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The Stanford Relativity Gyroscope Experiment (C): London Moment Readout of the Gyroscope

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1. INTRODUCTION

It has always been an objective of the relativity gyroscope experiment that the gyroscope should be a near perfect uniform solid sphere. It should be featureless and balanced. Under these conditions, it appears that the number of methods to determine the direction of spin of the gyroscope is limited. The conventional approaches used for free precession gyroscopes employ either unequal moments of inertia in the gyroscope, patterning on the surface, or a flat area on a pole of the gyroscope that is tracked by optical means. Objections to these approaches are based on both unacceptable drift and inadequate sensitivity.

The relativistic precessions of the gyroscope are small, 6.6 and 0.042 seconds of arc respectively for the geodetic and motional precessions. To make a meaningful measurement of the motional effect requires that the drift and other errors in the gyroscope be much smaller. Gyroscope errors below 0.001 seconds of arc are reasonable both in terms of the scientific value of the measurement and from the practical point of what we believe can be achieved. This goal cannot be reached without an unrelenting search for, and understanding of the sources of, Newtonian torques on the gyroscope. It was in the course of calculating the torque from the magnetic moment of the superconducting niobium coating on the gyroscope that C. W. F. Everitt hit upon the idea of using the magnetic moment as the basis of the angular readout of the gyroscope. This magnetic moment, the London moment [1], appears along the axis of rotation in any spinning superconducting object. The corresponding magnetic field B_L inside the superconductor is given by

$$B_L = (2m/e)2\pi f_r \quad , \tag{1}$$

where m and e are the mass and charge respectively of the electron, and f_r the rotation rate of the gyroscope in hertz. Interactions of the London moment with magnetic fields or magnetic or diamagnetic materials can lead to torques on the gyroscope. With proper design of magnetic shielding around the gyroscope, such torques can be made insignificant while leaving the London moment as a measureable indicator of gyroscope angle.

In simplest concept, the London moment readout [2] consists of a magnetometer capable of determining the direction of the magnetic moment. In practice, conventional magnetometers have inadequate sensitivity to perform the function. However, a combination of the Josephson effect based SQUID magnetometer [3], superconducting signal pickup loops, and an experiment geometry that requires a relatively limited angular readout range provides a gyroscope readout system having the sensitivity to reach the desired experiment accuracy of 0.001 seconds of arc in its one-year lifetime.

2. THE LONDON MOMENT READOUT

The London moment readout is shown schematically in figure 1. The superconducting readout loop lies in a plane containing the roll axis of the satellite. The roll axis is chosen to point in the direction of the guide star Rigel, selected according to specific criteria, which serves as the inertial reference against which to measure the motion of the gyroscope. The apparent position of the guide star is dependent upon stellar aberration, relativistic bending of starlight near the sun, stellar parallax, and proper motion, but, with the exception of the last, these are either directly calculable or deducible from the data as desirable measurements. Proper motion must be obtained by independent means and at present amounts to an error of approximately 0.002 seconds of arc [4]. The gyroscope spin axis will initially be aligned to within 10 seconds of arc of the nominal line of sight to Rigel, and the maximum departure from the nominal position, primarily from stellar aberration, will be under 50 seconds of arc. Relativity data

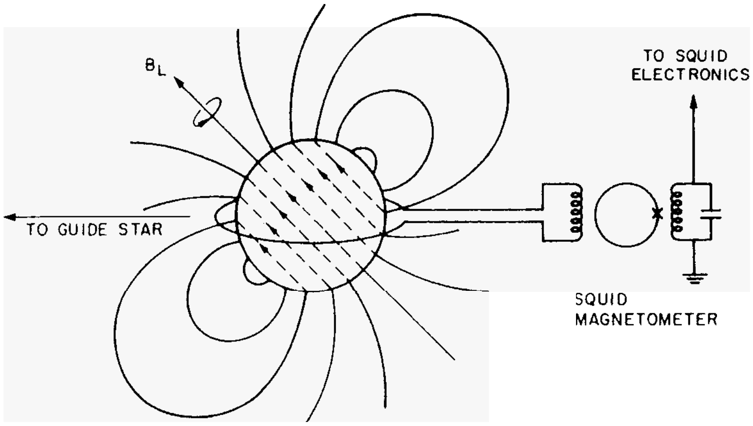

FIGURE 1. Basic configuration of the London moment readout.

are obtained from the difference between the line of sight to Rigel and the gyroscope spin axis as measured by the gyroscope readout system.

