965 was the start of our long and happy collaboration with Donald Davidson (then of Davidson Optronics Incorporated, later of Optical Instrument Design Co.), who built the fused quartz telescope for the experiment [56] and originated many of the ideas then and later for the design and manufacture of quartz gyro housings and magnetic shielding assemblies. Somewhat later, from 1968 on, Wilhelm Angele of NASA Marshall Center also contributed to gyroscope development, especially in his work on methods of manufacture for extremely spherical gyro rotors [57].

By 1968 we had developed many of the basic concepts for the experiment, but had not begun to have a working system. At this point John Lipa arrived from Australia on a CSIRO fellowship. He at once took charge of dewar and gyroscope development, which he led with brilliant insight and determination full time from 1969 to 1979 and part time thereafter. He was soon joined by John Anderson on gyro readout, Jack Gilderoy, Jr., on mechanical fabrication, and John Nikirk, who until his untimely death in 1975 held responsibility with Dick Van Patten for electronics development.

It would be invidious to try to apportion credit for what has been supremely a team effort, requiring tenacious application from many different people. In addition to those already mentioned, three physics graduate students deserve special recognition: Peter Selzer for developing the porous plug device for controlling the flow of liquid helium from a dewar under zero-g conditions [58], Daniel Wilkins for analyzing the correction to Schiff's geodetic formula arising from the earth's oblateness [59], and Blas Cabrera for developing ultralow magnetic field shielding techniques [60] which later he, as a member of the research staff, applied directly to the gyroscope experiment. Other research staff members who worked on the program up to 1981 were Robert Clappier, Frank van Kann, Graham Siddall, Brian Leslie, G. M. Keiser, Stephen Cheung and John Turneaure. Among aero-astro graduate students, work of especial note was done by John Bull, David Klinger, Richard Vassar, J.-H. Chen and Thierry Duhamel. In all, 23 graduate students have obtained doctoral degrees on the program, either in physics or in aeronautics and astronautics, another 11 have worked part time, and 15 undergraduates have made significant contributions, including several honors theses. One undergraduate, Charles Marcus, was selected as a 1984 national finalist for the American Physical Society's Apker award [61].

Recent developments in the group are discussed in the next subsection.

3.2 The Path to a Flight Program

The gyroscope experiment has the curious distinction of having had the longest running single continuous grant ever awarded by NASA, from November 1963 to July 1977. The closeout of this grant in 1977 [62], besides being a legal necessity, had symbolic significance in marking the end of the exploratory phase of the program.

NASA had begun examining the feasibility of a flight experiment in the early 1970's. In 1971 Ball Brothers Research Corporation completed a "Mission Definition Study" which contained a first look at the spacecraft layout and a program plan. The plan advanced there, with some prescience, was for a three-flight program with (a) a dewar test flight, (b) an engineering test flight of the gyroscopes, (c) the science mission. Our current plan has the dewar test flight already completed through the successful flight in 1982 of the IRAS (Infra-Red Astronomy Satellite) dewar, whose design was largely based on the one worked out for the gyroscope experiment by Ball Brothers Research Corporation and Stanford in 1971. The approach to engineering and science flights will be described in a moment.

To start a flight program in 1971 would have been premature, but by the late 1970's the situation had changed. In 1978 the Space Sciences Board appointed an *ad hoc* Gravitational Physics Committee under the chairmanship of I. I. Shapiro to formulate "A Strategy for Gravitational Physics in the 1980's"; its report, published in 1981 [63], put the gyroscope experiment as the number one priority, the only dedicated flight mission recommended by the board. In 1980 NASA conducted a major review of technological readiness under the chairmanship of Jeffrey Rosendhal and concluded that "the remarkable technical accomplishments of the dedicated Stanford experiment team give us confidence that, when they are combined with a strong engineering team in a flight development program, this difficult experiment can be done" [64].

With these and other endorsements, the stage was set for planning the flight program. NASA had already completed a Phase A study in-house at Marshall Center in 1980. In 1982 a much more extensive Phase B study was completed [65], also in-house, but this for a variety of technical reasons led to a somewhat large spacecraft (weight 5300 lb, power 576 W) and a too expensive mission. Accordingly, in 1983 we undertook an extensive restructuring of the program, which cut the weight to 2800 lb and the power to 143 W without sacrificing any of the essential science goals, and lowered the anticipated cost to about $130 M [66].

One concern throughout the Phase B study has been how to keep the risk of the program, especially the cost risk, within reasonable bounds. In 1983, after much thought, we settled on a plan that would separate the development costs of the high technology instrument from the "marching army" costs of spacecraft construction by proceeding in two phases, with the dewar and instrument being built first and tested in a 7 day engineering flight on shuttle in 1989, and then brought back for minor refurbishment and integration with the spacecraft for the science mission, to be launched in 1991. The first phase is called STORE (Shuttle Test Of the Relativity Experiment).

This plan was endorsed by the NASA administrator, James M. Beggs, in March 1984. In November 1984 Stanford selected Lockheed Missiles and Space Corporation as its aerospace subcontractor on STORE, with Stanford providing the central gyro package and Lockheed the dewar/probe and electronics packages. John Turneaure heads the Stanford hardware development group.

4. GENERAL APPROACH TO EXPERIMENT

4.1 A Near Perfect Gyroscope

In concept, the GP-B relativity gyroscope experiment is simplicity itself: an earth-orbiting spacecraft containing one or more precise gyroscopes referenced to a precise telescope pointing at a stable guide star. The difficulties lie only in the precisions needed. Doing a 1 m arc-sec/yr experiment calls for a gyroscope with an absolute drift rate of about 10^{-18} rad/sec, some nine orders of magnitude less than the absolute drift rates of very good inertial navigation gyroscopes, and six orders of magnitude less than the compensated drift rates obtained in the best such gyroscopes by modeling out predictable errors. More precise readouts are needed for both the gyroscopes and the telescope than in the best conventional instruments. Not surprisingly has the development of the experiment taken so long.

It might seem absurd, in view of the great effort and ingenuity applied to inertial navigation instruments over the past thirty years, that a university research team, however well supported by NASA and industry, should aim to produce a gyroscope with performance a million times better than the best hitherto available. But the task we are attempting is different. Inertial navigation gyroscopes have to operate in submarines, aircraft and missiles under very high g loads. They must be small, light, and—in commercial applications, at least—cheap. While cost is indeed a concern to us, sizing of the total package is less so, and, most important, the experiment operates not at levels of 1 g to 30 g as in submarines or missiles but in the nearly weightless environment of space. Since the limitations on certain inertial navigation gyroscopes, especially those using a suspended spinning sphere as the reference element, come principally from support torques, it is plain that the performance of such gyroscopes should greatly improve in space. To see the possibilities for improvement, it is only necessary to examine the simplest support-dependent torque: the mass unbalance torque Γ_u. Consider a spinning sphere that is perfectly round but not quite homogeneous. Let the mass of the sphere be M and the distance between its center of geometry and center of mass along the spin axis be δr. If the gyroscope is supported about its center of geometry and subjected to a transverse acceleration f, then $\Gamma_u = Mf\delta r$ and the resulting drift rate $\Omega_u = \Gamma_u/I\omega_s$ is

$$\Omega_u = \frac{5}{2}\frac{f}{v_s}\frac{\delta r}{r} \quad , \tag{2}$$

where r is the radius and v_s the peripheral velocity of the sphere (say, 2000 cm/sec). To do an experiment on earth, $\delta r/r$ would have to be 4×10^{-17}. In space with a 10^{-10} g acceleration, it need be only 4×10^{-7}.

Arguments of this type led us to the concept of a gyroscope in the form of a very round, very homogeneous fused quartz sphere, coated with superconductor, operating at cryogenic temperatures in the nearly zero-g environment of space. The gyroscope is weakly suspended by electrical fields in an evacuated spherical cavity, spun up initially by gas jets, and has for its angular readout the London moment readout scheme described in section 1.2. Details are given in paper (B) of this series by J. A. Lipa and G. M. Keiser. The marriage of cryogenic techniques with space techniques also solves, as we shall see, various problems in the operation of the reference telescope and the spacecraft control systems.

