Gradiometry Coexperiments to the Gravity Probe B and STEP Missions

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Abstract

The Gravity Probe B spacecraft, designed to test predictions of general relativity, will fly in the mid 1990's. It will carry four electrostatically suspended gyroscopes in a cryogenic environment and will have a drag-free control system to minimize disturbances on the gyroscopes. The Stanford Test of Equivalence Principle (STEP) spacecraft, to fly later, will carry a set of test masses under very similar conditions. This paper explores the possibility of using differential measurements of the GP-B gyroscope suspension forces and the STEP test mass displacement readout to form single-axis gravity gradiometers. We show that the noise in the suspension systems is sufficiently small in the relevant frequency range, and that enough information is collected to compensate for the spacecrafts' attitude motion. Finally, using Breakwell's "flat-earth" approximation, we compare these experiments to other geodesy experiments and predict the contribution they can make to the knowledge of the Earth's geopotential.

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INTRODUCTION

Two upcoming physics experiments will have application in the gradiometric study of geodesy: Gravity Probe B and the Satellite Test of the Equivalence Principle. In each case, geodesy is not a central feature, but is a coexperiment. However, because the requirements on the physics experiments are very tight, analysis of the error sources provides a large amount of data valuable for the study of Earth's gravity field. We analyze here the quality of the available geodetic information and make predictions about the results we can achieve using that information.

The Gravity Probe B mission is intended to test two predictions of Einstein's general theory of relativity: first, that a rotating mass such as the Earth causes a rotation of inertial space in its vicinity (the "frame-dragging" effect), and second, that the axis of a rotating mass orbiting in a gravitational field precesses in the direction of the orbit. To test these two predictions, GP-B will employ a set of four extremely sensitive gyroscopes, which will provide an inertial reference tied to local space, and a telescope fixed on a distant star, which will provide an inertial reference tied to distant space. The satellite will maintain a very stable environment about the gyroscopes: the temperature will be near absolute zero, and the acceleration will be less than 10^{-9} g.

The Satellite Test of the Equivalence Principle is fundamentally a tremendous extension of Galileo's famous experiment at the Tower of Pisa. It will measure the equivalence of gravitational mass and inertial mass to one part in 10^{17}. To do this it will use pairs of concentric cylinders, carefully aligned in a very disturbance-free environment, similar to the one aboard GP-B. It will measure any difference in the accelerations of the two cylinders during the course of several orbits by means of a differential accelerometer system using them as its proof masses. The spacecraft will carry three or more pairs of masses, with different centers.

A SHORT DESCRIPTION OF GRADIOMETRY (***or, SATELLITE GRADIOMETRY)

The Gravity Gradient (***not in shorter version; I have not read)

In Newtonian mechanics, one typically uses a potential function to describe the gravitational field exerted by a mass:

$$U(s) = \int G/(dr)\ dm$$

where U is the potential at a given point s, G is a universal constant, dm is a differential element of mass, and dr is the distance from dm to s. The integral nominally includes all the mass in the universe. Practically, of course, only nearby masses have much influence.

The gravitational force which a body feels is proportional to its mass multiplied by the gradient of the potential function;

$$F_i = M\ (\partial U / \partial x_i)$$

where i is a coordinate index, x_i refers to the i'th coordinate, and F_i is the force component in the direction of x_i. Since

$$F_i = M\ a_i,$$

we can set the two equations equal and cancel the mass. (The Krotkov-Roll-Dicke experiment has confirmed the equivalence of inertial and gravitational mass to approximately the precision we would need, but see below.) Therefore, units of acceleration, such as m/sec^2, describe a gravitational field at a point. The force on an object is a function only of the gravity potential gradient at the point it occupies. One can easily show that there are therefore very many configurations of mass in the universe which will produce the same force at a given point. For example, a given force in the x_1 direction could be produced by an infinite plane distribution of mass perpendicular to the x_1 axis and located anywhere along the positive x1 axis! Point masses of varying size located in the positive x_1 direction could also produce the same force.

We can get more information about the local mass distribution from the gravity gradient tensor. This tensor, G, is defined as the second derivative of the potential in two directions:

$$G_{ij} = \partial^2 U / \partial x_i \, \partial x_j.$$

where j is a coordinate index like i. The tensor has nine components (Figure 1), but only six of them are independent. The units of the Gravity gradient are acceleration per unit distance, or (m/sec^2) / m. Normally we cancel out distance and multiply by 10^{-9} for numerical convenience. The result is Eötvös Units, (EU); 1 EU = 10^{-9} /sec^2.

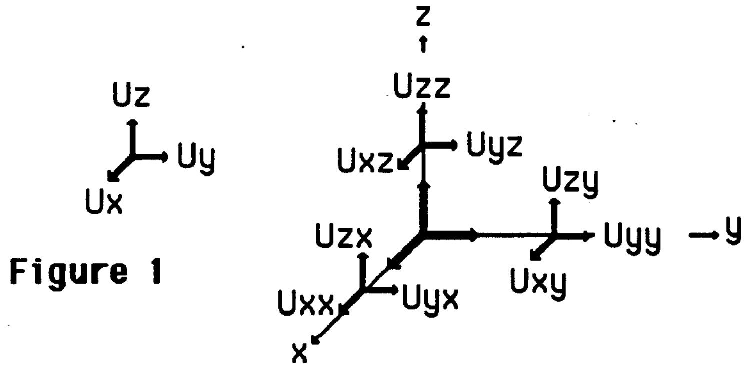

Figure 1

The 9 components of the Gravity Gradient Tensor

The gradient of the gravity field of an infinite plane is zero everywhere, so the gradient cannot distinguish the distance to such a plane. However, the gradient due to a point mass (in the direction of the mass) falls off as the cube of the distance; a small mass nearby will produce a larger gradient than a large one far off, even if both produce the same acceleration. This is generally true of most mass distributions, though spherical shells produce gradients identical to point masses. Nevertheless, for study of a mass distribution by examination of its gravity field, the gradient provides far more information than the acceleration alone.

