Relativity at the centenary

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When I was a first-term graduate student in the late 1960s, it was said that the field of general relativity was “a theorist’s paradise and an experimentalist’s purgatory”. There were some experiments – Irwin Shapiro, for instance, had just measured the effects of general relativity on radio waves as they passed the Sun – but the field was dominated by theory and by theorists. This seemed to reflect Einstein’s own attitudes: although he had a keen insight into the workings of the physical world, he felt that the bottom line was the theory. As he once famously said, when asked how he would have reacted if an experiment had contradicted the theory, “I would have felt sorry for the dear Lord. The theory is correct”.

Since that time the field has been completely transformed. Today, at the centenary of Einstein’s anno mirabilis, experiment has become a central component of gravitational physics. I know of no better way to illustrate this than to cite a paper by the LIGO Scientific Collaboration that was published in Physical Review D last year (see Abbott et al, in further reading). This was one of the papers reporting results from the first science run of the Laser Interferometer Gravitational-wave Observatory (LIGO), but with 374 authors from 41 institutions in 8 countries it is reminiscent of particle physics, not general relativity.

The breadth of current experiments – ranging from tests of classic general relativity such as the Shapiro delay and the bending of light, through space-based measurements of “frame-dragging” to searches for gravitational waves or violations of the inverse-square law – attests to the ongoing vigour of experimental gravitation. With all this data, can we still be sure that Einstein was right?

Testing the foundations

At the heart of the general theory of relativity is the equivalence principle – an idea that came to Einstein two years after he developed special relativity and led him to the dramatic conclusion that mass and gravity are intimately linked to the curvature of space–time (see figure 1 and box on page XX).

Put in simple terms, the equivalence principle states that gravity and acceleration are equivalent. Embellished over the years, this idea is now called the Einstein equivalence principle and encompasses three separate principles: the weak equivalence principle, and the principles of local Lorentz and local position invariance.

The weak equivalence principle states that test bodies fall with the same acceleration independent of their internal structure or composition: in other words gravitational mass (the m in $F=Gm^2/r^2$), where $F$ is the gravitational attraction between two masses a distance $r$ apart and $G$ is the Newtonian gravitational constant) and inertial mass (the $m$ in $F=ma$, where $a$ is the acceleration caused by any force $F$) are the same. There is also a strong version of the equivalence principle that goes beyond the weak version by stating that gravitational energy will fall with the same acceleration as ordinary matter and other types of energy in a gravitational field (see box on page YY).

The principle of local Lorentz invariance states that the outcome of any local non-gravitational experiment carried out in a freely falling reference frame is independent of the velocity of that frame, while the principle of local position invariance holds that the outcome of any local non-gravitational experiment is also independent of where and when in the universe it is performed. In this context “local” means confined to a suitably small region of space and time, while “freely falling” means falling freely under gravity with no other forces acting.

Although Einstein used it to derive general relativity, his equivalence principle implies only that gravitation must be described by a “metric theory” – a theory in which matter responds to the geometry of space–time and nothing else. However, general relativity is not the only metric theory of gravity, and other examples include the “scalar–tensor” theory developed by Carl Brans and Robert Dicke at Princeton University in 1961, building on earlier work by Markus Fierz and Pascual Jordan.
When Einstein introduced the concept of “relativity” in 1905 – the notion that there is no absolute motion in the universe, only relative motion – he overthrew ideas that had been in place since the time of Newton some 200 years before. In addition to $E=mc^2$, special relativity predicted various novel effects that occurred when bodies moved at close to the speed of light: time slowed down (an effect known as time-dilation) and lengths became shorter (Fitzgerald contraction). With the general theory Einstein then went on to show that we do not reside in the flat (Euclidean) space and uniform time of everyday experience, but in curved space–time instead.

Special relativity helped us to understand the microworld of elementary particles and interactions, while general relativity revolutionized our view of the universe by predicting astrophysical phenomena as bizarre as the Big Bang, neutron stars, black holes and gravitational waves.

The theory of relativity is a single, all-encompassing theory of space–time, gravity and mechanics, although special relativity and general relativity are often viewed as being independent. Special relativity is actually an approximation to curved space–time that is valid in sufficiently small regions called “local freely falling frames”, much as small regions on the surface of an apple are approximately flat, even though the overall surface is curved.

Einstein’s great insight was to realize that gravity and acceleration are equivalent in free fall, and he then went on to show that the laws of physics, such as the equations of electromagnetism, should have built-in local Lorentz and local position invariance.

