DO COSMIC STRINGS VIOLATE THE EQUIVALENCE PRINCIPLE ?

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ABSTRACT

We examine the compatibility of cosmic strings with the equivalence principle. In particular, we look for mass dependent accelerations of particles moving in the field of a cosmic string , and also of cosmic strings moving in background gravitational fields . Contrary to recent suggestions , it apppears that there should be no detectable violation of the equivalence principle .

1. Introduction

Cosmic strings are sources of rather unusual gravitational fields1. This has given rise to arguments wanting them to have unequal inertial (μ_{in}) and gravitational (μ) mass densities , and generally to violate the equivalence principle2 . In this article we re-examine the problem . Two different situations are considered : i) "Test" particles moving in the gravitational field of cosmic strings ; ii) Cosmic strings as "test" objects in given background gravitational fields . (The term "test" object above, means that there is no significant disturbance to the background field induced by it) .

In the second case, it appears that there is no fundamental reason imposing deviations from the equivalence principle . The acceleration of long string segments due to the interaction with the wake3 forming behind them, when they move through a gas, depends explicitly on their linear mass . This however, does not contradict the equivalence principle, as it has been claimed2, since such dependence is characteristic of systems with variable mass .

Test particles in the spacetime of a cosmic string on the other hand, have mass dependent accelerations which cannot be explained in a similar way . The presence of other masses in the Universe alters this effect however .

We only consider the case of a straight gauge cosmic string , since the phenomena of interest arise due to the conical singularity1 of the spacetime around it . Analogous - but not identical - singularities, are exhibited by the spacetimes around other topological defects such as monopoles3 and textures4, and we would expect similar results concerning their consistency with the equivalence principle. Remember that this principle states that all test bodies fall in a gravitational field with the same acceleration regardless of their mass or internal structure. It is very important to emphasize that here we look for violations within General Relativity

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coming only from mass dependent accelerations . Our equations appear in units where $c = 1$, $G = \frac{1}{M_{Planck}2}$, G is Newton's constant.

2. Test particles in the field of cosmic strings

Consider an infinite , straight , gauge string ; in cylindrical coordinates (r, θ, z) the spacetime around it , is given by[1] :

$$ds2 = dt2 - dr2 - (1 - 4G\mu)2r2d\theta2 - dz2 \tag{1}$$

This is known as a conical spacetime and it is locally flat .. As a result the gravitational force on test particles in it vanishes in $O(G)$. In $O(G2)$ however , there is a force . For a particle of mass m , at a distance r from the string :

$$F \propto -G\frac{(8\pi G\mu)m2}{r2} \tag{2}$$

the constant of proportionality being $\simeq .03$, and it can be interpreted as the attraction between the initial mass and its image in the conical metric[2] .

We can see from eq(2) that the acceleration of the particle induced by this force is mass-dependent , more specifically :

$$\frac{du}{dt} \propto (\frac{m}{m_{in}})m \tag{3}$$

and therefore it appears that the equivalence principle is violated . Note that for GUT cosmic strings , i.e $G\mu \simeq 10-6$, and a mass of say 1000 kg , at a distance 1 meter from the string, the force on the mass as given by eq.(2) will be $\simeq 5 \times 10-11$ N , and its acceleration $\simeq 5 \times 10-12\frac{cm}{sec2}$ which is within STEP's[5] aimed precission of $10-14cm/sec2$.

The presence of other masses in an actual "Galileo experiment", however, would change things . As M.Rees et al.[2] pointed out a particle will see in the string the image of every other mass in the Universe , not only its own . This will alter eq.(2) making F proportional to $m \times M_{im}$ instead of $8\pi G\mu m2$, where M_{im} is the image mass of the Universe ($M_{im} \gg 8\pi G\mu m$) , hence we no longer get the previous explicit mass dependent acceleration, but instead :

$$\frac{du}{dt} \propto (\frac{m}{m_{in}}) \tag{4}$$

which is the usual result for the motion of particles in gravitational fields . (Note that $\frac{du}{dt}$ can still not be a constant for particles of different internal structure).

We therefore see that for the case of a test particle moving in the gravitational field of a straight gauge cosmic string , although in principle we expect violation of the equivalence principle it ought to be unobservable due to the presence of other masses in the Universe .