Measurement of the tensor (***bold-face type was not contained in shorter versions)

One typically measures the tensor by physically measuring acceleration due to gravity at different locations around the point of interest. Since the tensor has six independent components, a total of six measurements should suffice, but generally more measurements are needed. The reason for this is that unmodeled rotation of the instrument frame can produce accelerations indistinguishable from gravity gradients with only six measurements.

The gradient of the gravity field of the Earth is very small compared to its acceleration. At the surface of the Earth, the vertical component of gravity decreases by $3.1 * 10^{-6}$ m/s^2 per meter of altitude; thus the corresponding component of the gradient tensor (the largest one) is 3100 EU. Measurement of such a small quantity would probably be fairly hopeless were it not for the fact that the measurement is relative; but because acceleration measurements may be differenced to obtain the gradient, it is possible to achieve fairly high accuracy. Earth-bound gradiometers obtain a sensitivity of 1 EU and better by using rotating instrument arrays and multiple differencing to separate the gradient from the constant gravity term. We expect to achieve at least 0.1 EU accuracy in the acceleration-free environment of STEP and GP-B.

Neither experiment is designed to be a gradiometer, so neither will have enough accelerometers to measure the full nine-component tensor, or even six components of it. Nevertheless, measurement of even a single component will provide enough information to substantially improve modeling of the Earth's gravity potential. Post-mission analysis will use the data to perform a best least squares fit for the parameters describing the spherical harmonic expansion of the Earth's potential.

(***This sentence opened the ¶ in the other version; you need to look it over.) Gradiometer measurements are better than traditional position or Doppler velocity measurements of the satellite trajectory in two ways.) **Traditionally, satellite geodesy has centered on the use of position or doppler velocity measurements of the satellite trajectory. Gradiometer measurements are better in two ways.** First, they provide more information at any single point, **as discussed above.** Secondly, they are much more sensitive to higher degree coefficients in the geopotential spectrum, because the position of a satellite is the second time integral of the space derivative of the gravity potential, but the gradient is its second space derivative. By removing the smoothing effect of the two time integrals, we can more easily measure the high frequency components.

The fact that gradiometry removes the time integrals works against it at low degrees; there, traditional position measurements have more validity due to the long integration times. (This is also why position measurement schemes such as GPS are so effective at low degree [Tapley, 89; Breakwell, 89].) **The point at which one system becomes more effective than the other depends on many factors. However, its existence has a very useful consequence for us; gravity gradient measurements from a satellite are most useful above a certain degree. The time frequency corresponding to this degree is generally about 0.1 radians per second. Constant gradients and very low-frequency changes are not important because the information they provide is more easily obtained with traditional satellite tracking. This is very fortunate, because it greatly simplifies the problems of calibration and common mode rejection.** (***That section was replaced by this one: Constant gradients and very low-frequency changes are not interesting to gradiometry because that information is more easily obtained with traditional satellite tracking. This greatly simplifies problems of calibration and common mode rejection. Signals with a frequency below about 0.1 rad/sec, including biases, are can be removed from the solution by filtering.)

THE GP-B AND STEP SCIENCE EXPERIMENTS

GP-B

To measure the frame-dragging effect, GP-B will compare the axis of rotation of four very sensitive gyroscopes to the direction to a distant star (Rigel). The entire spacecraft will roll slowly about the same axis to simplify readout of the gyroscopes' axis alignment (Figure 2). The predicted drift of the gyros due to the frame-dragging effect is only 0.042 arcseconds per year, so the gyros need to be extremely good. To attain this level of quality, the GP-B spacecraft will carry a drag-free control system to minimize suspension forces (and hence torques) on the gyros. However, there will still be some residual force on the gyros, so a suspension system must be present.

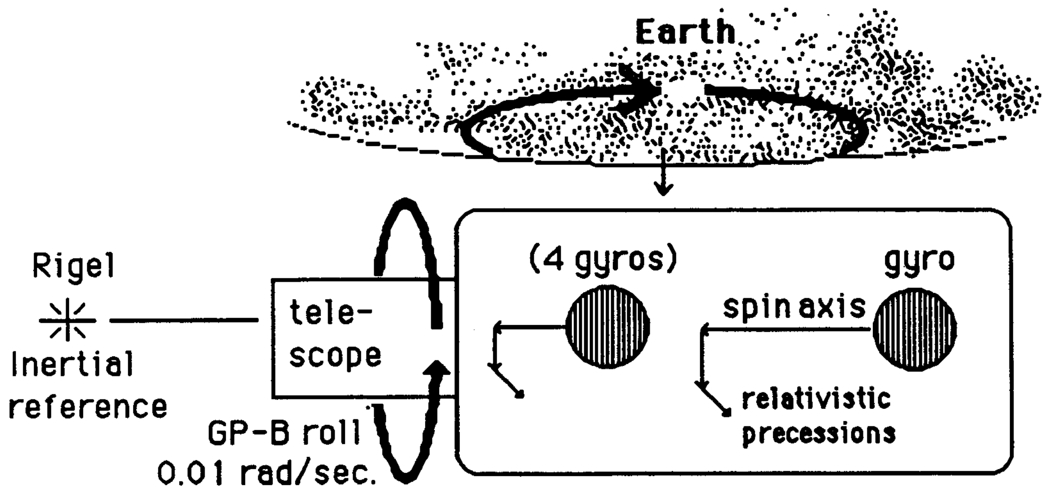

Figure 2: GP-B experiment arrangement

The suspension will be simple in concept but complex in execution. It is an alternating current electrostatic suspension with a capacitive pick-off feedback control. The gain in the feedback loop is switched according to the current level of the acceleration disturbances measured by the drag-free system. During the experiment, the system will center the gyro in its cavity to within $1.2 * 10^{-5}$ cm. The feedback signal in the suspension loop is proportional to the force on the gyro. By comparing the feedback signals between the four gyros, we can obtain the relative magnitudes of the forces acting on them, and hence the relative accelerations at their locations. There are three axes of suspension, each with a pair of electrodes, none of which are lined up along the axis of rotation.

STEP

Axial readout

The STEP experiment will have two positioning systems, one acting in the radial direction and one acting along the axis of the experiment (Figure 3). The radial suspension will be a superconducting magnetic bearing with a secondary electrostatic system.

