OBSERVATION OF THE LONDON MOMENT AND TRAPPED FLUX IN PRECISION GYROSCOPES*

Y.M. Xiao, W. Felson, C.H. Wu, G.M. Keiser and J.P. Turneaure
W.W. Hansen Experimental Physics Laboratory
Stanford University, Stanford, CA 94305, USA

Abstract--Precision gyroscopes are under development for an experimental test of general relativity. The spherical gyroscopes are made of fused quartz and are sputter coated with a Nb film. The superconducting Nb film generates the London magnetic dipole moment proportional to the spin speed that is used to magnetically readout the direction of the gyro spin axis. Here, we report observations of the London moment of an operating gyroscope consistent with the expected magnitude and proportional to the spin speed. The magnetic flux trapped in the film interferes with the London-moment readout unless its magnitude is significantly smaller. Values of the dipole component of the equivalent trapped magnetic field as low as 1.5×10^{-11} T have been observed.

I. INTRODUCTION

In 1960, L.I. Schiff [1] predicted, based on Einstein's general theory of relativity, that a gyroscope in orbit about the earth would experience a precession of its spin axis relative to the "fixed stars", see Fig. 1. The expected precession contains two components, commonly referred to as the geodetic precession and the frame-dragging precession. For a gyroscope in a 650 km altitude polar orbit with its spin axis initially pointed towards a reference star lying in the orbit plane and with a declination within a few degrees of zero, the geodetic precession rate is 6.6 arc-sec/yr about the normal to the orbital plane, and the frame-dragging precession rate is 0.042 arc-sec/yr about the earth's rotation axis. Measurements of these precession rates are expected to provide one of the most accurate tests of the theory of general relativity [2]

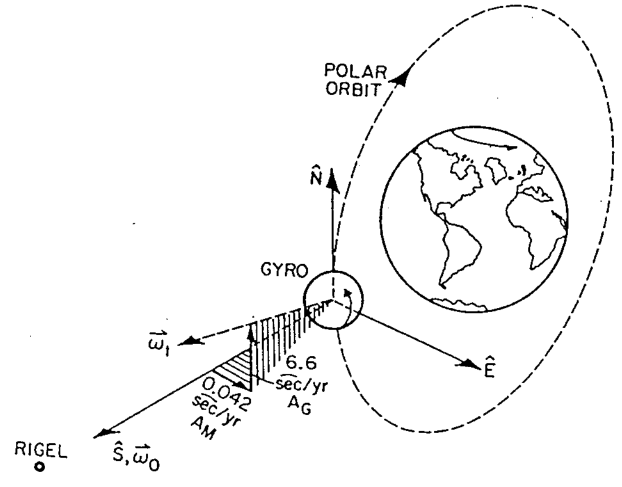

Fig. 1 Diagram Illustrating the Precessions due to General Relativity of a Gyroscope in Polar Orbit about the Earth.

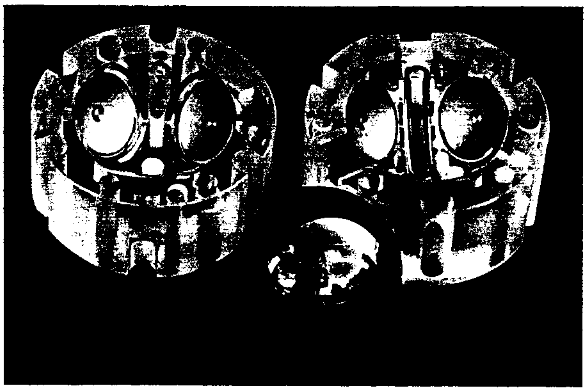

Fig. 2 Photo of Gyroscope and its Housing.

Four gyroscopes are the heart of the satellite instrument designed for measuring the geodetic and frame-dragging precession rates. Gyroscopes whose absolute Newtonian drift rates are less than 10^{-4} arc-sec/yr, when operated in a drag-free satellite, are being developed [3]. In this paper, we report ground-based measurements of the London magnetic dipole moment in a flight-type gyroscope with very-low trapped magnetic flux.