4.2 Experiment Configuration

Figure 3 is a view of the instrument, the main structural element of the spacecraft. The dewar has an annular helium well with capacity 1580 L, supported from 12 "passive orbital disconnect struts" (PODS) within a shell of 69 inch diameter and 106 inch overall length. A neck tube joins the helium well to the outer shell of the dewar so as to form a continuous

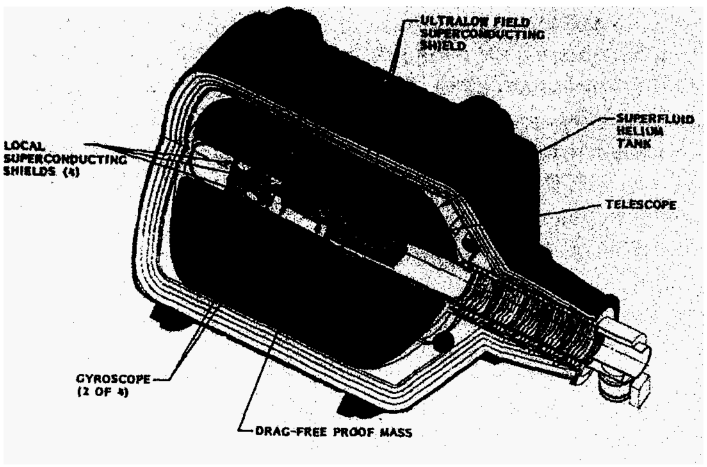

FIGURE 3. General view of GP-B instrument and dewar.

enclosed cavity of 10 inch inner diameter into which the instrument fits. The dewar is superinsulated and has four vapor-cooled radiation shields arranged to provide cooling also for the neck tube. The dewar is designed to operate at 1.8 K and hold helium for approximately two years, the proposed duration of the GP-B mission. Boiloff of the helium is controlled by the porous plug device to which we have already referred. The escaping helium gas is vented to space through proportional thrusters, also developed at Stanford, which provide control authority for pointing the spacecraft, as well as for a translational control system referenced to the drag-free proof mass.

The instrument comprises a quartz block structure, including the reference telescope, four gyroscopes and drag-free proof mass, enclosed in an evacuated cylindrical chamber 10 inches in diameter and 96 inches long, with its own insulating neck tube, all forming an independent assembly that can be inserted or removed as a unit in the inner cavity of the dewar. The gyro-telescope structure is held together in molecular adhesion by "optical contacting" for maximum mechanical stability.

The four gyroscopes are in a straight line on the instrument axis with their spin axes aligned parallel to the line of sight to the guide star, Rigel, two rotating clockwise and two counterclockwise. This configuration, as mentioned earlier, puts the spin vectors parallel to the line of sight causing the two precession effects, Ω_G and Ω_M, to appear simultaneously in each gyroscope. The resulting total precession is $\sqrt{\Omega_G^2 + \Omega_M^2}$, with a phase angle

ϕ with respect to the plane formed by the earth's axis and the line of sight given by $\tan\phi = \Omega_M/\Omega_G$. Since the spacecraft rolls about the line of sight with an angular velocity ω of 2.5×10^{-4} rad/sec (10 minute period), the gyro readout will record a sinusoidal signal of amplitude $\sqrt{\Omega_G^2 + \Omega_M^2}$ and frequency ω, whose phase can be determined separately by means of a "star blipper," attached to the spacecraft, which picks up signals each revolution from one or more suitably bright stars situated at points on the celestial sphere nearly 90° away from the guide star. The roll greatly reduces certain drift torques, eliminates the effect of 1/f noise in the SQUID, and eliminates errors from null drifts in the gyro and telescope readout.

Operation in space reduces the support torques on the gyroscopes, but does not make them zero. With a satellite of typical area/mass ratio in a 650 km orbit, the average residual acceleration on the spacecraft (and hence on the gyroscopes) from air drag, solar radiation pressure, *etc.*, is a few times 10^{-8} g. Low as this is, it is not low enough, so a drag-free proof mass is placed near the center of mass of the spacecraft. The proof mass is located in a cavity in the quartz block as shown in figure 3. The control system provides signals to the same proportional thrusters that are used for pointing the spacecraft. The level of translational control needed is about 10^{-10} g, a factor of 20 less stringent than that already demonstrated by the DISCOS controller for TRIAD.

4.3 Magnetic Shielding

Crucial to the success of the London moment readout of the gyroscope are two constraints on magnetic fields. (a) The gyroscope must operate in a very *low* magnetic field (less than 10^{-7} G) to keep trapped flux in the rotor at an acceptable level. (b) The gyroscope must operate in a very *stable* field (effective variations less than 2×10^{-13} G) to prevent changes in the external field from disturbing the readout.

Trapped flux in the rotor appears in the readout as an ac signal at the 170 Hz spin speed superimposed on the essentially dc signal from the London moment. In itself this ac signal is not deleterious; indeed, it can be a useful aid as a diagnostic for gyro torques and a calibrating signal for the gyro scale factor. If, however, the amplitude exceeds the linear range of the readout amplifiers, rectification offsets will occur. Hence the 10^{-7} G requirement.

The requirement is met by making use of one of the expanded balloon ultralow magnetic field shields conceived by W. O. Hamilton [67], developed by B. Cabrera, and described by Cabrera elsewhere in this volume. The shield, located between the gyro probe and the inner well of the flight dewar as shown in figure 3, is 70 inches long and 10 inches in diameter and closed at the lower end. It is made of 2.5 mil thick lead foil. To protect the superconductor at the open end from going normal in the earth's field, a mu-metal shield (not shown) is incorporated in the dewar.

Since the lead bag has an open end, the earth's magnetic field enters it and may affect the gyroscopes. In fact, the combination of mu-metal shield and lead bag attenuates the transverse component of the field by about seven orders of magnitude, leaving a need for nearly six orders of magnitude additional attenuation to reduce the field variations as the satellite orbits the earth to the 2×10^{-13} G requirement. The extra attenuation is achieved by (a) surrounding each gyroscope with a transverse cylindrical superconducting shield (figure 3), which provides nearly four orders of magnitude of attenuation through a combination of direct shielding and symmetry, and (b) exploiting the self-shielding effect of a gyro readout loop tightly coupled to the superconducting rotor such that the external field penetrates only the narrow annular gap between the ball and the loop.

The inner shields also eliminate cross talk between the gyroscope readouts. For further details see paper (C) of this series.

4.4 Telescope and Data Instrumentation System

The star-tracking telescope has to give a precise reference with resolution, linearity and null stability adequate to avoid errors of a milliarc-sec during the lifetime of the experiment. A scheme is also needed to subtract and process the gyroscope and telescope signals into a form suitable for storage and transmission to ground. The designs of the telescope and data instrumentation systems are bound up with the design of the attitude control system of the spacecraft. The proposed telescope is linear to 0.3 m arc-sec over a range of ±60 m arc-sec. Provided the telescope and gyro readouts are scaled correctly, the requirement is to point the telescope within this range of the apparent star position. Pointing to 60 m arc-sec is feasible with a two-loop control system. The proportional thrusters point the spacecraft to rather better than an arc-sec; within the instrument there are cryogenic actuators which tilt the gyro-telescope structure with respect to the dewar, and keep the telescope pointed within the desired 60 m arc-sec range.