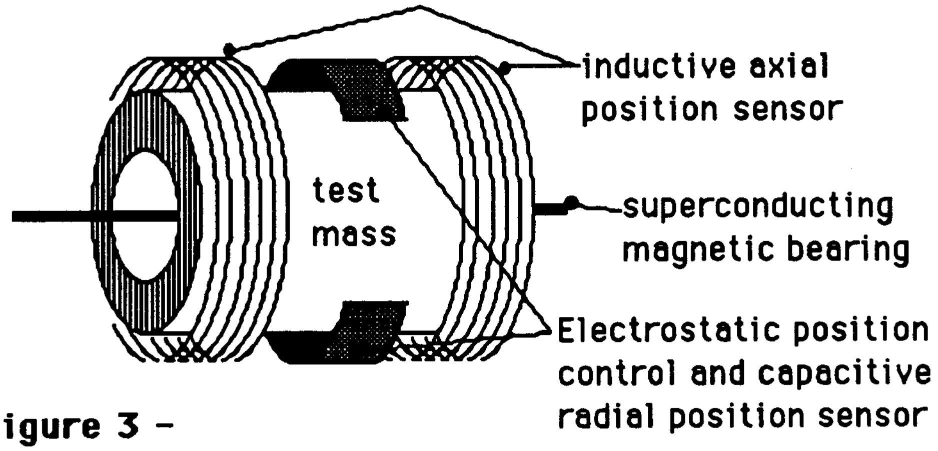

Figure 3 - STEP Position Sensing and Control

The measurement system of the experiment provides axial positioning. Displacement of a mass produces a magnetic reaction force in the measurement pickups, which tends to re-center the mass. The force also causes current to flow thorough a superconducting quantum interference device (SQUID) which provides the displacement measurement. Because periods of oscillation are matched very closely between concentric masses, the system will be exquisitely sensitive to differential mode acceleration. The sensitivity to common mode acceleration will also be very good. By comparing common mode acceleration between pairs of concentric masses, we obtain the axial gradient across the experiment.

Lateral suspension

The lateral suspension is a superconducting magnetic bearing, a passive system, supplemented by an electrostatic suspension with capacitive position sensing. The magnetic bearing should provide most of the centering force, with the electrostatic system employed for calibration and control during maneuvers. However, the capacitive pick-off should be constantly operational. By comparing the measured radial displacement between different pairs of masses, we can obtain the transverse gradient across the experiment.

NOISE SOURCES AND MAGNITUDES OF DISTURBANCES

Measurement noise

The noise in the measurements come from several different sources, including electronic noise, rotation, and spurious gradients caused by spacecraft components. The electronic noise is the least significant in most cases.

The feedback system for GP-B has already been tested, using a fixed capacitance as a dummy load. The noise has a spectrum with strong 1/f characteristics, but in the frequency range of interest (0.1 rad/sec to 0.02 rad/sec) the distribution is very low. The test results indicate a noise corresponding to an acceleration of between $3 * 10^{-13}$ cm/sec and $2 * 10^{-12}$ cm/sec^2 in that frequency range. This accuracy should be the same in each of the three suspension axes. As we shall see, this error is low enough that it should be completely dominated by other errors.

Due to uncertainties in the roll rate and orientation of the spacecraft, radial measurement of acceleration has greater uncertainty than axial. Therefore, it is critical that we be able to resolve acceleration accurately into radial and axial components. **Again, static errors are not significant; they can be modeled and removed from the data during post-mission analysis.** The most significant source of error will be variations in the capacitance of the pickup due to motion perpendicular to the axis of the pickup. This can be calculated based on the readings of the transverse pickups, but electrode spericity errors will contribute to errors in the position determination. (See Figure 4)

Several means of approaching this problem are available. One (*****solution**) is to map the asphericity of the electrodes during pre-mission calibrations, by applying known accelerations and observing the reactions and measurements resulting from the four gyros. In any case, we must require **(based on results to follow)** that the magnitude and direction of accelerations be separable into axial and transverse components to an accuracy of 1 part in 10^4. **(The other paper says 5 parts in 10^3)**. We anticipate little difficulty in meeting this requirement.

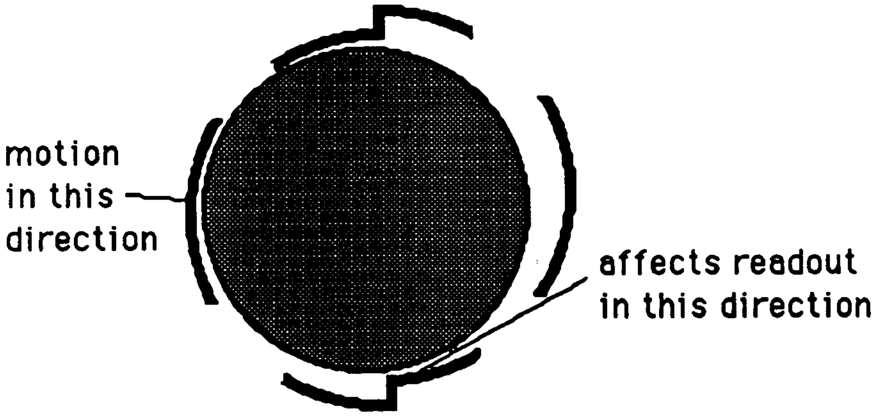

Figure 4: Effect of Electrode Asphericity on position readout.

The axial measurement system for STEP is the fundamental measurement for the experiment itself; but for gradiometry we need to measure the acceleration common to concentric pairs of masses as compared to other pairs (instead of the differential acceleration of a single pair). This measurement has a much lower accuracy in the current design, because different pairs are not well calibrated against one another. However, the common mode rejection ratio (CMRR) for any pair of masses is 10^{-5}. Thus, to measure a differential acceleration of 10^{-14} cm/sec^2, the goal of STEP, the common acceleration of any pair must be measured and controlled to an accuracy of at least 10^{-9} cm/sec^2. Ccalibration and tuning of different pairs of masses could improve this, but this might increase the complexity and cost of the mission. If we accept that the limiting acceleration sensitivity will be 10^{-9} cm/sec^2, a fundamental requirement to the mission, we can calculate the sensitivity of the gradiometer in the direction of the axes by simply dividing by the distance between mass pairs, which we assume to be 10 cm, and converting to Eötvös units. The result is a sensitivity of 0.1 EU. If instead we argue that adequate calibration could improve matching between pairs of test masses by an order of magnitude (a preliminary guess is that even two orders of magnitude will not be unreasonable), then we could achieve 0.01 EU.