II. THE GYROSCOPE

Figure 2 is a photograph of the gyroscope and its housing. The gyroscope is a 38 mm diameter sphere of homogeneous fused quartz with a uniform layer of Nb coated on its surface. The coating thickness is typically 2.5 μm. The gyroscope is assembled inside of a housing composed of two halves also made of fused quartz. Each housing half has three pairs of electrodes for electrostatic suspension of the gyroscope and raised support lands around each electrode to prevent electrical contact (when not suspended) between the gyroscope and the electrodes. In addition, the readout half has a thin-film Nb pickup loop located on its parting plane; and the spin-up half has a spin-up channel with an inlet and an outlet for He gas spin up of the gyroscope.

During operation, the gyroscopes are cooled to 2 K. The gyroscopes are electrostatically suspended and then spun up to about 170 Hz. Two characteristics of the superconducting Nb play important roles in the performance of the gyroscope. First, the Nb coating makes an ideal equipotential surface for the electrostatic suspension; secondly, the London magnetic dipole moment provides a means to read out the direction of the gyro spin axis.

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III. LONDON-MOMENT READOUT

When a superconducting sphere of radius R is spun up at a temperature below its superconducting transition temperature, a magnetic dipole moment aligned with the spin axis and proportional to the spin frequency is generated. The magnetic dipole moment is commonly referred to as the London moment. Within the sphere, the magnetic field B_L is uniform in the direction of the gyro spin vector and has a magnitude given by

$$B_L = (2m/e)\,\omega_g = 1.14\times10^{-11}\,\omega_g \qquad (1)$$

where ω_g is the gyro angular frequency, m and e are the electron mass and charge. For a gyroscope with an spin frequency of 170 Hz, B_L is 1.22×10^{-8} T.

The London magnetic dipole moment is used to determine the direction of the gyro's spin axis as shown in Fig. 3. The magnetic flux through the pickup loop is sensed by a dc SQUID magnetometer. For the planned satellite configuration, the gyro's spin axis is initially aligned within a few arc-sec of the line-of-sight to the reference star. The plane of the pickup loop is also aligned within a few arc-sec of the line-of-sight to the reference star. The entire satellite and pickup loop are rolled about the line-of-sight to the reference star with a 600 s period. The magnetic flux Φ through the pickup loop, which is modulated at the roll frequency $\omega_r = 1.67$ mHz, is

$$\Phi = \Phi_0 + C_G B_L[(NS+A_G t)\sin\omega_r t+(EW+A_M t)\cos\omega_r t] \qquad (2)$$

where C_G is a constant scale factor, A_G and A_M are the geodetic and frame-dragging precession rates, and NS and EW are the initial positions of the spin axis relative to the reference star. The dc SQUID converts the flux modulation at its input into a voltage signal.

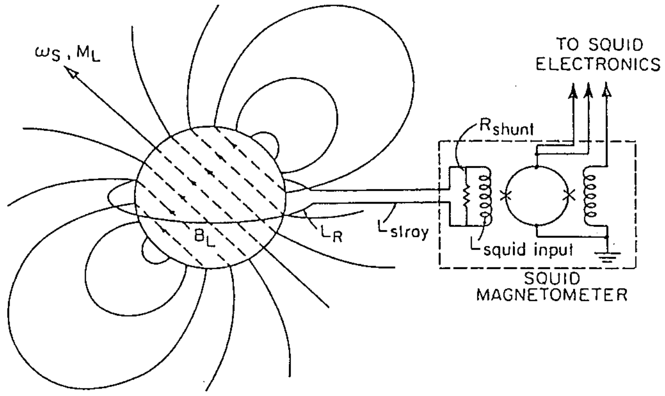

Fig. 3 Schematic Diagram of the London-Moment Readout.

The noise in the dc SQUID of the gyro readout system is the dominant source of noise in the data analysis for finding the geodetic and frame-dragging precession rates and their uncertainties. Since the gyro readout signal appears at the input of the dc SQUID at roll frequency, the SQUID noise at 1.67 mHz is the relevant noise quantity. For the baseline gyro pickup loop configuration and for measured dc SQUID noise at 1.67 mHz, a gyro angle of 10^{-3} arc-sec can be resolved in 5 hr of integration time [4,5]. Covariance analyses using dc SQUID noise of this magnitude and other less important noise sources indicate that the geodetic and frame-dragging precession rates will have an uncertainty of less than 0.01% and 1%, respectively in a 1 yr experiment [6].