In special relativity the “distance” between two points in space–time is given by the line element, $ds^2 = c^2 dt^2 + dx^2 + dy^2 + dz^2$, where $t$ is time and $c$ is the speed of light in a vacuum. In the curved space–time of general relativity $ds^2 = g_{\mu \nu} dx^\mu dx^\nu$, where $x^1, x^2$ and $x^3$ are the three spatial dimensions, $x^0 = ct$, and $g_{\mu \nu}$, which is called the metric, is a function in space–time. The right-hand side of the equation must be summed over all values of $\mu$ and $\nu$ between 0 and 3.

General relativity provides a set of field equations that allow us to calculate the space–time metric (i.e. the amount of curvature) from a given distribution of matter – something that is not defined by the equivalence principle. Einstein’s aim was to find the simplest field equations that made this possible. The result was a set of 10 equations, symbolized by the seductively simple equation $G_{\mu \nu} = 8\pi T_{\mu \nu}/c^4$, where $G_{\mu \nu}$ is Einstein’s curvature tensor, which can be obtained directly from $g_{\mu \nu}$ and its derivatives, and $T_{\mu \nu}$ is the stress-energy tensor of normal matter. Sweating the details hidden in this equation has kept generations of relativists occupied.

In the past it was customary to speak of the three classical tests proposed by Einstein: the deflection of light by a massive body; the advance of the perihelion of Mercury; and the gravitational redshift of light (although this is actually a test of the Einstein equivalence principle rather than general relativity itself). Many new tests have been developed since Einstein’s time: in 1964 Irwin Shapiro, then at the Massachusetts Institute of Technology, predicted a delay in the propagation of light past a massive body; and in 1968 Kenneth Nordtvedt Jr of Montana State University showed that theories other than general relativity do not necessarily obey the equivalence principle in certain situations. One of the most striking predictions of general relativity is the black hole: when a massive star collapses under its own gravity it can warp space–time to such an extent that nothing, not even light, can escape. There is now convincing observational evidence for these objects.

One of the outstanding problems in physics is to unify general relativity, which is our best theory of gravity, with the quantum field theories that describe the three other fundamental forces. Although this challenge defeated Einstein, it should not surprise us that all the leading candidates for a unified theory – string theory, branes and loop quantum gravity – are all fundamentally geometrical.

When it comes to testing metric theories of gravity, we need to distinguish between the weak-field limit, which is valid in the solar system (see figure 2 and box on page 22), and the strong-field regime that is needed to describe regions where gravity is extremely strong, such as in the vicinity of a black hole or a neutron star. If we are being really ambitious, we might also try to describe situations where gravity is strong and quantum effects are important, such as during the Big Bang, but that is a separate story (see “Welcome to quantum gravity” Physics World November 2003 pp27–47).

In non-metric theories matter can respond to something other than the geometry of space–time, and this can lead to violations of one or more pieces of the Einstein equivalence principle. For instance, in the string theories that seek to unify gravity with the other three forces of nature, the equivalence principle is violated because matter can respond to additional long-range fields. Searching for violations of the Einstein equivalence principle is therefore a good way to search for new physics beyond the standard metric theories of gravity.
A completely different test of the weak equivalence principle involves bouncing laser pulses off mirrors on the lunar surface to check if the Earth and the Moon are accelerating toward the Sun at the same rate. Lunar laser-ranging measurements actually test the strong equivalence principle because they are sensitive to both the mass and the gravitational self-energy of the Earth and the Moon. The bottom line of these experiments is that bodies fall with the same acceleration to a few parts in $10^{13}$ (see figure 1).

In the future, the Apache Point Observatory for Lunar Laser-ranging Operation (APOLLO) project, a joint effort by researchers from the University of Washington in Seattle and the University of California at San Diego, will use enhanced laser and telescope technology, together with a good, high-altitude site in New Mexico, to improve the lunar laser-ranging test by as much as a factor of 10 (see Williams et al. in further reading and Physics World, June 2004 p.9).

The next major advance may occur in space, if two satellite missions are successful. MICROSCOPE, which could be launched in 2008, aims to test the weak equivalence principle to 1 part in $10^{15}$, while a later mission called the Satellite Test of the Equivalence Principle (STEP) could improve on this by a factor of 1000. These experiments will compare the acceleration of different materials moving in free-fall orbits around the Earth inside a drag-compensated spacecraft. Doing experiments in space means that the bodies are in perpetual fall, whereas Earth-based experiments at “drop towers” are over in seconds, which leads to much larger measurement errors.