Conservation of magnetic flux in a closed superconducting path requires that the flux in the superconducting circuit consisting of the pickup loop and the SQUID magnetometer input coil remain a constant, independent of the amount of magnetic flux threading the pickup loop from the London moment of the gyroscope. Flux conservation is maintained in the presence of an applied field by a flux of the opposite sense generated by a shielding current in the readout circuit. If it is assumed that the gyroscope initially lies in the plane of the pickup loop and that there is no trapped flux in the readout circuit, then the current I_p that flows in the readout circuit owing to a deflection θ of the gyroscope spin axis is

$$I_p = \frac{\phi}{L_{tot}} \sin\theta \quad , \tag{2}$$

where

$$\phi = \pi r_0^2 B_L \sum_i^N (r_0/r_{li}) \quad , \tag{3}$$

and where r_0 is the radius of the gyroscope, r_{li} are the radii of each of the pickup loop turns, L_{tot} is the total inductance of the readout circuit, and N is the number of turns on the pickup loop. The ratio of radii is a geometric factor that comes about because the pickup loop is necessarily larger than the rotor radius and does not intercept all of the flux originating in the rotor. For the small angles under consideration, it is sufficient to replace $\sin\theta$ by θ. Although equation (2) contains the scaling of I to θ, in practice a direct calibration of the SQUID output to θ can be more accurately obtained from both daily and annual stellar aberrations of 5 to 20 seconds of arc respectively. The presumed constancy of the magnetic signal from an almost inevitable small amount of trapped flux spinning with the gyroscope can be used to provide short-term gain calibrations [5].

In its earliest concept, the London moment readout was to have a single-turn pickup loop. It was held that while the flux increased as N and the inductance as N^2, the coil energy, which is proportional to ϕ^2/L, did not depend upon N. In practical terms, however, the increase in inductance is useful. A single-turn pickup loop has $\sim 0.2\ \mu$H inductance, while typical SQUID's under consideration have $\sim 2\ \mu$H. The multi-turn pickup loop provides efficient energy transfer to the SQUID without the signal losses associated with a matching transformer. Further, by the proper spacing of turns, one can actually achieve a sensitivity increase. Spacing the turns a modest 0.02 r_0 significantly decreases the mutual inductance between turns so that for a small number of turns the inductance increases more slowly than N^2 without any significant loss of intercepted flux. Less of the field energy in the effective volume enclosed by the pickup loop is lost to energy stored in the mutual inductance between turns.

It is useful to have a perspective on the relevant numbers. The ratio of the maximum angle to be sensed to the system resolution is

$$\begin{aligned} \text{dynamic range} &= 100''/0.001'' \\ &= 10^5 \quad . \end{aligned}$$

This is not an extraordinarily high level of precision although it does require considerable care. Had the experiment geometry not been arranged to give readout angles close to null, the dynamic range would have been $\sim 2 \times 10^8$. However, in terms of sensitivity, the actual magnetic field corresponding to 0.001 seconds of arc is extremely small. The full London moment at a gyroscope rotation rate of 170 Hz is 1.2×10^{-4} gauss. To sense the field change for a gyroscope precession of 0.001 seconds of arc requires a field sensitivity of 7×10^{-13} gauss!

If a wide dynamic range were needed, then the quantization of magnetic flux could be applied to the readout to extend its range of precision [6]. Flux quantization is central to the operation of the SQUID sensor. In the absence of linearizing feedback from the SQUID electronics, the SQUID output would be periodic with each quantum of flux generated within the SQUID by the input current. By operating the SQUID and electronics in a linear mode but resetting the electronics to zero each time the SQUID contains one full flux quantum, the readout current can be measured in precise, countable units and fractions thereof over an extended range. The stability is determined by the mechanical and electrical stability of the SQUID at liquid helium temperatures, which are excellent. The scaling of the readout for the gyroscope experiment would be ~ 1300 seconds of arc per flux quantum, or ~ 240 flux quanta for one quadrant.