The London moment for hollow Be gyroscopes has been observed in previous experiments [7]. Here we report ground-based measurements of the London moment for fused quartz gyroscopes of the flight design.

IV. GROUND TEST

Experiments were carried out in a unique ground test facility which was designed to provide the conditions necessary to observe the London moment of the spinning gyroscope. This facility provides the gyroscope an ambient field of about 1×10^{-11} T, similar to that of the flight experiment. This low ambient field is produced by first expanding a folded superconducting Pb foil shield within a normal magnetic shield and then expanding successive superconducting Pb shields within the previous shield [4]. The tests are typically carried out at a temperature of about 4.2 K and in a pressure of about 2.5×10^{-5} Pa. Both the temperature and the pressure of the ground test are higher than that of the flight experiment. Since we are interested primarily in the London moment and the trapped flux, we consider these conditions adequate. Three sets of pickup loops mounted outside of the gyro housing were used to detect the magnetic signals generated by the gyroscope. The axes of the three pickup loops were mutually orthogonal to each other. For the ground tests, the pickup loops are fixed in the laboratory frame. Two of the three pickup loops were in the Helmholtz configuration with their axes in the horizontal plane. The third pickup loop was configured to sense only the quadrupole and higher multipoles of the magnetic field associated with the gyroscope. The mounting structure and other experimental details are similar to our previous work [8].

Previous tests have demonstrated that at moderate gyro spin frequency (10 Hz), the motion of the gyroscope in the laboratory is dominated by the rotation of the earth. To reduce the effects of drift in the SQUID output, the thickness of the sputter deposited Nb film on the gyroscope was deliberately increased on one side of the sphere. This additional Nb film increases the magnitude of the mass unbalance (the vector between the center of geometry and the center of the mass) to 1.4 μm for the gyroscope used in the ground-test experiments reported here. If the mass unbalance vector is aligned with the gyro spin vector, then this mass unbalance will cause the gyroscope to precess about the local vertical at a rate of 300 ^{o}/hr at a 10 Hz spin frequency. To further simplify the data analysis, the initial direction of the gas spin-up torque is placed in the horizontal plane. In this configuration, the flux through the two pickup loops whose axes are in the horizontal plane is expected to be sinusoidal in each loop at the gyro's precession frequency. The amplitudes in the two loops are proportional to the London moment and hence also to the gyro spin frequency. Since the axes of the two pickup loops are perpendicular to one another, the two signals at the precession frequency are expected to be 90° out of phase.

The data acquisition was computer controlled. The outputs of the magnetometers were directly connected to A/D converters. For the data presented below, the sampling rate was at 256 Hz. For every 10 s, about 8 s of data were sampled and analyzed. Part of the data analysis was carried out on-line by the acquisition program. The on-line analysis includes converting the data to the Fourier domain with an FFT, searching signals above the noise floor of the Fourier spectrum, and interpolating between the bins of the FFT to find the amplitude of the spin frequency signal and the harmonics of the spin frequency. From each 8 s of data, one obtains the dc average value, the gyro spin frequency, and the amplitudes and phases of the signals at harmonics of the gyro spin frequency.

The electrostatic suspension system senses the position of the rotor using a 1 MHz capacitance bridge and supplies the high voltage to each of the six electrodes as required to keep the gyroscope suspended. On the ground, the dc charge needed to support the gyroscope is about 6×10^{-8} C and the current needed to sense the position of the gyroscope is about 1 mA. Current flowing into each of the six electrodes induces currents in the pickup loops, which, in turn, are sensed by the SQUIDs. In addition, electrostatic coupling between the suspension system and the input circuit to the SQUIDs can exist. For these reasons, a superconducting transformer, with a copper shield between the primary and the secondary, was used between the pickup loop and the input of the SQUID. The copper shield attenuated the high frequency signals and provided an electrostatic shield for the input circuit of the SQUID. Shunt resistors across the input terminals of the SQUID were also used to further attenuate the high frequency signals. Even so, the SQUIDs were operated near their slew rate limit and the SQUIDs' low-frequency noise and drift are significantly higher than when the gyro suspension system is off. The white noise (above 0.2 Hz) of the magnetometers is about 7×10^{-12} T/Hz$^{1/2}$, limited by the dissipation in the copper shield and the shunt resistance.