The radial suspension system for STEP is less sensitive. The spring constant (220 dyne/cm) of the passive magnetic bearing will limit the response of the mass to radial accelerations. The capacitive pickup will have a sensitivity of 0.001 micron (10^{-7} cm). For a 1.0 kg mass, this produces an acceleration sensitivity of $2.2 * 10^{-8}$ cm/sec^2. It is doubtful whether the accuracy of the capacitive pickup can be much improved, due to a combination of electronic noise and constraints on electrode spacing. Reducing the spring constant of the magnetic bearing would improve sensitivity but would endanger the equivalence principle experiment. This represents the limiting factor in transverse acceleration measurement on STEP. If we make the same assumptions as above, the quality of the transverse gradiometry is 2.2 EU.

(***Other replaces first sentence with this one: Resolving acceleration measurements on different proof masses into identical axes to find the gradient depends on the alignment of the sensors on the different masses.) The separation of measurements into the appropriate axes depends on the alignment of the sensitive axes. The specifications for the STEP experiment call for an alignment correct to 10^{-5} radians; this implies a carryover of only $2.2*10^{-4}$ EU from the less sensitive axis to the more sensitive one, well below the limiting accuracy.

Spacecraft dynamics

GP-B

The GP-B spacecraft is oriented with the experiment axis pointing toward Rigel, which serves as a reference. The spacecraft rolls about this axis with a period of ten minutes, to allow spectral separation of the (nearly secular) gyro drift from the 1/f SQUID readout noise. (***Other combines ¶)

The roll contributes strongly to errors in the acceleration measurement. Since the gyros are nominally on the spin axis (the specifications call for a deviation of no more than 0.5 mm), the roll should have no effect on them. However, **due to the small size of the accelerations under consideration,** even tiny deviations from the centerline can produce significant variations in acceleration. There are also effects due to variations in the rate of roll which produce accelerations on non-centered gyros.

The most obvious problem is that gyros fixed in the spacecraft but not on the centerline will experience a centrifugal acceleration indistinguishable from a gravity force except for its ten-minute period. Furthermore, and more serious since it is impossible to model, any variation in the roll rate will produce a variation in the centrifugal force. Current models of a roll control subsystem indicate a root mean square (RMS) roll rate error of $4.4 * 10^{-6}$ rad/sec. For a gyro 0.1 cm (***other says 0.5 mm) from the axis of rotation, this variation in the roll rate creates a variation of $9 * 10^{-9}$cm/sec^2, (***other says $4.4 * 10^{-9}$cm/sec^2) or a gradiometer sensitivity of 0.9 EU. (***0.44 EU) In this calculation the largest term is the product of the roll rate with its variation.

The variations in roll also produce a tangential acceleration on the gyro with similar characteristics. The same model of roll control implies a roll rate variation whose RMS value is $4.3 * 10^{-9}$ rad/sec^2. Again assuming a gyro 0.1 cm (***0.5 mm) from the roll axis, this produces an acceleration of $4.3 * 10^{-10}$ cm/sec^2, or 0.043 EU. (*** $2.2 * 10^{-10}$ cm/sec^2, or 0.022 EU)

"Pitch and yaw" variations, or those about axes perpendicular to the direction to Rigel, are also serious. The specifications require a pointing capability of $0.15\ 10^{-6}$ rad, which limits the size of excursions to a very low value. This in turn limits the rate, if the frequency of oscillation is known. A further requirement that the bandwidth of the controller be less than 0.03 rad/sec will serve to limit maximum acceleration enough to produce a 0.1 EU sensitivity in the radial direction. Considering both the low authority of the gas jets and the stable nature of attitude disturbances, this may be possible, but it will impose severe limitations on the control system.****

The centrifugal (axial) force produced by pitch and yaw motion is less significant. Assuming the same bandwidth as before, and given the 10-cm separation between gyros, we get an axial acceleration of $2.5 * 10^{-13}$ cm/sec^2, or $2.5 * 10^{-4}$ EU. Clearly, the largest problems with spacecraft rotation are those caused by unknown centrifugal and tangential forces, all acting in the radial direction. The axial measurement will therefore be the most sensitive, but will be contaminated by errors in the radial direction unless the accelerations can be well resolved into axial and radial components.

STEP

The STEP spacecraft will be maintained in a constant orientation in inertial space, except for a one degree per day drift needed to keep it aligned with (***what about **in, or on**) its sun-synchronous orbit. This is small enough that the effects of roll discussed above are negligible. Since there is no large constant roll rate, the terms containing the product of roll rate and variation of roll rate, troublesome in GP-B, become much less significant. The specifications require a pointing accuracy of less than $2.5 * 10^{-5}$ radian, with a noise of 10^{-5} rad/sec/$\sqrt{\text{Hz}}$. **This is necessary to meet the requirement of 10^{-9} cm/sec^2 acceleration on any pair of test masses, as we have discussed before.** In the baseline configuration of the experiment, the differential acceleration measurement is used to assist in meeting this requirement. This would make the data reduction more complicated for purposes of gradiometry. One alternative (**I think ***alternate*** would be better) configuration, depicted in Figure 5, would remove this necessity (***by placing all the accelerometers on the same axis. In this configuration, pitch and yaw movement would not produce accelerations along the sensitive axis of the experiment. This would greatly simplify the gradiometer measurement. It would also greatly improve the geodesy benefit from the gradiometer.***I think you can make some modifications to make that last couple of sentences better still.)**and greatly simplify the gradiometer measurement. As shown below, it would also greatly improve the geodesy benefit from the gradiometer.**

Dynamic self-gradients

The presence of gravity gradients within the spacecraft due to components of the spacecraft itself also causes error. Again, constant gradients can be calibrated or modeled and removed from the data in post-mission processing. This allows us to confine our arguments to two classes of disturbances. First are variations in acceleration felt by a test mass due to motion of the test mass within its cavity. For GP-B the gyros will move less than $1.2 * 10^{-5}$ cm from the center of the cavity. If we assume a field of 800 EU within the experiment package (equivalent to an unbalanced 1-kg mass at 0.033 m distance), and a position uncertainty of $1.2 * 10^{-6}$ cm, we obtain an acceleration uncertainty of $1 * 10^{-10}$ cm/sec^2, or 0.01 EU.