Clearly the most significant issue to secure the success of the London moment readout is obtaining adequate magnetometer sensitivity. Additionally, one must be concerned with magnetic torques generated by the readout. As the environment of the gyroscope is one filled with electromagnetic noise from the electrostatic suspension system, the magnetometer must in some way be protected from this source of interference. We shall see that these issues are interrelated.

3. SENSITIVITY

To do a meaningful measurement of a quantity implies the ability to obtain a statistically valid number on a time scale no longer than the allowable duration of the measurement based upon other factors. These factors may be either technical or practical, such as the lifetime of a transient event like a supernova or the lifetime of the experimenter. In the case of the gyroscope experiment, the lifetime is set by the holdtime for the liquid helium in the dewar, which is a minimum of one year. It is a design objective that data and noise accumulated from one year of operation give a sensitivity of 0.001 seconds of arc. The dominant source of noise is that from the gyroscope readout system, but, as will be shown below, it is sufficiently low to reach the experiment objectives.

If the satellite is in a nonregressing orbit, the relativistic gyroscope precession signal is a slowly building ramp. Spacecraft roll was initially introduced to reduce torques on the gyroscope, but serves also to sinusoidally modulate the relativity signal. This shifts the spectral content of the relativity signal from frequencies on the order of once per year to once per satellite revolution, which currently is 10 minutes. The importance of this can be seen from the SQUID noise curves shown in figure 2 [7,8,9,10]. SQUID's, like all electronic measuring instruments, have what is known as 1/f noise. The noise power rises inversely with frequency below some corner frequency. The benefit of shifting the readout signal frequency spectrum to higher frequencies is that the SQUID noise is lower. The corner frequency in many SQUID systems is on the order of 1 hertz. It would be desirable to modulate the relativity signal at greater than 1 hertz, but practical satellite balance and control authority limitations preclude roll frequencies greater than about 0.017 Hz (10 minute roll period).

On any signal measured, the amount of noise associated with the measurement increases with the bandwidth of the measuring instrument. For "white noise," in which the noise power per unit bandwidth is constant, the amplitude of the noise increases as the square root of the bandwidth of the measurement. For other noise distributions, one must integrate the noise

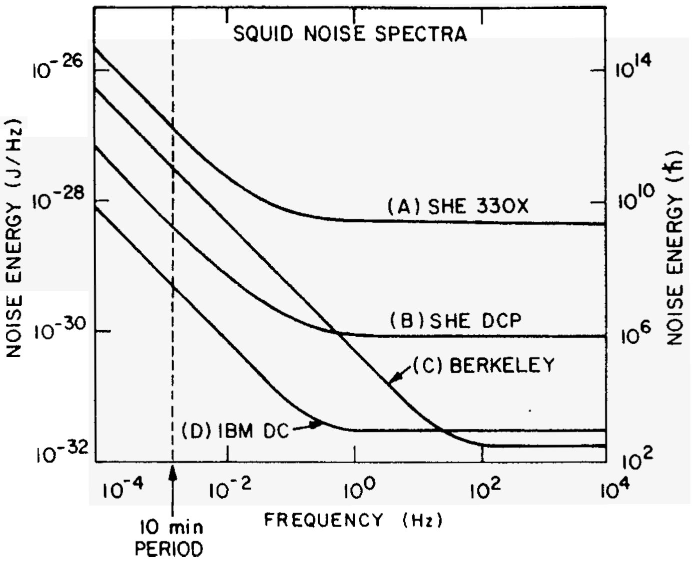

FIGURE 2. Simplified power spectra for several SQUID's. With the exception of A, these are dc SQUID's. The sources of the data for curves A through D are, respectively, references [7] through [10]. The simplification that has been used is the elimination of the statistical roughness of the data by the approximate fitting of a model consisting of a 1/f section and a white noise section.

density over the measurement bandwidth. The usual strategy for making a measurement minimally corrupted by noise is to reduce the measurement bandwidth. For dc measurements, this is usually accomplished by putting the signal through a low-pass filter. In a simple measurement such as checking the voltage of a battery, there is an implied low frequency limit to the measurement which is on the order of the reciprocal of the time duration of the measurement. If one measures the battery voltage for 1 second, the data contain little information on how the battery voltage varies over a 1000 second period. It makes little sense to filter a 1 second measurement through a 0.001 Hz filter; the measurement itself becomes just high frequency noise that is removed by the filter. Thus all measurements are in fact made through a bandpass filter. The least noise corruption of the measurement comes from the minimum bandpass that can be used consistent with the frequency content of the signal.