The telescope is illustrated in figure 4. It is a folded Schmidt-Cassegrain system of 150 inch focal length and 5.6 inch aperture, held together entirely by optical contacting. The physical length is 13 inches. The addition of the tertiary mirror to the conventional Cassegrain design puts the focal plane at the front of the telescope. This is done primarily for structural convenience; but it also gives an opportunity to improve baffling of stray light. Image division occurs in the light-box contacted to the corrector plate. The light first passes through a beam-splitter near the focal plane to give two star-images, one for each readout axis. Each image then falls on the sharp

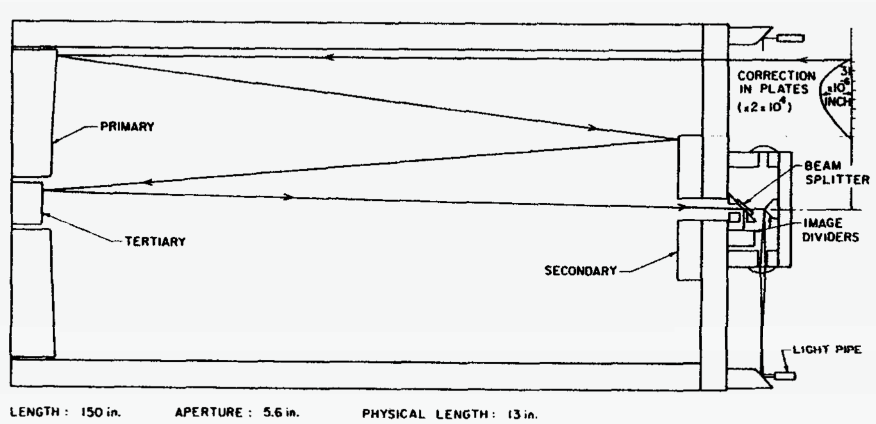

FIGURE 4. Design layout of cryogenic star-tracking telescope.

edge of a roof prism, where it is again subdivided and passed through light-pipes to a chopper and photodetector at ambient temperature.

The ultimate limit to sensitivity of the telescope is set by photon noise from the starlight. The noise equivalent angle θ_t for each readout axis is calculated from the fluctuation in intensity of the light falling on the photocell during each signal period. For a star of magnitude M and color temperature $\bar{\lambda}$, the result is approximately

$$\theta_t = 2 \times 10^{-6} \frac{\delta}{D} \frac{\sqrt{2.51^{-M} \Delta\nu}}{\bar{\lambda}\epsilon\eta} , \qquad (3)$$

where δ is the image diameter, D the telescope aperture, ϵ the effective light loss in each channel, η the quantum efficiency of the detector, and $\Delta\omega$ the bandwidth. For diffraction limited optics, δ may be taken as 2.44 λ/D. A telescope of 5.6 inch aperture, having $\epsilon\eta$ about 0.001 (which is very conservative), resolves the direction of a first magnitude star to 10 m arc-sec in 0.1 seconds of time. Rigel is of 0.16 magnitude.

Figure 5 shows the principle of the science data instrumentation system. The gyro and telescope signals are subtracted and summed with the final signal in the precision summing amplifier Σ_1 and then filtered in the integrating data loop represented by the heavy lines on the diagram. The output of Σ_1 is an amplitude modulated suppressed carrier alternating current signal; it is processed in a sampling demodulator and filter to give a direct current output with extremely low offset, and then integrated by means of an 18 bit up-down binary counter, which contains the read-out signal for storage and telemetry. The integrating loop is closed by an 18 bit digital-to-analog converter summed into Σ_1. Call the gyro ouput G, the telescope output T, and the signal in the up-down counter R. The summing amplifier yields the function $(T - G + R)$, which is maintained at

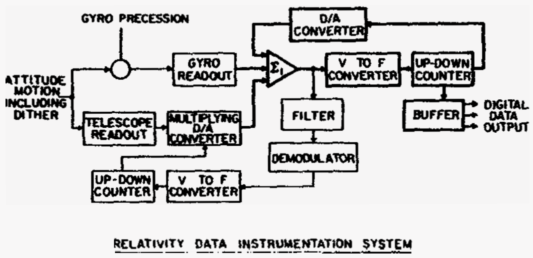

FIGURE 5. Instrumentation system for processing the GP-B science data.

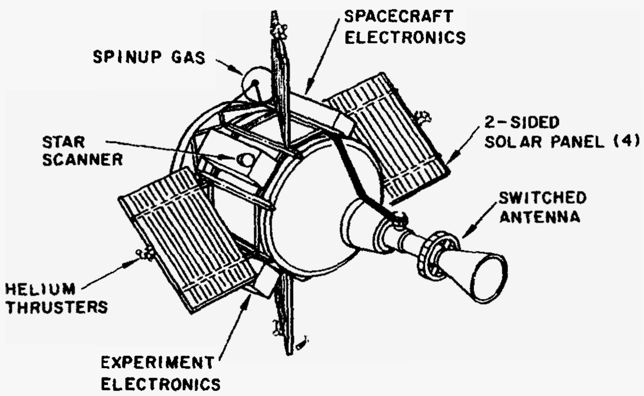

FIGURE 6. GP-B spacecraft, with telescope sun shield and other support equipment.

TABLE 1. PRELIMINARY WEIGHT BREAKDOWN OF SPACECRAFT (Kg).

Structure	273
Dewar (including helium)	743
Instrument	155
Control systems	25
Command and data management system	77
Electrical power	273
Thermal control	16
Total	1562

null, making the final signal R equal to $(G - T)$, the quantity of interest in the experiment.

The scale factors of the gyro and telescope readouts have to be nearly matched; otherwise, a pointing error of 50 m arc-sec might cause a null shift in R indistinguishable from the relativity signal. In the flight experiment, the match should be to about 2%. It is achieved by introducing a low frequency dithering motion in the pointing control to make the gyro-telescope package swing back and forth across the line of sight to the star with an amplitude of about 30 m arc-sec at about 1 minute period. If the scale factors of the two readouts are not equal, a signal appears at the output of Σ_1, where it is synchronously detected and used to drive an automatic gain control on the telescope output by the second loop shown in figure 6.

4.5 Spacecraft

Figure 6 shows the layout of the spacecraft. The extension at the front end is the sunshield for the telescope. The solar arrays provide an initial power of 230 W and a final power after one year's lifetime of 200 W. The overall diameter of the spacecraft (excluding solar panels) is 8 feet, the length including the sunshield is 14 feet 8 inches. It is designed to stand upright on a single shuttle pallet mounting and so occupy one-fifth of the shuttle bay. Table 1 shows the weight breakdown. The power requirement is about 150 W.

5. REDUCTION OF THE DATA

5.1 Explanation

A gyroscope moving in an ideal polar orbit experiences two principal relativity effects Ω_G and Ω_M, both, as we have seen, causing linearly increasing changes in direction of the spin vector, with the geodetic precession Ω_G lying in the plane of the orbit and the motional precession Ω_M lying in the plane of the celestial equator. These, being at right angles, are easily separated. In nonpolar orbits the terms become mixed in a way that seems troublesome but in fact is not.

The succeeding sections cover:

(1) procedures for separating the two Schiff terms in any orbit

(2) the inclusion of three smaller relativistic effects in addition to the Schiff terms

(3) a method of calibrating the scale factor of the experiment absolutely through starlight aberration signals

(4) the combination of (1), (2), (3) in a Kalman filter covariance analysis of the data

(5) an application of the experiment to improve the measurement of the distance to Rigel and potentially to improve our knowledge of the nearby distance scale for the universe

(6) the effect of uncertainties in the proper motion of the guide star.

For convenience we take (2) first.

5.2 Five Relativity Effects

An experiment at the milliarc-sec/yr level will detect five distinct relativity effects, Ω_G and Ω_M from a spherical earth, and three additional terms:

- the de Sitter-Fokker solar geodetic precession
- a correction to the Schiff terrestrial geodetic precession due to the earth's oblateness
- the deflection of the light from the guide star by the gravitational field of the sun.

For the starlight deflection term one is, as it were, turning the experiment around and using the gyroscope as a reference for the telescope.

The first of these additional effects was drawn to my attention by Blackett in 1962 in the same letter [6] in which he recounted his reflections on the experiment in the 1930's. Schiff when thinking of a 10 m arc-sec/yr experiment had dismissed it as negligible [5]. Published discussions of its importance in the gyroscope experiment were given simultaneously by Barker and O'Connell [68] and by Wilkins [59]. The second effect was investigated independently by Wilkins [69] and O'Connell [70]; in 1978 J. V. Breakwell [71] gave the elegant treatment published for the first time in paper (F) of this series. The third was first pointed out by O'Connell and Surmelian [72], while a method of extracting it from the data *via* the Kalman filter covariance was derived by Duhamel [73]. Table 2 lists the five relativity effects, with numerical values for Ω_G, Ω_M and the oblateness correction to Ω_G computed for a satellite in a 650 km polar orbit. The solar geodetic and starlight deflection effects are independent of the orbit. Each of the three additional terms can be treated satisfactorily in data reduction, though each has a different logical status. The explicit formulae relating the oblateness and solar geodetic effects to the Schiff geodetic and motional coefficients being determined by the experiment are given below in section 5.4.