3. Cosmic strings in external gravitational fields

A procedure widely used to compute the motion of a cosmic string in a fixed background gravity field is to approximate its action with the area of the world sheet generated by the string in this spacetime[6] (Nambu action prescription). This method however already pre-supposes that the response of cosmic strings to the gravitational field is the same for strings of different mass densities , so that a geometric description for the gravity field is valid for the string-field interaction.

For the case of a string and a body of mass M_b say , in the weak field limit, their interaction can be put in the usual Newtonian form[2] . It is in $O(G)$:

$$F^{(1)} \propto -G\frac{\mu M_b}{r^2} \tag{5}$$

where F denotes force per unit string length - the superscript indicates the order in G - and r is the distance between the body and the string. Furthermore, in $O(G^2)$ we have :

$$F^{(2)} \propto -G\frac{8\pi G\mu M_b^2}{r^2} \tag{6}$$

which is the image interaction we discussed before . On writing the force on a string segment as its inertial linear mass times its acceleration both (5) and (6) yield :

$$\frac{du_s}{dt} \propto (\frac{\mu}{\mu_{in}}) \tag{7}$$

u_s being the speed of the segment , which shows no explicit mass dependence .

Consider now the situation where a long gauge cosmic string segment moves through a thermal gas . A planar overdensity will form behind it (wake). The gravitational interaction between the string and its wake has served as an example of EP violation by cosmic strings[2]. It turns out that both direct and image interactions of the wake with the string result in forces proportional to μ^2- (see eqs.(7), (11) and (15) in Ref.2)-i.e the acceleration of the string will have the form :

$$\frac{du_s}{dt} \propto (\frac{\mu}{\mu_{in}})\mu \tag{8}$$

which indeed seems incompatible with EP . The problem is that here we have a system, of variable mass . The longer the string moves through the gas the larger the extent of the wake behind it and the stronger the interaction of it with the string. Explicit mass dependence of the accelaration of ojects in variable mass systems is a standard characteristic and it is compatible with the equivalence principle, the motion of a rocket being the obvious example . To illustrate this more clearly , consider two masses, say M and m. Take M to varry with time proportional to some power of m(m =const.) e.g.

$$\frac{dM}{dt} = Cm^p, \qquad 1 < p \in N, \qquad [C] = [M]^{p-1}[T]^{-1} \tag{9}$$

Newton's law then gives for the acceleration of the mass m :

$$\frac{du_m}{dt} = CGt(\frac{m}{m_{in}})mp - 1 \tag{10}$$

which is mass dependent . Note that this would still be true even if we used a set of test masses that have shown say no composition dependence , ie $m = m_{in}$ and have -to our best accuracy - agreed to a universal free-fall in the Earth's field .

Finally it has also been argued2 that "Even in $O(G)$ cosmic strings violate the equivalence principle , since they do not attract masses but there is a non-vanishing inertial mass associated with the string ". This argument is based on the relation

$$0 = \mu \neq \mu_{in} \tag{11}$$

which has been formally derived by V.P.Frolov et.al.(1989)1, where it is also shown that the gravitational and inertial mass of a bent gauge cosmic string coincide in the weak field limit .

Here we run into difficulties with the definition of the equivalence principle . According to the definition we employed in the first page there is no contradiction. We saw in §2 that in $O(G)$ all test masses in the field of a straight gauge cosmic string have zero -i.e. the same- acceleration . In the case of cosmic strings in fixed external gravitational fields, the relevant situation would be an initially static, strictly straight, infinite gauge string in a homogeneous gravitational field ; the string will remain still, independently of its inertial mass and again we are consistent with our definition of equivalence principle .

On the other hand if we accept the equivalence principle definition : $M = M_{in}$ for all bodies we get a violation . I believe that the definition of page 1 is a better one to use; however the whole concept of equivalence principle for bodies "neutral" to gravitational fields is ill.

For practical purposes if test particles were lacking a universal free fall in the field of cosmic strings, the accretion of matter around the strings would be affected. Potential lensing images of cosmological objects in the field of strings would be distorted too, as the string would act as a prism creating series of monochromatic images . On the other hand since in most viable theories cosmic strings form as a network at the same time , they have common mass6 . As a result any mass-dependent response to external fields would be unimportant .

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