Direct current measurements and some very low frequency measurements are affairs of the mind. Because of the very long time scales involved, one is often inclined to think of them differently from events occurring at frequencies of approximately 1 Hz or above. They are not different. In the case of the gyroscope experiment, the roll frequency of the satellite, and hence the primary frequency contained in the readout signal, is 0.0017 Hz, but the same signal-processing rules and techniques applicable at 1 kHz or 1 MHz are applicable there as well. Therefore the noise on the readout signal can be reduced by passing it through a bandpass filter centered on the satellite roll frequency with as narrow a passband as possible. The minimum bandwidth is the reciprocal of the observation time, which in this case is the one year hold-time of the dewar. Conceptually, it is meaningful to calculate the frequency or observation time to reach a given resolution, 0.001 seconds of arc being the obvious choice here.

In most SQUID applications, the SQUID input coil can be treated as a lossless inductance. As such, it draws no signal power but it does store signal energy. The input of a dissipative device draws signal power, and the noise of such a device is referred to the input in units of power/unit bandwidth. Correspondingly, when the input is an energy, the noise is in units of energy/unit bandwidth, hence the units in figure 2. The energy in the SQUID input coil is

$$E = \frac{1}{2}\left(\frac{L_i I_p^2}{2}\right) \quad , \tag{4}$$

where L_i is the SQUID input inductance. The additional factor of one-half comes from the fact that the rolling of the satellite produces a sine wave from a dc signal. The bandwidth B to detect this signal is found simply from

$$B = \frac{E}{e_n} \quad , \tag{5}$$

where e_n is the energy noise density at the frequency of interest.

In figure 3 is shown a realization of the readout circuit with typical circuit values. The addition of the filter resistor is of practical rather than fundamental importance. It is needed to remove high frequency magnetic noise which otherwise would impair the operation of the SQUID system. Signals for which the inductive reactance of the readout circuit is greater than the resistance of the filter resistor are shunted by the resistor. The source of the noise is suspension system currents at 20 kHz, 1 MHz, and their harmonics in and around the gyroscope. Laboratory versions of the readout have used a transformer and "damping cylinder" to inductively couple the pickup loop to the SQUID. The damping cylinder is a resistive cylinder interposed between the two windings of the transformer. The induced currents flowing in the cylinder at high frequencies shield the transformer winding attached to the SQUID, but at low frequencies they are damped by the resistance of the cylinder, which allows the dc and low frequency signals to pass. Improvements to the suspension electronics will eliminate the need for this relatively inefficient filtering method.

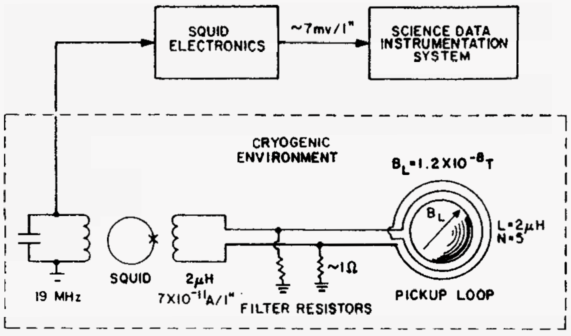

FIGURE 3. Typical circuit values for a practical implementation of the London moment readout.

Using the values given in figure 3 and equations (1) through (4), the energy in the SQUID input coil for a gyroscope angle of 0.001 seconds of arc is 2.5×10^{-33} J. From curve A of figure 2, the SQUID energy noise density at 0.0017 Hz is 1.6×10^{-27} J/Hz; since the bandwidth will be small, this can be treated as a constant over the bandwidth of interest. The required bandwidth is 1.6×10^{-6} Hz. The reciprocal of B gives the observation time. In fact, since we presume to know the phase of the relativity signal (the satellite roll phase), the noise in the out-of-phase component can be rejected so that the observation time is reduced by one-half. The approximate observation time is thus 89 hours.