◦ *de Sitter-Fokker effect*: This effect, which lies in the plane of the ecliptic, being identical in character with the terrestrial geodetic precession, is computed from the same formula with the mass and distance of the sun

TABLE 2. THE FIVE RELATIVITY EFFECTS.

Schiff geodetic effect Ω_G	6600 m arc-sec/yr
Schiff motional effect Ω_M	42.0 m arc-sec/yr
DeSitter-Fokker solar geodetic effect Ω_{DS}	19.0 m arc-sec/yr
Oblateness correction to Ω_G	−7 m arc-sec/yr
Starlight deflection (with Rigel as guide star)	+14.4 m arc-sec maximum

substituted for the mass and distance of the earth. The logical relation is a bootstrap one in which a combination of the terrestrial geodetic precession and a component of the solar geodetic precession is determined experimentally, and then used to calibrate the remaining component of the solar geodetic precession. See changes given below in equations (6) and (7).

∘ *Oblateness correction*: This is computed from Breakwell's formula, based on a generalized application of Schiff's geodetic expression $\boldsymbol{\Omega}_G = 3(\mathbf{g} \times \mathbf{v})/2c^2$ derived from equation (1). Logically the calculation rests on the same assumption as the one underlying the computation of the de Sitter-Fokker effect, but the relationship of the oblateness correction to Ω_G is a more direct one, since the effect comes from the same source and (for a polar orbit) lies in the same plane.

∘ *Starlight deflection*: This causes a deflection away from the sun in two axes, making the apparent position of the star describe the curve shown in figure 7 with a maximum deflection of 14.4 m arc-sec on June 10. Since the time signature of the effect is different from any of the gyroscope precession terms, it can be separated from them in the data reduction. The precision depends on the choice of SQUID magnetometer in the gyro readout. Duhamel [74] has shown that a conventional SHE Incorporated 19 MHz SQUID yields a 4% measurement of the starlight deflection coefficient and a Clarke double-junction SQUID a 1.4% measurement.

5.3 Aberration of Starlight and Scale Factor Calibration

A critical issue in the experiment is the calibration of the scale factor of the gyroscope. We have seen (section 4.4) how the telescope scale factor

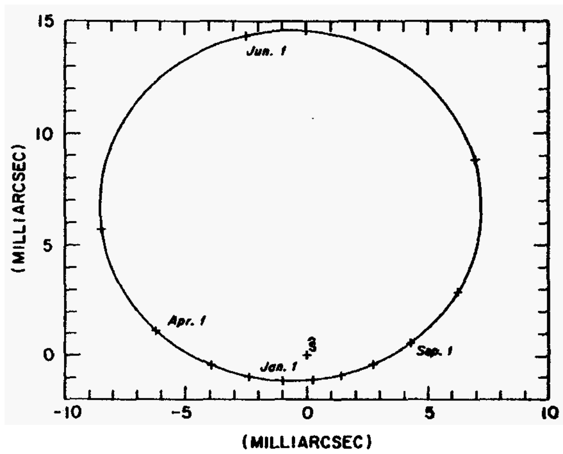

FIGURE 7. Starlight deflection measurement.

is forced to agree with that of the gyroscope by means of the dithering technique, but no process has been described that will insure that a given voltage out of the gyro readout circuit will correspond to a given angular displacement of the spin axis. Nor, in the laboratory, is it easy to find such a method of calibration.

In space, however, by remarkable good luck, nature has supplied a ready-made yardstick. Superimposed on the relativity terms to be measured, and completely distinguishable from these terms, are other signals of known amplitude and phase from the aberration of starlight. As the earth orbits the sun, there is a ±20.408 arc-sec variation in the apparent position of the guide star due to the motion of the telescope across the line of sight; as the satellite orbits the earth, there is a corresponding ±5.5 arc-sec variation in the orbit plane. The annual aberration is known from JPL ephemerides data to 0.07 m arc-sec or 3 parts in 10^6, and the orbital aberration from tracking data to about the same precision. These signals appear in the subtracted gyro-telescope output of the experiment and establish an absolute scale factor for it.

The role of aberration in scale factor calibration was first pointed out in 1968 [75]. Occasionally these signals have been regarded as an obstacle to a successful relativity experiment; in reality, they provide a further illustration of the principle enunciated earlier that an obstacle seen rightly may turn into an advantage.

The standard expression for the angular deflection from aberration is $(v \sin\theta)/c$, where $v \sin\theta$ is the velocity of the telescope across the line of sight and c is the velocity of light. An intriguing point made independently by P. Stumpff [76] and T. Duhamel [77] is that in computing the annual aberration at the milliarc-sec level, it is necessary to include a special relativistic correction $-(v^2 \sin 2\theta)/4c^2$ in the calculation.

5.4 Inclined Orbits and the Kalman Filter Covariance Analysis

So far I have only discussed effects in an ideal polar orbit. In an inclined orbit a component of the geodetic precession becomes superimposed on the motional precesson, and if the coinclination angle $(90° - i)$ exceeds a certain value, there is the further complication that certain Newtonian torques on the gyroscope resulting from the gradient in the earth's gravitational field cause drift rates in excess of 1 m arc-sec/yr. These gravity gradient disturbances are of two kinds: a *direct* term from the interaction of the earth's monopole field with the quadrupole mass moment of the spinning gyro rotor, and *indirect* terms from the effects of gravity gradient accelerations that act on the gyroscopes because they are not at the center of mass of the spacecraft. The latter produce gyro drifts through the mass unbalance torque of equation (2) and the various suspension torques discussed in section 6 of paper (B) in this series of papers.

There is yet another complication. In a nonpolar orbit the earth's oblateness makes the right ascension of the ascending node of the orbit regress at a rate

$$\omega_n = -\frac{3}{2} J_2 \left[\frac{R_e}{a}\right]^2 \bar{\omega}_o \cos i \quad , \tag{4}$$

where R_e and J_2 are the mean radius and oblateness coefficient of the earth, and a, $\bar{\omega}_o$ and i are the semimajor axis, mean motion and inclination of the orbit. At 650 km altitude $\omega_n = -7.03 \cos i$ deg/day. In the discussion which follows, "near polar" refers to an orbit for which ω_n is a few degrees or less per year, and "nonpolar" to one for which ω_n is many degrees per year.

A first impression would be that all these complications make the doing of an experiment in anything other than an ideal polar orbit hopelessly complicated. Once again, however, an apparent obstacle (the regression of the orbit plane) turns out to be an advantage [78]. The regression modulates the effects in such a way that the relativity effects can be distinguished uniquely from gravity gradient terms.

A full discussion is given elsewhere [79,80]. Here it is sufficient to write down what happens in a near polar orbit and state the results for other orbits qualitatively. The results are expressed in terms of the time variation $\dot{\mathbf{n}}_s$ of the unit vector $\mathbf{n}_s$ along the direction of spin of the gyroscope (related to the precession vector $\boldsymbol{\Omega}$ by the usual formula $\dot{\mathbf{n}}_s = \boldsymbol{\Omega} \times \mathbf{n}_s$) and coefficients A_G, A_M and A_g corresponding to the numerical values from geodetic, motional and gravity gradient precessions of the gyroscope, A_G and A_M

being the quantities we want to determine from the experiment and compare with Schiff's predictions. The experimentally significant quantities are the rates of change of $\mathbf{n}_s$ in the north-south and east-west planes and the second derivative of $\mathbf{n}_s$ in the north-south plane. Let us write these as $(\dot{\mathbf{n}}_s^{ns})_{\mathrm{MEAS}}$, $(\dot{\mathbf{n}}_s^{ew})_{\mathrm{MEAS}}$ and $(\ddot{\mathbf{n}}_s^{ns})_{\mathrm{MEAS}}$, it being understood that each term is averaged over many orbits. If we work out the expected gyroscope precessions in a near polar orbit *including the effects of the oblateness and solar geodetic terms* (but assuming that the starlight deflection term has been removed from the data by virtue of its different time signature), we find, after stripping the equations of all terms smaller than a milliarc-sec, that

$$A_g = \frac{(\ddot{n}_s^{ns})_{\mathrm{MEAS}}}{\omega_n} , \tag{5}$$

$$A_G = (\dot{n}_s^{ns})_{\mathrm{MEAS}} \left[1 - q \sin I + \frac{9}{8} J_2 \left(\frac{R_e}{\bar{r}} \right)^2 \right] - (\ddot{n}_s^{ns})_{\mathrm{MEAS}} \frac{\phi_0}{\omega_n} , \tag{6}$$

$$A_M = (\dot{n}_s^{ew})_{\mathrm{MEAS}} \cos\delta + (\dot{n}_s^{ns})_{\mathrm{MEAS}} (i' - q \cos I) . \tag{7}$$