Many of the techniques developed to test the weak equivalence principle have been adapted to search for possible violations of the inverse-square law of gravity at distances below 1 mm. Such violations could signal the presence of additional interactions between matter or “large” extra dimensions of space. No deviations from the inverse-square law have been found at distances between 100 μm and 10 mm, but there are enough well-motivated theoretical predictions for new effects at these distances to push experimentalists towards better sensitivities and shorter distances.

**Tests with atomic clocks**

The predictions of general relativity can also be tested with atomic clocks. Local position invariance requires that the internal binding energies of all atoms, and thus the time given by atomic clocks, must be independent of their location in both time and space when measured in a local freely falling frame. However, if two identical atomic clocks are placed in different gravitational potentials, they will be in different local frames and, according to the Einstein equivalence principle, they will give slightly different times.

In 1976 Robert Vessot, Martine Levine and co-workers at the Harvard Smithsonian Astrophysical Observatory and the Marshall Space Flight Center compared a hydrogen maser clock on a Scout rocket at an altitude of 10,000 km with one on the ground, and verified Einstein’s 1907 prediction for this “gravitational redshift” to a few parts in $10^{11}$. This redshift actually has an impact on our daily lives because it must be taken into account (along with the time dilation associated with special relativity) to ensure that navigational devices that rely on the Global Positioning System (GPS) remain accurate. Relativistic effects mean that there is a 39 ms per day difference between ground-based atomic clocks and those on the GPS satellites.

Recent clock-comparison tests of local position invariance at the National Institute of Standards and Technology (NIST) in Boulder, Colorado, and the Observatory of Paris have shown that the fine-structure constant – which determines how fast the atomic clocks “tick” – is constant to 1 part in $10^{15}$ per year. The NIST team compared laser-cooled mercury ions with neutral caesium atoms over a two-year period, while the Paris team compared laser-cooled caesium and rubidium atomic fountains over five years. Plans are being developed to perform such clock comparisons in space, possibly on the International Space Station.

Atomic clocks can also be used to test the two pillars of special relativity – Lorentz symmetry and position invariance. At the centenary of special relativity, it is useful to recall that acceptance of this theory was slow in coming – Einstein’s 1921 Nobel Prize was for the photoelectric effect, another of his 1905 triumphs, not for relativity. However, special relativity is now such a foundation for modern physics that it is almost blasphemy to question it, although that has not stopped a growing number of theoretical and experimental physicists searching for violations of Lorentz and/or position invariance (see “A very special centenary” on page XX). In earlier times, such thinking would have been called “crackpot”, but these new ideas are well rooted in attempts to find a quantum theory of gravity and, ultimately, a unified theory of
Einstein became a public celebrity when Arthur Eddington and colleagues measured the deflection of light by the Sun during the solar eclipse of 1919 and found that their results agreed with the predictions of general relativity. Measurements of the deflection (top) – plotted as $(1 + \gamma/2$, where $\gamma$ is related to the amount of spatial curvature generated by mass – have become more accurate since 1919 and have converged on the prediction of general relativity: $(1 + \gamma/2 = 1$. The same is true for measurements of the Shapiro time delay (bottom). "Optical" denotes measurements made during solar eclipses (shown in red), with the arrows pointing to values well off the chart; "radio" denotes interferometric measurements of radio-wave deflection (blue); while Hipparcos was an optical-astrometry satellite. The left-most data point is the measurement made by Eddington in 1919, while the arrow just above it refers to the value obtained by his compatriot Andrew Crommelin (see “Einstein and the eclipse” on page XX). The best deflection measurements (green) are accurate to 2 parts in $10^7$ and were obtained with Very Long Baseline Radio Interferometry (VLBI); see Shapiro et al. in further reading). A recent measurement of the Shapiro time delay by the Cassini spacecraft, which was on its way to Saturn, was accurate to 1 part in $10^9$ (see Bertotti et al. in further reading).