A typical set of data obtained in a recent 20 hr test is shown in Fig. 4. The gyro spin frequency, see Fig. 4A, was intentionally damped for this test so that the dependence of the London moment on the gyro spin frequency could be demonstrated within a reasonable time. Figures 4B and 4C show the dc average values from each 8 s burst of data for the two horizontal pickup loops. The two signals are 90° out of phase with one another, have an amplitude which decreases with gyro spin frequency, and a period which decreases inversely with the gyro spin frequency. A preliminary analysis shows that the data are consistent with the expected London-moment signal from a gyroscope which is precessing about the local vertical with an average mass unbalance parallel to the gyro spin axis of 0.58 μm. This result is consistent with the total mass unbalance determined from coating thickness measurements of 1.4 μm and an average orientation of the gyro spin axis with respect to the mass unbalance of 65°. This preliminary analysis and a preliminary calibration of the scale factors of the SQUIDs show that the magnitude of the horizontal component of the equivalent London-moment field is 60% of the total value of the expected equivalent London-moment field. A comparison of the measured and expected magnitudes of the London-moment requires an accurate determination of the orientation of the gyro spin axis with respect to the local vertical.

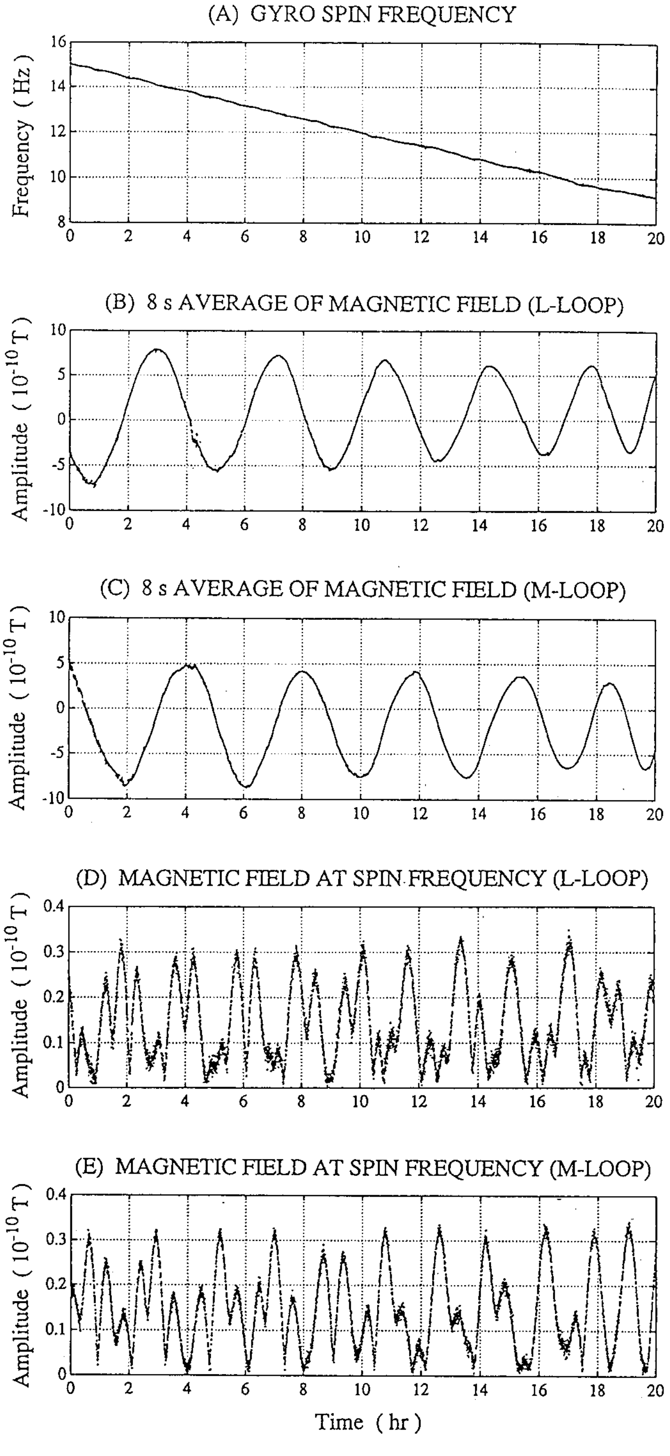

Fig. 4 Spin Frequency and Magnetic Signals of a Superconducting Gyroscope. The magnetic signals are observed with two perpendicular pairs of Helmholtz coils (L-loop and M-loop) whose axes lie in the horizontal plane. The plotted quantity is the magnetic field equivalent to a uniform field within the spherical gyroscope.

The absolute amplitudes of the spin frequency signals from each burst of data are presented in Figures 4D and 4E. Note that the vertical scale of these two figures is 1/50 as large as that of Figures 4B and 4C. Assuming that the signals at the spin frequency are due to the dipole component of the trapped magnetic field, the amplitude and phase of these signals may be used to derive the orientation of the gyro spin axis in the laboratory, the orientation of the spin axis relative to the dipole moment, and the magnitude of the trapped magnetic field. This analysis of the trapped magnetic field is in progress and should provide more detailed quantification of the gyro dynamics and the London moment.