Here J_2 and R_e are as before the oblateness coefficient and mean radius of the earth, δ is the declination angle of the guide star from the celestial equator, $\bar{r}$ is the mean radius of the satellite orbit (see paper (F) below), i' is the coinclination of the orbit plane ($i' = 90° - i$), ϕ_0 is the initial misalignment angle between the orbit plane and the line of sight to the star, I is the inclination angle of the earth's axis to the ecliptic plane, and q is given by

$$q = \frac{M_s}{M_e} \left[\frac{\bar{r}}{\bar{r}_s} \right]^{5/2} , \tag{8}$$

where M_e and M_s are the masses of the earth and the sun and $\bar{r}_s$ is the mean distance from the earth to the sun.

Equations (5), (6) and (7) are the heart of the data reduction process in a near polar orbit. Three points need to be made, illustrated numerically for a 650 km orbit with 0.1 deg coinclination i' and 0.1 deg initial misalignment ϕ_0. The nodal regression rate for an i' of 0.1 deg is 4.5 deg/yr.

First, and most important, the determination *via* equation (7) of the smaller, scientifically more significant of the quantities, the motional coefficient A_M, is notably free from pollution. Neither the gravity gradient precession terms A_g nor the oblateness correction Ω_Q affect it. Nature is on our side. The corrections for the terrestrial and solar geodetic precessions Ω_G and Ω_{DS} *are* appreciable, being respectively 11.5 and 17.7 m arc-sec/yr, but both are known with extreme precision from the 1 part in 10^4 measurement of the geodetic coefficient A_G effected *via* equation (6). They can be removed from the data with great confidence.

The one significant complication in measuring A_M is the uncertainty in proper motion of the guide star. See section 5.5.

Second, the numerical value of the gravity gradient coefficient A_g (lumped together from all sources) is known on other grounds to be of order 100 m arc-sec/yr. Cross multiplying equation (5) to get $\ddot{n}_s^{ns} = A_g \omega_n$, we find that in a 650 km orbit with coinclination 0.1 deg, the cumulative precession from this term is about 4 m arc-sec after one year and 16 m arc-sec after two years, all in the north-south plane. Thus the effect is small, and distinguishable from the relativity terms in data reduction through its quadratic form.

Third, although the last term of equation (6) shows a contribution to the north-south precession from the initial misalignment of the orbit plane with the line of sight to the star, with an A_s of 100 m arc-sec/yr and an i' of 0.1 deg, this term contributes only 0.17 m arc-sec/yr and can therefore be dropped from the equation in most instances, further simplifying data reduction. The corrections to A_G from $q \sin I$ and $J_2(R_e/\bar{r})^2$ yield drift rates of 6.8 and 7.0 m arc-sec/yr, respectively. Since the quantities q, I, J_2, R_e and $\bar{r}$ are each known to a part in 10^6, the computational margin is enormous.

Thus the approach to data reduction in a near polar orbit is simple and direct: A_G is determined from equation (6) and A_M from equation (7), and the only significant contribution of the gravity gradient effects is a small quadratic drift in the north-south plane which is readily separable from A_G and has no effect on A_M. Indeed, if the determination of A_M in a near polar orbit were the only goal, we could probably relax some of the manufacturing constraints on the gyroscope stated in paper (B) below. We choose, however, to retain them.

In nonpolar orbits the separation of terms is more complex but still unambiguous. Before giving results we must briefly consider the Kalman filter covariance analysis developed by John Breakwell, Richard Vassar and Thierry Duhamel [79,81]. The Kalman filter may be regarded as an extension of the Gaussian least squares method of data fitting, which takes into account parameter variations, for example variations in the roll rate of the satellite and the scale factor of the gyroscope. It provides the technique for utilizing the starlight aberration signals to calibrate the gyro scale factor, and also for separating the relativity signals from the gravity gradient signals in the regressing orbit.

The relativistic precessions comprise terms that either are linear in time or depend on the first harmonic of the nodal regression rate ω_n; the gravity gradient precessions comprise in addition terms depending on higher harmonics of ω_n. The direct gravity gradient torque, the mass unbalance torque and the odd harmonic suspension torque all yield precessions of the same signature containing terms in $2\omega_n$; the even harmonic suspension torques yield precessions containing terms in $2\omega_n$, $3\omega_n$ and $4\omega_n$. The covariance analysis finds coefficients for these effects from the higher frequency components and uses them to compute uncorrupted relativity signals, a linear east-west term of rate

$$\bar{\dot{n}}_s^{ew} = \left[A_G \cos i - \frac{1}{2} A_M (1 + 3 \cos 2i) \right] \cos \delta \quad , \tag{9}$$

and two periodic terms, one in the north-south direction of amplitude

$$\Delta n_s^{ns} = \frac{1}{\omega_n} (A_G - 3A_M \cos i) \sin i \quad , \tag{10}$$

the other in the east-west direction of amplitude $\Delta n_s^{ew} = \Delta n_s^{ns} \sin \delta$. Since the nodal regression rate ω_n varies as $\cos i$, the amplitudes of Δn_s^{ew} and Δn_s^{ns} both depend on inclination angle as $(A_G \tan i - 3A_M \sin i)$. Equations (9) and (10) may be solved to determine A_G and A_M.

Use of a nonpolar orbit has two intriguing consequences. First, it partially eliminates proper motion errors. Since the proper motion of the guide star will be sensibly uniform, the sinusoidal components of the relativistic precessions will not be affected by it and the coefficient $(A_G - 3A_M \cos i)$ can be determined absolutely. Second, surprisingly, certain nonpolar orbits yield a more precise measurement than a polar orbit. Assume, as is indeed the case, that the precision is limited by noise in the gyro readout. In a polar orbit the line of sight to Rigel is occulted by the earth for nearly half of each orbit and no data can be taken then. Now take a slowly precessing orbit chosen so that at the beginning and end of the year the ascending node lies in a plane 90° away from the line of sight. At these times, which, being at the extremities of the measurement curve, are the most critical, the star is continuously visible, and nearly twice as much data is available for fixing the shape of the curve. Figure 8 illustrates the variation of resolution with inclination found for simulated gyro readout noise by Richard Vassar [82]. The curve has two local minima for orbits with inclination angles of 78 deg and 86.25 deg, corresponding to nodal regressions of 577 deg and 182 deg. The expected error in the 86.25 deg orbit is slightly lower than in an ideal polar orbit.

Another outcome of the covariance analysis is the evolution of the measurement error with time. Naively one would expect a measurement limited by gyro readout noise to improve with time as $t^{-3/2}$ [83], since the relativity signals (in polar orbit) increase linearly with t and the SQUID noise averages as $t^{-1/2}$. Reality is more subtle. The Kalman filter makes use of the readout data for three distinct purposes: (a) to measure the gyro angle, (b) to compare the relativity signals with the starlight aberration signals, (c) to aid in processing the satellite roll phase information. All three contribute to the overall noise of the measurement. Figure 9, also due to Vassar

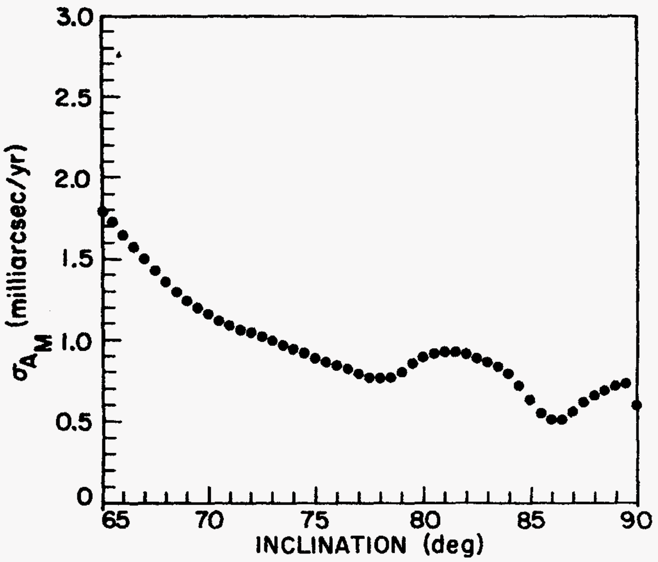

FIGURE 8. Expected error in measuring the motional precession *versus* inclination, assuming optimum launch month and known proper motion ($i = 65 - 90$ deg).