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Special relativity and $E = mc^2$ tell us that energy and mass are essentially the same. The mass of a proton and an electron is greater than that of a hydrogen atom because energy must be supplied to break the electromagnetic bond in the atom. The weak equivalence principle asserts that this difference will change both the gravitational mass and the inertial mass by the same amount. This means that all forms of energy at microscopic scales – electromagnetic, strong and weak – respond to gravity in the same way. But what about large bodies like the Earth and Sun, or even extreme gravitational bodies like black holes, which also have measurable gravitational binding energy? The strong equivalence principle goes beyond the weak version by stating that gravitational energy falls with the same acceleration as ordinary matter and other forms of energy in a gravitational field. Although the gravitational self-energy contained in the gravitational forces that hold the Earth together only changes its total mass energy by less than 1 part in a billion, lunar laser-ranging experiments (see main text) can achieve a precision of 1 part in $10^{25}$ and can therefore test the strong equivalence principle. General relativity obeys the strong equivalence principle, whereas the Brans–Dicke theory and many other alternative theories do not.

Does space–time do the twist?

A central prediction of general relativity is that moving matter generates a gravitational field that is analogous to the magnetic field generated by a moving charge. Thus, a rotating body produces a “gravitomagnetic” field that drags space–time around with it, and this “frame-dragging” may play an important role in the dynamics of matter spiralling into supermassive black holes in quasars and other active galaxies. Frame-dragging might also be partly responsible for the collimated relativistic jets seen in such systems.

The Gravity Probe B satellite is currently measuring this effect near the Earth. Launched on 20 April 2004, its goal is to measure the precessions of four gyroscopes relative to a telescope trained on a nearby guide star called IM Pegasi over the course of a year (until the liquid helium that is used to cool the experiment runs out). The gyroscopes are spheres that are perfect to a few parts in 10 million and are coated with a thin layer of superconducting niobium. When the spheres rotate, the superconducting films develop magnetic moments that researchers found no effects down to a few parts per $10^{26}$.

These “clock anisotropy” experiments are latter-day versions of the classic Michelson–Morley experiments of 1887. In the Michelson–Morley experiment the “clocks” being compared were defined by the propagation of light along each of the two perpendicular arms of an interferometer. Einstein took the null result of these experiments for granted in his 1905 paper on special relativity, although he never referred to them by name.

Looking to the future, the discreteness of space–time at the Planck scale that is found in some quantum theories of gravity could also lead to effective violations of Lorentz invariance. However, a wide range of experiments, including tests of CPT (charge–parity–time) symmetry in particle-physics experiments and careful observations of gamma rays and synchrotron radiation from astrophysical sources, have ruled these out to a high-level of precision.

The four fundamental forces of nature.

Various string theories, for instance, allow for the possibility of long-range fields that are linked to the average matter distribution of the universe. If these fields couple weakly to local matter, they could lead to effects that can be observed in experiments. In particular, we know from observations that the Earth moves through the cosmic background radiation at a speed of 350 km s$^{-1}$ per second. With the right kind of long-range field, this motion could produce an effective interaction that has a preferred direction associated with it. If this long-range field were then to couple weakly to, say, electromagnetism, then the electromagnetic fields in atoms could be changed by an amount that depends on the orientation of the atom relative to our direction of motion through the universe.

During the late 1980s researchers at Seattle, Harvard and NIST looked for these effects by checking if atomic transition frequencies change over the course of a year as their orientation changes relative to our cosmic velocity. Exploiting the then newly developed techniques of atom trapping and cooling, the
are precisely parallel to their spin axes. This means that any precession of the spins can be measured by monitoring changes in the magnetic flux through superconducting current loops fixed in the spacecraft.

General relativity predicts that frame-dragging will lead to a precession of 41 milliarcseconds per year, and the Gravity Probe B team hopes to measure this with an accuracy of 1%. The experiment will also measure the “geodetic” precession caused by the ordinary curvature of space around the Earth. General relativity predicts a value of 6.6 arcseconds per year for this effect. Gravity Probe B has been designed so that these precessions are perpendicular to one another, and the first results from the mission are expected in early 2006 (see figure 3).

Meanwhile, last October Ignazio Ciufolini of the University of Lecce in Italy and Erricos Pavlis of the University of Maryland used techniques in which laser beams were reflected from satellites to make a measurement of frame-dragging on the orbit of a satellite. Their result agreed with general relativity, with errors at the level of 10% (see Physics World November 2004 p7).

**The binary pulsar**

In 1974 Russell Hulse and Joseph Taylor, then at the University of Massachusetts, discovered a binary pulsar called PSR 1913+16 that was to play a crucial role in tests of general relativity. Pulsars emit pulses of radio waves at very regular intervals and are thought to be rotating neutron stars. PSR 1913+16 was special because it was a pulsar that was in orbit around another compact object.