The choice of curve A of figure 2 for the noise data as a baseline is guided by the considerable operational experience that has accumulated for that SQUID. One prefers to choose mature components for a space mission where possible. However, the quality of the experiment, system performance, and perhaps experiment content are enhanced by having a SQUID sensor of considerably lower noise. As shown in figure 2, lower noise SQUID's are available. With the best of these in the 1/f noise region, the IBM SQUID (curve D of figure 2), the observation time to reach 0.001 seconds of arc is 12 seconds! At present, most of the advanced SQUID's are either more complex to operate, suitable only for specialized applications, or not generally available, but we anticipate the eventual incorporation of advanced SQUID technology into the gyroscope readout.

That one can actually measure such a small signal is shown in figure 4 [11,12]. The prominent peak is the simulation of a gyroscope signal obtained by producing in a SQUID the same magnetic flux as would be produced by the current in a single-turn readout loop for a 0.020 seconds of arc gyroscope angle and a satellite roll rate of 0.017 Hz (1 minute period). The amplitude of the noise at frequencies away from the roll frequency is ~ 0.0007 seconds of arc rms. Data were taken for 140 hours and analyzed

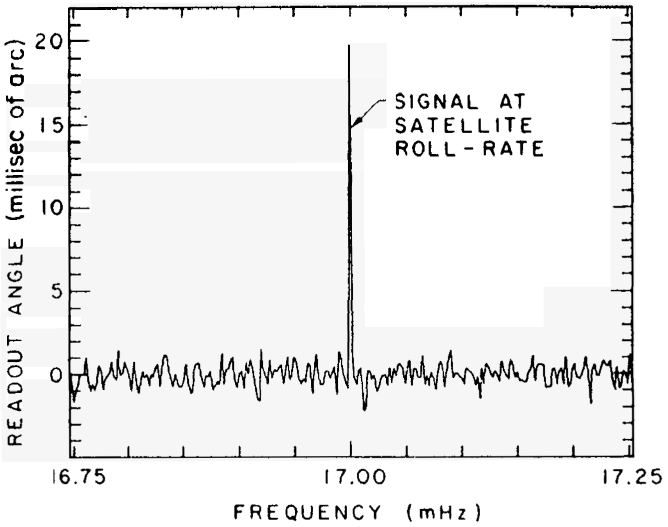

FIGURE 4. Noise spectrum of the SQUID output for a signal input corresponding to that from the readout loop for a 0.020 seconds of arc simulated gyroscope signal. The amplitude of the noise, 0.0007 seconds of arc rms, at frequencies away from that of the signal, is a measure of the sensitivity at the signal frequency.

to recover the signal. The noise level and observation time are in agreement with equation (5) and the observed noise spectrum of the SQUID used in this test.

Other methods have been suggested to improve the sensitivity of a magnetic gyroscope readout. Everitt has examined the use of a spinning ferromagnetic material [13], which produces a magnetic moment known as the Barnett moment. Although the field could be relatively large, the presence of the ferromagnetic material in the pickup loop would act to attract stray magnetic fields into the loop, thereby degrading the signal-to-noise ratio. The London moment, while small, is generated in a diamagnetic material which provides shielding of the pickup loop. Hendricks has considered a low-inductance pickup loop consisting of a relatively wide band close to the gyro rotor that includes the SQUID [14]. There are technical problems in integrating the SQUID with the gyro, and possibly in null stability. One approach that appears promising is the use of trapped flux in the gyro rotor. In such a case, the flux is now locked to the rotor surface rather than to the spin axis, so the operation is somewhat different from that of the London moment readout. The primary advantage is that by choosing to observe the rotating component of the trapped field, the readout can take advantage of the much lower SQUID noise at the gyroscope rotation speed of 170 Hz instead of the SQUID noise at the satellite rotation speed of 0.0017 Hz. Perhaps a larger field could be used than that provided by the London moment, but other considerations may limit its size. Assuming the use of one of the advanced SQUID's, observation times of minutes instead of tens of hours would be needed to reach 0.001 seconds of arc. At the present level of understanding, the ac trapped flux readout requires complex signal processing. However, the possibility of an exceptional improvement in sensitivity demands that it be studied further. Some operational experience will be gained through its use in the gimbaled gyroscope test apparatus now being built.