[84], illustrates the time development of the uncertainties σ_{A_M} and σ_{A_G} in the determinations of the motional and geodetic precessions, obtained for a polar orbiting mission with a September launch date, assuming an intrinsic readout noise of 1 m arc-sec in 70 hours, as discussed by Anderson in paper (C) of this series. Superimposed on the expected smooth improvement in resolution is a six-month periodicity consequent upon a frequency doubling of the annual aberration signal used in scale factor calibration.

Breakwell and his students have extended the Kalman filter covariance analysis to investigate many different aspects of the data reduction process. One already discussed is the computation of the relativistic deflection of starlight from the experiment. Others include studies of the effects of the polhode motion of the gyro rotor [85], methods of handling interruptions of data, including ones resulting in temporary malfunction of the gyroscope [86], and a method of improving the measurement resolution by making use of the trapped flux in the rotor to aid in scale factor calibration [87].

In conclusion, I remark that intriguing as nonpolar orbits are, it is wise, in the first instance anyway, to stick with the near polar orbit. Doing so minimizes the burden of data reduction and hence the possibility of error in the reduction process. Particularly important is the fact that in a near polar orbit gravity gradient torques have no influence on the determination of the motional precession of the gyroscope.

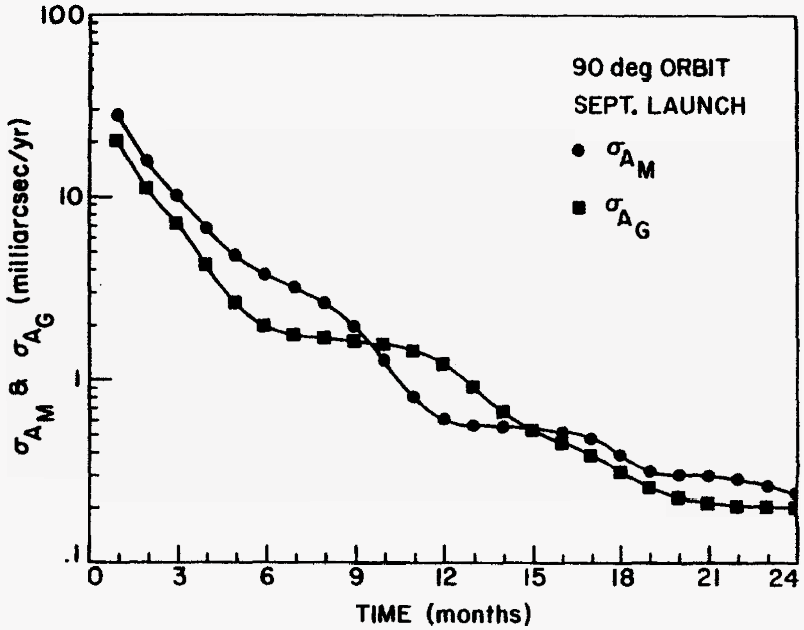

FIGURE 9. Time history of σ_{A_M} and σ_{A_G} for a 90 deg orbit, September launch.

5.5 The Proper Motion Error

The largest known source of error in the experiment is, as already remarked, the uncertainty in proper motion of the guide star. A current best estimate for the uncertainties in the absolute proper motion of Rigel is 0.9 m arc-sec/yr in declination and 1.7 m arc-sec/yr in right ascension [88]. The uncertainty in right ascension contributes a 4% error to the determination of the motional coefficient A_M.

Ultimately the proper motion error is of no great concern because knowledge of proper motions will improve with time and can be applied retroactively to improve the result. Data from the European Space Agency's HIPPARCOS astrometric satellite and from the Hubble Space Telescope may have appreciably reduced the uncertainty by the time the experiment flies. Beyond that there are (in addition to the partial solution for an inclined orbit described in the preceding subsection) two possibilities. One, suggested by R. H. Dicke [89], is to design a special earth-based instrument to measure the motion of Rigel with respect to a local background field of distant stars rather than to solve the general astrometric problem. The other, suggested in 1965 by I. I. Shapiro [90], is to compare results from two gyroscope experiments flown at different orbit altitudes.

5.6 Ranging to Rigel, and a Potential Method for Improving the Nearby Distance Scale of the Universe

The apparent position of Rigel varies not only through aberration but also through parallax. Being about 250 parsecs from the solar system, its annual parallactic variation is about 4 m arc-sec, and this variation, being 90 deg out of phase with the annual aberration, produces a phase shift ϵ ($\sim 2 \times 10^{-4}$ rad) in the measured aberration signal. Currently the distance to Rigel, inferred from a distance scale based on its brightness and star type, is known to about 25 percent. Vassar and Duhamel [91] have shown that the distance to Rigel can be fixed to 3% in a gyroscope experiment with readout based on the 19 MHz SHE SQUID, and to 0.7% in one with a readout based on the Clarke double junction SQUID.

If the method were extended to measure ranges to the group of Cepheid variables within our galaxy, the experiment could help significantly in improving the overall distance scale for the universe.

6. PROSPECTIVE: SIX CONCEPTS FOR IN-FLIGHT CALIBRATION OF THE EXPERIMENT

A measurement as difficult and important as the one we are undertaking demands, whatever its outcome, exceptionally severe scrutiny. Only a modest knowledge of the history of physics is needed to make one aware how easily results in agreement with a cherished theory are accepted and those in disagreement explained away. However great our confidence in the error analysis, any spirit of idle complacency about it would be an abrogation of scientific responsibility. A rigorous plan of in-flight check and countercheck is a *sine qua non* of the gyroscope experiment.

Systematic consideration of in-flight calibration followed upon a discussion initiated by Rainer Weiss of MIT and David Wilkinson of Princeton during the visit of the NASA Technology Review Committee to Stanford in August 1980. At that meeting two countervailing principles were brought out. One, emphasized by Weiss and Wilkinson, is the physicist's principle: "Vary everything you can." To assure that the instrument is working as planned, the gyro operating conditions (pressure, support voltage, spin speed and so forth) should be varied widely during the course of the experiment. The other, emphasized by Israel Tabeck, former project manager of the Viking mission at NASA Langley Center, is the engineer's principle, "If it ain't broke, don't fix it." The following discussion, which is a long way from being final, outlines an in-flight calibration plan in terms of six concepts: *redundancy, variation, enhancement, separation, continuity* and *absolute relationships.*

The flight instrument has four gyroscopes, each of which is designed to measure all five relativity effects. If all the gyroscopes agree to 1 m arc-sec or better throughout the year, one has gained *redundancy* of the most valuable kind, because it is a redundancy combined with *variation.* The four gyroscopes are independent not just in being separate but in the deeper sense of having different characteristics and operating conditions. Two spin clockwise and two counterclockwise (which makes the direct, but not the indirect, gravity gradient torques on them yield drifts of opposite sign); each rotor has a different shape and mass distribution (which makes the support and mass unbalance torques on each slightly different); each gyroscope is at a different location with respect to the center of mass of the spacecraft (which makes the indirect gravity gradient effects on each one different). If with all these variations the four gyroscopes yield identical relativity signals, as they ought to, our confidence in the result is much higher than it would be if the gyroscopes were simple replicas of one another.