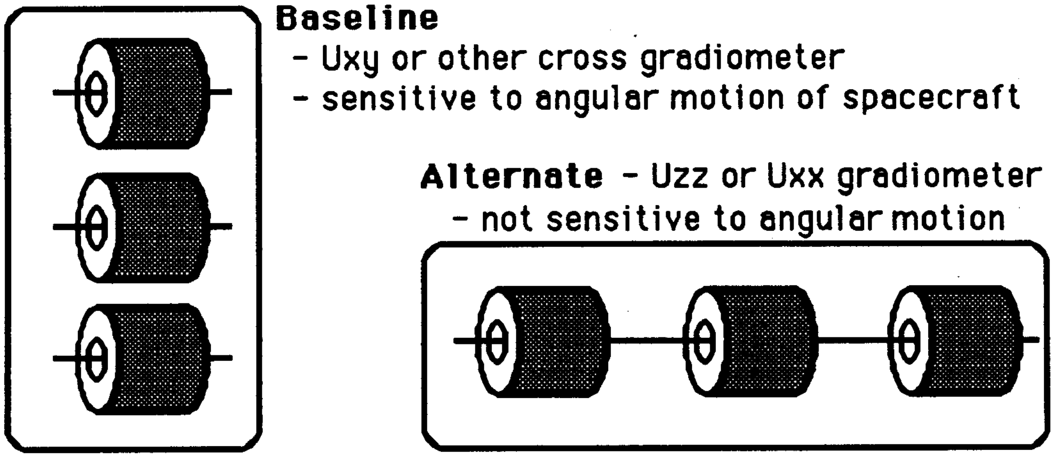

Figure 5
Accelerometer arrangement for Baseline and Alternate configurations for STEP spacecraft

For STEP, the axial positioning system must produce an uncertainty of less than 10^{-7} cm. Therefore, the uncertainty in gradient due to mass motion is more than 10 times better in this direction than in GP-B for the same internal gradient. Similar calculations give the result (***show) that radial position errors of $1.2 * 10^{-6}$ cm give gradient errors of 0.012 EU. Constant internal gradients will not be a problem for either spacecraft, due to the precise positioning of their masses.

The second class (***of errors), varying internal gradients, are another matter, however. (***This section was moved in the other to after the discussion of helium.) **Moving solid masses within both spacecraft will be very rare, due to the severe effects of vibration or unexpected motion on the science experiments. Both spacecraft will contain mass trim and reaction systems, but the motions of these systems will be very smooth and well known. Therefore their effect on gradiometers can be removed**

from the data with great confidence. Thermal distortions of the spacecraft are less well known, and are under consideration. For GP-B, the requirements on maximum allowable distortion are already fairly tight, because distortions would adversely affect attitude motion due to the roll. For STEP, thermal distortions may be a limiting factor.

Each spacecraft will contain large dewars of liquid helium surrounding the science package, which maintain very cold and stable temperatures and provide reaction mass for the control systems. Motion of the helium due to tides or spacecraft motion can produce gradients large enough to completely mask the geodesy signal and which are completely unknown. (***cannot be modeled?) The best measurements on GP-B will probably be about 0.1 EU. A signal of this size could be produced by 30g of helium at a distance of 25 cm from the mass.

For STEP, the best measurement could have a sensitivity of 0.01 EU. A signal of the same size could be produced by only three grams of helium at a distance of 25 cm from the mass. It is therefore absolutely (***is that word necessary?) critical to the success of the gradiometry coexperiments that the helium location be well known or well controlled or both.

Control of the helium can be accomplished in several ways. GP-B's rotation will serve to keep the helium against the outside wall by centrifugal force. At the planned rate of rotation, the centrifugal force will be larger than the tidal force by many orders of magnitude. Both spacecraft will have the primary heat flow into the dewar through the external walls. Since liquid helium is strongly attracted to heat, this will also serve to draw the helium to the walls (and away from the experiment). Both spacecraft could easily accommodate a very porous gel or ceramic insert in the helium dewar; this would serve to bind the helium into place by its surface tension.

As noted above, we have assumed that inertial and gravitational mass are identical in our discussion of gravity gradients. However, this assumption is the subject of the STEP experiment, so we should consider whether a violation could affect our coexperiments. A bias would appear in the data at a frequency of only one revolution per orbit; thus it is (***orbit, below) below the frequency range we are interested in. The magnitude of the bias, assuming Roll-Krotkov-Dicke to be the limit on the size of the violation, could be as large as $3 * 10^{-8}$ cm/sec^2. This corresponds to 3 EU and would probably be detectable by either coexperiment. It is unlikely to occur in GP-B because the gyros are all made of the same material; but in STEP, the different test masses will be made of very different materials. Fortunately, in STEP the effect will be much better determined by the main experiment than by the coexperiment. Equivalence principle violations should therefore contribute very little to gradiometer error.

ORIENTATION OF BEST EXPERIMENT AXES

GP-B

For GP-B, we have shown that the uncertainty in the radial acceleration corresponds to a sensitivity of about 0.9 EU (***0.5 EU), taking all sources into account. The magnitude of the radial acceleration is largest; the centrifugal force on a mass may be as large as 10^{-5} cm/sec. (***other $5*10^{-6}$ cm/sec)(This is larger than the allowable average acceleration on the gyros; however, because it is averaged over a circle, it will not affect the long-term pointing of the gyros. Hence, it is not necessarily a conservative assumption.) If these radial acceleration components produce an error in the axial direction which is reduced by 10^4 (*** $5*10^3$) from the radial component, the result is still 10^{-9} cm/sec^{2,} or 0.1 EU in the axial direction. Of course, improvements in either the centering or the resolution of components will improve this, but either will be difficult to do. This error dominates the error in the axial direction, so that the best measurement we expect from GP-B is 0.1 EU. This measurement represents the change in axial acceleration with distance in the axial direction.