By carefully measuring small changes in the rate of the pulsar “clock”, Hulse and Taylor were able to determine both non-relativistic and relativistic orbital parameters with extraordinary precision. In particular they were able to measure three relativistic effects: the rate of advance of the periastron (the analogue of the perihelion in a binary system); the combined effects of time-dilation and gravitational redshift on the observed rate of the pulsar; and the rate of decrease of the orbital period.

If we assume that general relativity is correct and make the reasonable assumption that both objects are neutron stars, then all three relativistic effects depend on the two unknown stellar masses. Since we have, in effect, three simultaneous equations and just two unknowns, we can determine the mass of both objects with an uncertainty of less than 0.05%, and also test the predictions of general relativity. If we assume that the orbital period of the system is decreasing due to the emission of gravitational waves, then theory and experiment agree to within 0.2%. Hulse and Taylor shared the 1993 Nobel Prize for Physics for this work.

Binary pulsars can also be used to distinguish between different theories of gravity because they have very strong internal gravity (see Stairs in further reading). Indeed, several tenths of the rest-mass energy of a neutron star is contained in the gravitational forces that hold the star together, while the orbital energy only accounts for $10^{-6}$ of the total mass energy of the system. In the Brans–Dicke theory this internal self-gravity leads to the prediction that binary pulsars should emit both dipole and quadrupole gravitational radiation, whereas general relativity strictly forbids the dipole contribution. The emission of dipole radiation would have a characteristic effect on the orbital period of the system, but such an effect has not been seen. Several recently discovered binary-pulsar systems may allow new tests of general relativity.

**Gravitational waves**

One of the outstanding challenges in physics today is to detect gravitational waves, and new gravitational-wave observatories in the US, Europe and Japan hope to achieve this, possibly before the end of the decade. In addition to exploring various astrophysical phenomena, these observatories might also be able to carry out new tests of fundamental gravitational physics (see “The search for gravitational waves” on page XX).
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General relativity makes three predictions about gravitational radiation that can be tested: gravitational waves have only two polarization states, whereas other theories can predict as many as six; gravitational waves travel at the speed of light, while other theories may predict different speeds; and the emission of gravitational waves acts back on the source that is emitting them in a characteristic manner.

For example, as described above, scalar–tensor theories and general relativity make different predictions for the nature of the gravitational waves emitted by binary pulsars, and it may be possible to detect these differences. Moreover, if gravitational waves with long wavelengths travel more slowly than those with shorter wavelengths, then it might be possible to observe this behaviour — which is generally associated with massive (as opposed to massless) elementary particles — in the gravitational radiation from binary systems.

Although the collision of two compact objects to form a black hole is too complex to allow precision tests of general relativity, analysis of the gravitational waves produced in the collision will reveal information about the masses and spins of the compact objects themselves, and also about the mass and angular momentum of the final black hole. Such observations will therefore reflect dynamical, strong-field general relativity in its full glory.

Making firm predictions for this situation involves solving Einstein’s equations in a regime where weak-field methods fail, and therefore requires large-scale numerical computations. This challenging task has been taken up by many “numerical relativity” groups around the world. The discovery and study of the formation of a black hole through gravitational waves would provide a stunning test of general relativity.

Relativity and beyond

Einstein’s special and general theories of relativity altered the course of science. They were triumphs of the imagination and of theory, with experiment playing a secondary role. In the past four decades we have witnessed a second triumph for Einstein, with general relativity passing increasingly precise experimental tests with flying colours. But the work is not done. Tests of strong-field gravity in the vicinity of black holes and neutron stars need to be carried out. Gamma-ray, X-ray and gravitational-wave astronomy will all play a critical role in probing this largely unexplored aspect of the theory.

General relativity is now the “standard model” of gravity. But as in particle physics, there may be a world beyond the standard model. Quantum gravity, strings and branes may lead to testable effects beyond general relativity. Experimentalists will continue to search for such effects using laboratory experiments, particle accelerators, instruments in space and cosmological observations. At the centenary of relativity it could well be said that experimentalists have joined the theorists in relativistic paradise.

Further reading

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C M Will 2001 The confrontation between general relativity and experiment Living Reviews in Relativity www.livingreviews.org/lrr-2001-4
J G Williams, S Turyshev and T W Murphy Jr 2004 Improving LLR tests of gravitational theory Int. J. Mod. Phys. D 13 567

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