Two other possible redundancies deserve consideration. One which we wished for but have now abandoned would be for each gyroscope to have two orthogonal readout loops. J. M. Lockhart [92] has shown that the extra loops give data of only marginal usefulness and are best removed on grounds of simplicity. The second redundancy, still under study, hinges on whether the four precise readout loops for the gyroscopes are coplanar or whether two are referred to the x-axis readout of the telescope and two to the y-axis readout. The latter arrangement, though a complication in design, has operational advantages and also has the potential for allowing cross-checks on certain kinds of telescope error. If adopted, it is an exemplification also of the principle of *separation* to be discussed below.

Our next concept for in-flight calibration is *enhancement.* A cardinal principle of physical experimentation, to my knowledge first systematically applied by Henry Cavendish [93] during his torsion balance measurement of the gravitational constant G in 1798, is that if one is uncertain how large some disturbance on an apparatus is, one should deliberately increase it and see how big it is. The gyroscope experiment depends preeminently on making a great number of effects "near zero." A corollary is that at some period in the mission one should invert the strategy and make the effects large.

To serve as a useful diagnostic, *enhancement* requires *separation.* It is no good making all the disturbances large at the same time. Examples of productive enhancements follow.

o *Drag-free bias*: Control to 10^{-10} g appears to be essential in reducing the mass unbalance and primary *odd* harmonic suspension torques on the

Tests based on varying the preload and drag-free level and on stopping the roll would be done in phase (3), a test at reduced spin speed in phase (1).

An attractive approach is to utilize trapped flux in the gyro rotor to study gyroscope performance before spin up. Consider a torque Γ_i independent of spin speed. For the spinning gyroscope, $\Omega_i = \Gamma_i / I\omega_s$; for the nonspinning one, in time t the rotor will slew through an angle $\theta_i = \Gamma_i t^2/2I$. If Ω_0 is the desired performance, the maximum allowable slew angle is

$$(\theta_i)_{max} < \frac{1}{2}\Omega_0 \omega_s t^2 \quad , \tag{11}$$

which for an Ω_0 of 0.3 m arc-sec/yr (5×10^{-17} rad/sec) corresponds to a rotation of 38 arc-sec in a day. This is easily resolved. Since the trapped field at 10^{-7} G is three orders of magnitude less than the London moment field for a gyroscope spinning at 170 Hz, the resolution is found by substituting 1 arc-sec for 1 m arc-sec in the figures for gyro readout, *i.e.*, 1 arc-sec in 70 hours with an SHE 19 MHz SQUID and in 16 hours with a Clarke double-junction SQUID. A day is more than enough. In reality the gyroscope will have some initial slow spin established during levitation, but that is an advantage rather than a disadvantage.

Evidently a program of enhancement tests can be worked out for the nonspinning gyroscope.

What of tests during phase (2) of the experiment when relativity data is being gathered? Here we apply our fifth and sixth calibration concepts: *continuity* and *absolute relationships*.

Continuity may be thought of as a modest violation of the "near zero" principle. The use made of orbital aberration signals in calibrating the gyro scale factor is one example. Having such a precisely known signal with periodicity 98 min acting continually throughout the year is an elegant diagnostic. For usefulness in "continuity" an effect needs to be repetitive, distinguishable from other terms, large enough to be detected but still sufficiently "near zero" not to upset the experiment. Take four examples: (a) cyclic gravity gradient accelerations, (b) the quadratic component of gyro drift, (c) residual trapped flux in the gyro rotor, (d) the action of the earth's magnetic field on the gyro readout.

- *Cyclic gravity gradient accelerations*: The gravity gradient acceleration acting on a gyroscope ℓ cm from the center of mass of the spacecraft has a component of amplitude $1.7 \times 10^{-9}\ \ell\ g$ at twice orbital period lying in the orbit plane. (See equation (4) of paper (B).) This acceleration will appear in the output of the gyro suspension system; its continuity throughout the year is a check of the suspension system's performance and hence of the gyro performance.

∘ *Quadratic component of gyro drift*: As was explained in section 3.4, there is a small quadratic component of precession in the north-south plane arising from the changes caused in various torque terms by the changing gravity gradient in a regressing near polar orbit. Continuity of this effect throughout the year in proper relationship to the orbit plane checks the gyro performance and fixes the magnitude of the coefficient A_g for the sum of mass unbalance, direct gravity gradient and odd-harmonic suspension torques. It does more. The quadratic precession is the result of applying to the gyroscope a linearly increasing acceleration of known magnitude at right angles to the orbit plane. For a gyroscope 30 cm from the spacecraft center of mass, in an orbit regressing 4 deg in a year, the total change in acceleration over the year is 3.4×10^{-9} g, or 34 times the drag-free limit on the spacecraft. Once A_g has been found from the quadratic term, it can be combined with the drag-free limit to set an upper bound on the uncertainty introduced into the experiment through the action of the residual drag-free acceleration on mass unbalance, direct gravity gradient and odd-harmonic suspenion torques.

∘ *Trapped flux in the gyro rotor*: The use of trapped flux in assessing gyro performance prior to spin up has already been discussed. It also aids during the mission. The trapped flux signals provide a repeating pattern at the spin frequency, modulated by the polhoding of the body. Details are given elsewhere; three points are significant. (i) The carrier frequency gives the spin speed and spin down rate of the gyro rotor, (the spin down rate yielding incidentally the best measure of the gas pressure in the cavity). (ii) Measurement of the amplitude of the trapped signals is, as was mentioned in section 3.4, a useful aid in calibrating the scale factor of the gyro readout, specifically in reducing effects of short-term fluctuations [87]. (iii) Continuity in the polhode pattern (there should be no detectable variation throughout the year) gives insight into certain kinds of gyro disturbance [94].

∘ *Action of the earth's magnetic field*: To prevent signals at the 1 m arc-sec level getting into the gyro readout from the earth's magnetic field, the field has to be attenuated by some thirteen orders of magnitude. With less attenuation, a signal at twice orbital period, modulated at 24 hour period because of the earth's rotation, will appear in the output, but higher order terms will be negligible, being smaller and more strongly attenuated. Two points can be made. First, a small disturbance of this kind will not impair the relativity data because the doubly periodic signature is different from that of any of the relativity terms; one may if one chooses relax the shielding requirement slightly. Second, by keeping watch on signals doubly periodic with the orbit, one can check the integrity of the magnetic shielding continually throughout the mission.

Tact is needed in balancing "near zero" and "continuity." The right initial approach is a strenuous emphasis on "near zero." Our original rhetoric was first make each disturbing effect absolutely zero, then average it in every conceivable way possible; then, finally, the experiment may just become feasible. But "continuity" skillfully applied has its own potency. The secret is to have effects that are known with great assurance and sharply distinguished from each other. The use of the quadratic component of gyro drift to determine A_g is a good example. The applied acceleration, being fixed by geometrical considerations, is known very exactly. Other cases may require more thought. Thus the magnetic shielding effect seems unambiguous in that it is doubly periodic with the orbit; but suppose the doubly periodic gravity gradient acceleration disturbed the gyro readout in some way. Would this cause an ambiguity, and if so would the ambiguity matter? Probably not, because any gravity gradient effect should be 90° out of phase with the direct magnetic effect; but these are the kinds of issues that need to be thought through.

Lastly, *absolute relationships.* Two examples will suffice: (a) the absolute relationship between the planes in which the aberration signals occur and the planes in which the relativity signals are expected to occur; and (b) the absolute relationship between the magnitudes of the relativistic starlight deflection and the relativistic gyro precessions.

The orientation of the rolling spacecraft is established once every 10 min by the combination of rate-integrating roll-reference gyroscopes and a star blipper (see section 2.2). Appearing in the output are the orbital and annual aberration signals, each of which has a known roll phase tied respectively to the plane of the orbit and the plane of the ecliptic. A suggestion by T. M. Spencer [95], confirmed by R. Vassar and J. V. Breakwell, is that the aberration signals may themselves provide a roll reference, possibly even to the exclusion of the star blipper. In fact, Vassar has shown [96] that an experiment done thus without the blipper is degraded only by about a factor of two. It seems best, however, to keep the star blipper, not just to recover the factor of two, but because doing so provides an end-around check of the total process used in separating the two relativity terms. Admittedly, the deepest problem of the experiment, the error from the uncertainty in Rigel's proper motion, remains, but it is nice to bridge over the other uncertainties.