Since the axial direction is always aligned toward Rigel, it is sometimes vertical and sometimes horizontal with respect to the Earth's surface. The best signal is obtained when it is vertical; fortunately, this happens when the spacecraft is near the equator, so that over the course of the mission, a large fraction of the Earth's surface is measured in this manner.

STEP

For the STEP spacecraft, the axis of the accelerometers is the most sensitive axis. The errors are dominated by the angular motion of the spacecraft in the baseline configuration, and by the ability to reject common mode accelerations in the alternate configuration of Figure 5. In the baseline configuration the sensitivity is unlikely to exceed 0.1 EU; in the alternate, it may be 0.01 EU or better, depending on the calibration technique. The baseline configuration measures change in axial acceleration with distance in a radial direction, whereas the alternate measures change in axial acceleration with distance in the axial direction. Like GP-B, the orientation of STEP is fixed in space. Unlike GP-B, however, it may be reoriented several times during the course of the mission. This will allow optimum coverage over any part of the Earth if geodesy is a priority in determining the orientation schedule.

EXPECTED RESULTS

Using the flat-earth approximation described in Breakwell [79], we predicted the quality of the geodetic information produced by the two experiments.We assumed an altitude of 650 km for GP-B and 550 km for STEP. We assumed a duration of 18 months for GP-B and of 6 months for STEP. We used the 0.1 EU value for the quality of the GP-B gradiometer, and tried both the 0.1 EU and the 0.01 EU case for STEP. (***Other says We used the 0.1 EU value for both gradiometers.) We assumed for both systems a single measurement of the gradient every ten seconds.

The output of the procedure is the expected uncertainty, averaged over all orders at a given degree, in the potential harmonic coefficients describing the Earth's gravity field. Lower uncertainty implies more knowledge in general but cannot be specifically applied to any one coefficient. The upper limit of uncertainty is set on all plots by Kaula's rule, an empirical formula giving the expected average size of the coefficients. We assumed this size as an *a priori* bound on the uncertainty. The current state of the art (actually two years old) (***this will not be true for your dissertation) is represented by the GEM-T1 curve on each chart. GEM-T1 is a model of the Earth's gravity developed by Goddard Space Flight Center using only satellite tracking data. (Successors such as GEM-T2 and PTGF-89 typically also use information from radar altimeters to measure the ocean's surface.)

Figure 6 shows the results for two different orientations for GP-B. The Uxx curve applies to the case where the axis of the experiment is horizontal, as it generally is over the poles. The Uzz curve applies when the axis is vertical. The information obtained from the vertical case is clearly far more useful; the polar regions will probably be less well measured than the equator despite the higher density of ground tracks there.

Figure 7 shows the same two curves for STEP, with one further (***that word needed?) addition. In the Uxy case the displacement between accelerometers is perpendicular to their sensitive axes. This is the baseline configuration. The Uxx and Uzz cases correspond to the alternate configuration in horizontal and vertical orientations, respectively. The case for using the alternate configuration is very strong from a geodesist's point of view.

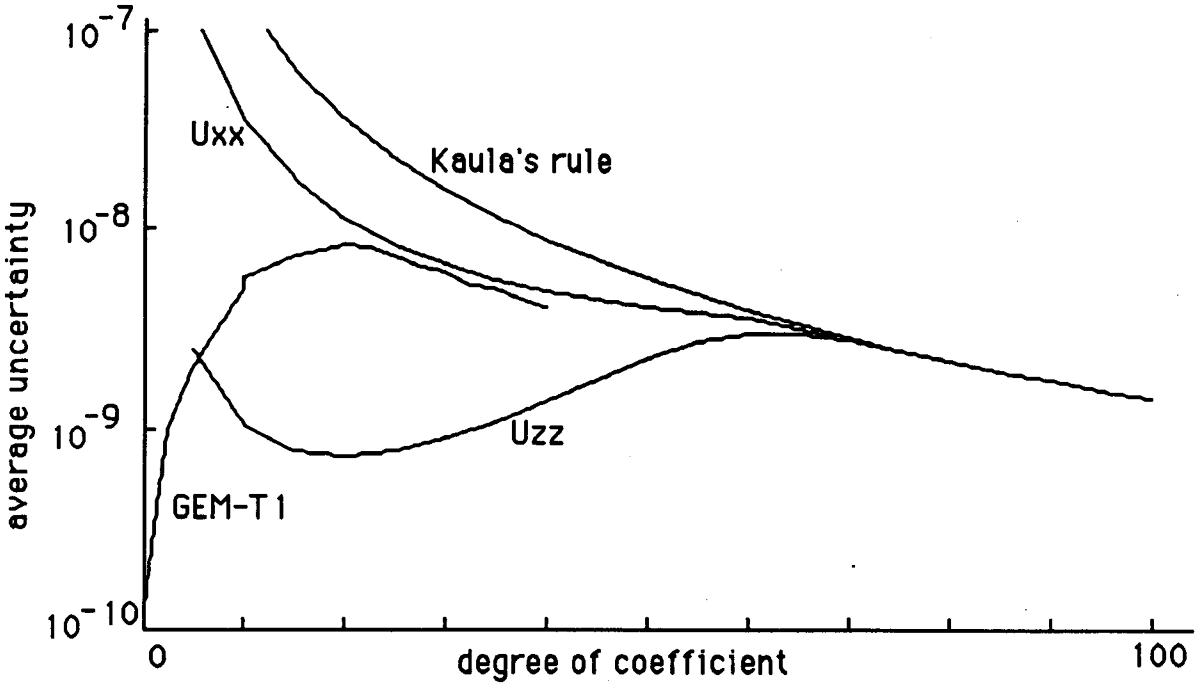

Figure 6

Geopotential Coefficient Uncertainties vs. Degree for 0.1 EU GP-B mission in horizontal (Uxx) and vertical (Uzz) orientations.