The variation in the trapped flux signal is due to the polhode motion and the precession. Note that the magnitude of the spin frequency signal goes to a minimum whenever the corresponding dc signal is at a maximum or mininum. At these points the gyro spin axis is nearly aligned with the normal to the pickup loop. In addition, there is a periodic signal with a period which slowly decreases with the gyro spin frequency. This signal is due to the polhode motion and indicates that the fractional difference in the moments of inertia is approximately 1.4×10^{-5} . The procedure used for coating the gyroscope will produce a mass unbalance which is aligned with the principal axis corresponding to the smallest moment of inertia. The other two moments of inertia should be approximately equal. Since the precession rate indicates that the gyro spin axis lies at an angle of 65° from the mass unbalance vector, the difference in moments of inertia cause the gyro spin axis to move through an angle of 130° in the gyro body frame. At some point in this polhode motion, the gyro spin axis will be perpendicular to the dipole component of the trapped magnetic field. Therefore, the peak values of the data in Figures 4D and 4E are equal to the magnitude of the dipole component of the equivalent trapped magnetic field. This magnitude is 3×10^{-11} T.

A magnetic field trapped in the gyroscope has been observed for all the tests carried out so far, including tests with gyroscopes having a copper coating over the Nb film [8]. The trapped field depends on many experimental details, including the ambient magnetic field and the process by which the Nb film goes through the normal to superconducting transition. Typically, the minimum observed trapped field is on the order of the ambient field. Since the trapped field could significantly complicate the data analysis and the instrumental design of the flight experiment, much effort has been devoted to reducing the amplitude of the trapped field. The materials which are used close to the gyroscope are carefully checked for residual magnetization and magnetic impurities. The amount of electrically conducting material close to the gyroscope is kept to a minimum since temperature gradients in electrically conducting materials may generate magnetic fields. In addition, the gyroscope is slowly cooled through its superconducting transition temperature. This operating procedure now routinely limits the trapped field to levels below 5×10^{-11} T. The lowest trapped field observed in the gyroscope so far is 1.5×10^{-11} T.

VI. CONCLUSION

The London-moment readout has been observed in flight quality gyroscopes and it has been demonstrated that it is possible to reduce magnetic field trapped in these gyroscopes to levels as low as 1.5×10^{-11} T. A preliminary analysis shows that the horizontal component of the London-moment signal is 60% of the total expected London-moment signal and is proportional to the gyro spin speed. Further analysis and experimental results will allow us to determine the absolute magnitude of the London moment, to study methods for further reducing the trapped magnetic flux, to determine the stability of the London moment and the trapped magnetic flux over periods as long as one year.

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