Similarly for the relativistic deflection of starlight. A properly working experiment measures starlight deflection to about 1 percent. But of course the deflection coefficient is known at least that well. Our best programmatic is to treat starlight deflection as a relativistic calibrating signal, whose relation to the relativistic gyro precessions adds a further end-around check on the experiment.

Proper motion is the one shaft that finds a real chink in our armor. Only by accepting the risks of an inclined orbit can we gain any in-flight calibration of that uncertainty, and then only a partial one. The need for certainty on this point is a pressing one.

The in-flight calibration process is made up of many interlocking pieces whose whole, if properly put together, is greater than the sum of its parts. The six concepts advanced here, ***redundancy, variation, enhancement, separation, continuity*** and ***absolute relationships***, are guides in the difficult process of establishing a safe, systematic and searching plan for the three program phases: initialization, gathering of relativity data, and post-experiment testing.

7. RETROSPECTIVE I: TEN FUNDAMENTAL REQUIREMENTS

To perform an experiment that will determine the relativistic drifts of a gyroscope to a precision of 1 m arc-sec/yr with respect to the inertial frame, ten separate requirements have to be met. An enumeration of these will usefully aid in retrospective summary:

(1) a gyroscope with drift rate less than 1 m arc-sec/yr,

(2) a gyro readout system that is linear and stable, and has a resolution better than 1 m arc-sec over a range of about ±100 arc-sec,

(3) a telescope readout that has a resolution better than 1 m arc-sec sufficiently stable and linear over a range of about ±60 m arc-sec,

(4) a gyro-telescope structure that is mechanically and optically stable in inertial space to better than a milliarc-sec/yr,

(5) a pointing system that keeps the telescope aligned with the reference star to within the telescope's linear range,

(6) a science-data instrumentation system capable of (a) subtracting the gyro and telescope signals from each other to a precision better than 1 m arc-sec over a total range of ±100 arc-sec, (b) ensuring that the scale factors of the gyro and telescope readouts are matched with the required accuracy over the range of the telescope readout, so that a subtraction made when the system is not pointing at the star remains an honest one,

(7) a means of eliminating effects of electronic, magnetic or optical bias drifts in the gyroscope and telescope readouts and in the science-data instrumentation system,

(8) a calibration of the combined scale factor of the gyro and science-data instrumentation systems, so that one knows that a particular digital count corresponds to a particular number of arc-sec, despite the long-term and random short-term variations in these scale factors,

(9) a means of separating the geodetic and motional precessions observed by each gyroscope,

(10) adequate knowledge of the absolute proper motion of the guide star.

If any of the foregoing ten requirements is not met, the experiment will fail, although inadequate present knowledge of (10) could be corrected for by retroactive extrapolation of later improved data.

In the gyroscope experiment, (1) is achieved by space operation combined with high vacuum and low magnetic field techniques and spacecraft roll, (2) by the London moment readout with an appropriate feedback loop, (3) by the cryogenic optically contacted quartz telescope with roof prism image dividers, manufactured in a particular manner, (4) by operating at low temperatures in the vacuum of space, (5) by a precision pointing control system with proportional thrusters, (6) part (a) by an 18 bit integrating data loop, and part (b) by an automatic gain control loop which forces the telescope scale factor to match the gyroscope scale factor through injecting a "dither" signal into the pointing system and looking synchronously in the output of the summing amplifier of the data integration loop for a mismatch signal which can be used for reference in automatic gain control, (7) by rolling the spacecraft about the line of sight to the star with a 10 min roll period, which chops the signal and eliminates any drifts with frequency longer than that for the roll (care is needed to scrutinize possible rectification effects at 10 min, due, for example, to temperature dependent null drifts in the electronics systems), (8) by making use of (a) the annual aberration signals to provide absolute long-term scale factor calibration, (b) the orbital aberration signals to provide both absolute short-term and absolute long-term scale factor calibration, (c) trapped flux signals which provide the best relative short-term scale factor calibrations, (9) from spacecraft roll through measurement of roll phase with the external gyroscopes and star-blipper. Requirement (10) is at present the weakest link in the chain because no one is sure how far to trust the astrometers.

With the present uncertainty in the absolute proper motion of Rigel ascension, the resultant uncertainty in determining the 44 m arc-sec/yr motional precession is 3.8%, and this at present is the dominant error in the experiment.

8. RETROSPECTIVE II: THREE INTRINSIC AND SEVEN EXTRINSIC "NEAR ZEROS"

I remarked earlier that the gyroscope experiment preeminently illustrates the "near zero" principle. If anything can be regarded as "near zero" it is the 10^{-11} deg/hr absolute drift which is required of this gyroscope.

The 10^{-11} deg/hr limit on drift rate is an *intrinsic* "near zero," a requirement the gyroscope has to meet, whatever the details of its design. Two other intrinsic "near zeros" follow from the fundamental one, related to the observation (section 1.2) that the two real difficulties in the experiment are (a) reading the direction of spin and (b) spinning the gyroscope to begin with. Take spin up. To spin a gyroscope one must apply a torque Γ_s parallel to the axis of spin. Assuming that Γ_s is constant and that there are no drag torques, one finds that the total angular momentum $I\omega_s$ of the gyroscope is just equal to $\Gamma_s t_s$, where t_s is the spin time. After spin up, Γ_s is reduced, hopefully to zero. Suppose there remains a small component of torque Γ_r at right angles to the spin axis, then since $\Omega_r = \Gamma_r / I\omega_s$, we get the following requirement on torque switching:

$$\frac{\Gamma_r}{\Gamma_s} < \Omega_0 t_s \quad , \tag{12}$$

which for a gyroscope with an Ω_0 of 5×10^{-17} rad/sec and a spin time of 2000 sec means Γ_r/Γ_s has to be less than 10^{-13}... truly a "near zero" requirement, but one that is indeed achieved by the gas spin up system in a rolling spacecraft.

The third intrinsic "near zero" applies to the reaction torque on the gyroscope from the readout system. The readout current acts back on the London moment causing a torque; there is also a differential damping torque from the trapped flux in the rotor. Both are negligible. In a rolling spacecraft the readout current torque for a gyroscope whose spin axis is misaligned with the readout plane by an angle α is given approximately by

$$\Omega_{rr} = \frac{15}{16\pi}\left(\frac{mc}{e}\right)\frac{\omega_s}{\rho r L K}\sin 2\alpha \quad , \tag{13}$$

where ω_s and (mc/e) are, as before, the gyro spin rate and the mass-to-charge ratio for the electron (in electromagnetic units), ρ and r are the density and radius of the ball, L is the inductance of the readout loop (in electromagnetic units), and K is the loop gain of the feedback servo. For small angles, equation (13) reduces to

$$\Omega_{rr} = 3 \times 10^{-15}\left(\frac{\omega_s}{\rho r L K}\right)\alpha = 2 \times 10^{-15}\frac{\alpha}{K} \quad , \tag{14}$$

so for a gyroscope misaligned by 20 arc-sec, the drift rate from this source is $1.2 \times 10^{-3}/K$ m arc-sec. Since the servo gain is 10^3 or 10^4, the resultant effect is very near zero indeed.

The three intrinsic "near zeros" lead in turn to a different category of "near zeros" which may be called *extrinsic*. These are the particular design constraints that have to be met to reach the desired gyro performance; there are seven of them as listed in table 3. All are attainable. Note the complementary character of the constraints on electric and magnetic torques. The gyroscope has a nonzero magnetic dipole moment (the London moment) and must therefore operate in near zero magnetic field, but has nonzero electric fields around it (the suspension field) and must therefore have near zero electric dipole moment.

Wisdom for the gyroscope experiment rests on these seven pillars.

TABLE 3. SEVEN EXTRINSIC "NEAR ZEROS."

	Requirement	Reason
	Gyro rotor	
Inhomogeneity	3×10^{-7}	Mass unbalance and gravity gradient torques
Out of roundness	5×10^{-7}	Suspension torques
Electric dipole moment	10^{-10} e.s.u.	Electric torque
	Environment	
Temperature	1.8 K	Superconductivity Mechanical stability
Acceleration	$10^{-10}\,g$	Suspension and mass unbalance torques
Magnetic field	$10^{-7}\,G$	Readout magnetic torques
Gas pressure	10^{-10} torr	Gas torques

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