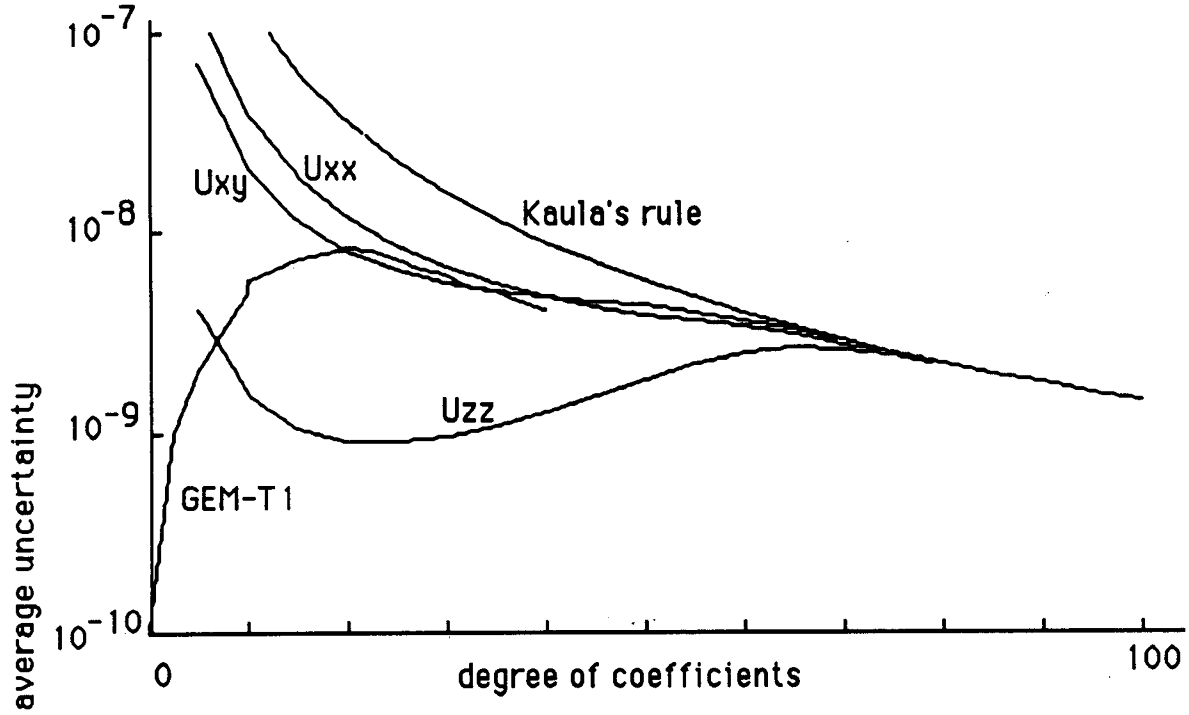

Figure 7

Geopotential Coefficient Uncertainties vs. Degree for 0.1 EU STEP mission in horizontal (Uxx), horizontal transverse (Uxy), and vertical (Uzz) orientations.

The third plot (Figure 8) shows the same thing as the second plot, but assuming a gradiometer quality of 0.01 EU for STEP instead of 0.1. This presumes that the accelerometers can be calibrated to improve common mode rejection between different pairs of masses by a factor of 10. The potential for improvement over the current state of the art is obvious.

Finally, we conducted a brief study to determine the geodetic optimum inclination for the STEP orbit given that GP-B was to be in a polar orbit. Making the STEP orbit polar also will duplicate too much information over the poles; making it equatorial will prevent it from measuring much of the Earth. We based our analysis on the total amount of groundtrack coverage in each one-degree latitude band from the equator to the pole. We assumed equivalent gradiometer qualities in the two spacecraft and made the same assumptions as above regarding mission length and altitude.

We present a plot (Figure 9) showing the cumulative uncertainty (over all latitude bands) as a function of STEP inclination for each of five different degrees. The minimum of each curve represents the optimum inclination at that degree. From the chart, we can see that the optimum inclination increases with increasing degree. Depending on which degree is judged most interesting, the optimum could be anywhere from 50 to 90 degrees. For this analysis, inclinations greater than 90 degrees are the same as their complements. We note that STEP has a sun-synchronous orbit, with an inclination of 97 degrees (here the same as 83 degrees), as its baseline plan. This produces information only a few percent worse than the optimum for any degree.