4. TORQUES INDUCED ON THE GYROSCOPE BY THE READOUT

A measurement, whether quantum limited or macroscopic, alters the state of the system being measured. In the former the Heisenberg uncertainty principle offers a guide to the ultimate accuracy of the measurement, while in the latter such pedestrian effects as the current drawn by a voltmeter will influence the measurement. In the case of the gyroscope, the perturbation of interest is the torque the readout causes. The source of the torque is the interaction between the London moment of the gyroscope and the magnetic moment of the readout loop from any current flowing in it. Given a maximum drift rate of 0.001 seconds of arc/year, the maximum torque from all sources must be less than $\Gamma_{max} = 1.5 \times 10^{-18}$ N m. For purposes of illustration, we will allocate all of this to the readout. The torque Γ_l on the gyroscope from a current I_l in the readout loop is given by

$$\Gamma_l = \pi r_0^2 I_l B_L \sum_i^N \frac{r_0}{r_{li}} \cos\theta \tag{6}$$

If I_l is an ac current, proper attention must be paid to the phase. Since the satellite is rolling, a trapped current in the readout loop produces an alternating torque on the gyroscope, and no net drift. The readout current itself does, however, produce a drift. While the current through the SQUID alternates polarity as the satellite rolls, the magnetic field generated by the current is always in opposition to that of the gyroscope. For gyroscope angles of less than 1600 seconds of arc, the torque produced by the readout current will be less than Γ_{max}, a factor of 32 greater than the angular range required by the experiment. As shown in figure 3, the readout will use current feedback to the readout input circuit, reducing the readout circuit current and thereby reducing the torque on the gyroscope from this source by the loop gain of the feedback system. It can be shown that the readout reaction torque is the angular derivative of the energy in the readout circuit. Therefore any method of increasing readout sensitivity that relies upon increasing the signal in the readout loop also increases the readout reaction torque. However, the use of current feedback will, for most situations, reduce the torque to acceptable or negligible values. Those methods that rely upon improving the sensor noise do not generally influence the torque.

Trapped flux in the gyroscope rotor provides another source of torque. The flux rotating with the rotor within the pickup loop acts an an ac generator. Whereas the dissipation of the readout circuit at dc is zero, it is not generally so above dc. The filter resistor shown in figure 3 provides the largest source of dissipation, but most SQUID's have resistive shunts built into them either for L/R filtering or to prevent Josephson junction hysteresis. Torques from this source fall into a broad category known as differential damping torques, which act selectively on different components of the gyroscope spin. In this case, there is no torque generated by the trapped flux rotating around the component of spin lying perpendicular to the plane of the pickup loop.

For small angles, the drift from ac trapped flux is given by:

$$\Omega = \frac{15}{64} \frac{\left(B_{ptf} \sum_i^N r_0/r_{li}\right)^2 F(f_r, f_0)}{\rho r_0^5 L_{tot}} \theta \quad , \tag{7}$$

where B_{ptf} is the average perpendicular component of the trapped flux, and ρ is the rotor density. $F(f_r, f_0)$ contains the frequency dependence, and is given by

$$F(f_r, f_0) = \frac{1}{f_r} \frac{(f_r/f_0)}{1 + (f_r/f_0)^2} \quad . \tag{8}$$

If the trapped field in the rotor differs significantly from that of a dipole, the additional harmonic content of the trapped flux signal increases the drift by an amount that is found by summing the contributions from each of the harmonics. For readout loop configurations under consideration, trapped fields ~ 50 times the London moment field would cause unacceptable drift, but this is many orders of magnitude larger than any that could be tolerated on other grounds. Current feedback reduces the drift by the square of the feedback gain, so that this source of torque is also negligible.

In practice, none of the sources of torque from the readout appear to present any difficulty.

Acknowledgments

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