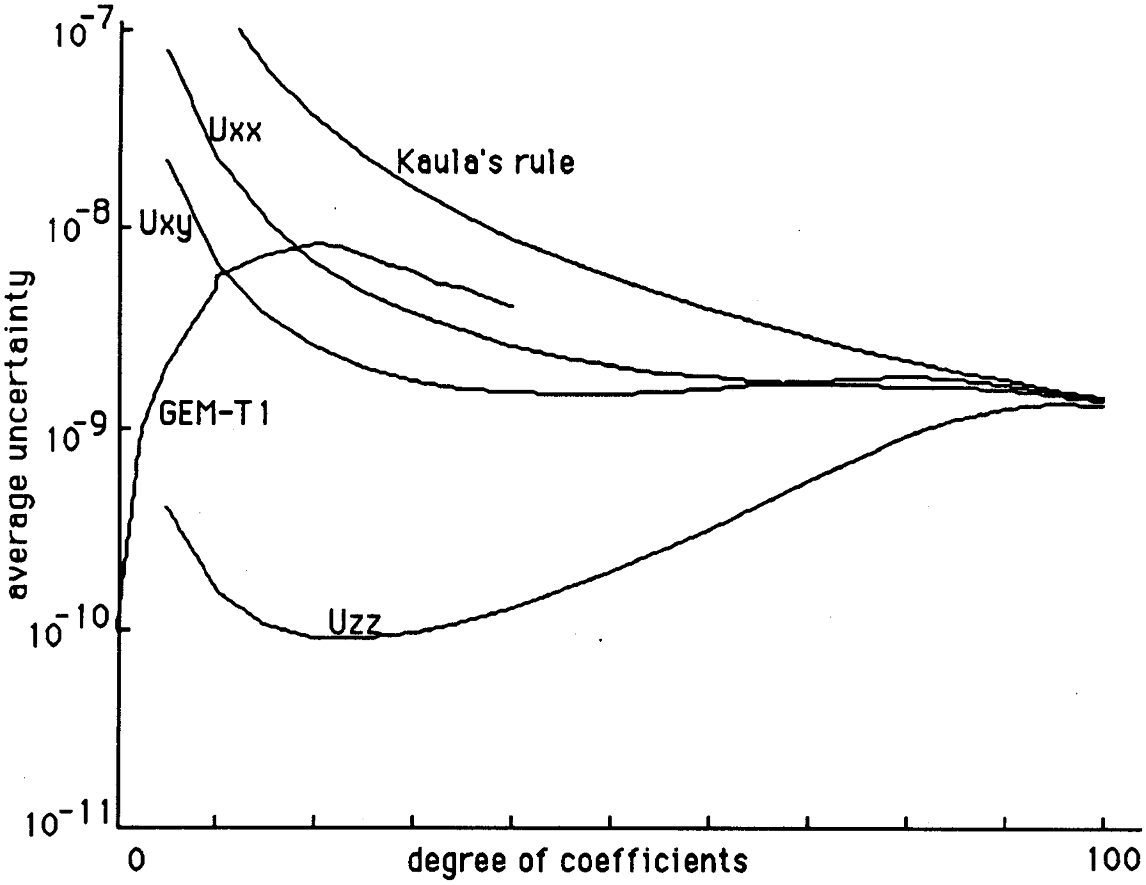

Figure 8

Geopotential Coefficient Uncertainties vs. Degree for 0.01 EU STEP mission in horizontal (Uxx), horizontal transverse (Uxy), and vertical (Uzz) orientations.

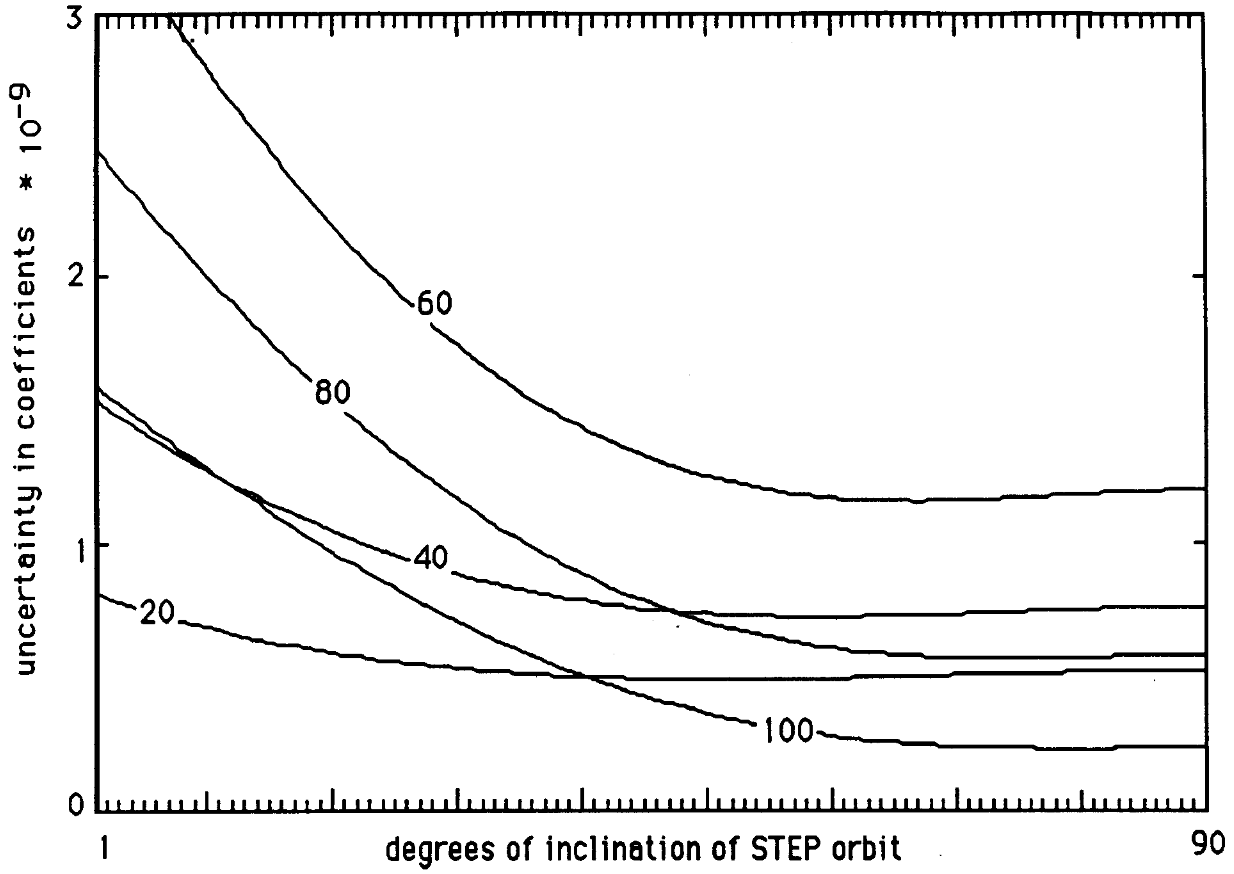

Figure 9
Geopotential uncertainty averaged over all latitude bands vs. inclination of STEP orbit for several different coefficient degrees.

CONCLUSION

STEP and GP-B are both ambitious physics experiments. The extremely high quality environment they provide to their science packages also gives gradiometer coexperiments an unusually good platform. Much can be accomplished on these missions at very little added expense. The alternate configuraion for STEP is far superior for geodesy. (***I think that sentence could be tied in better.) The current state of knowledge of the Earth's geopotential can be improved by several orders of magnitude in the best case. Further, the information the gradiometers will provide is complementary to that which the GPS system will provide, thus allowing good broad-spectrum coverage of the geopotential frequency content. We regard these missions as a very valuable opportunity to advance the sciences of both gradiometry and geodesy.

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(***I copied the bibliography over; you can modify it as you wish. I didn't compare those of the two documents.)

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