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An Optimal Thruster Configuration Design and Evaluation For Quick STEP

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Abstract. A technique to calculate the margin of safety of a multi-thruster system is defined. This technique is based on normalizing the thruster configuration matrix by the expected disturbance environment. An technique based on maximizing the margin of safety is outlined yielding an optimal configuration of a set of thrusters. The technique assumes a fixed number of thrusters, some geometrical constraints, a specific thruster controller and a specific thruster limit. Two thruster controllers are presented for one-sided thruster systems. A nonlinear pseudoinverse controller based on a modified Newton Raphson iteration is shown to have higher control authority than one based on a biased pseudoinverse controller. These controllers as well as the thruster configuration optimization technique are applied to the design of a thruster system for the Quick STEP (Quick Satellite Test of Equivalence Principle) spacecraft.

Key Words: Actuator Configuration, Controllers, Optimization, System Evaluation, Redundancy.

1. INTRODUCTION

The designer of a thruster system for the attitude and translation control of a spacecraft is faced with many design issues: how many thrusters should there be, how should they be controlled, how should they be physically distributed in terms of their position and orientation (thruster configuration), and how to evaluate different thruster configurations. A technique to calculate the margin of safety of a thruster system is presented allowing the designer to evaluate and compare various thruster system designs. An optimization technique is also developed to automatically find a thruster configuration which maximizes the margin of safety given a fixed number of thrusters, a specific type of thruster controller, a specific thruster limit and some geometrical constraints. For illustration, these methods are applied to a simple two dimensional case and then to the Quick STEP satellite [Worden 1993], an orbiting physics experiment to test the equivalence of gravitational and inertial mass.

The instruments on-board the Quick STEP satellite are kept at cryogenic temperatures and are isolated from disturbances by a drag-free spacecraft in orbit around the Earth. The liquid helium which cools the experiment continuously boils off and a system of proportional thrusters directs this helium gas in specific directions to cancel the disturbances and also maintain the desired attitude. An optimal thruster configuration for the QuickSTEP spacecraft with 16 one-sided thrusters is found by maximizing the margin of safety.

Chen [1983] first introduced the concept of minimum control authority which corresponds to the force and moment generating capability of a thruster system in its weakest output direction. Wiktor [1992] developed mathematical techniques to calculate the minimum control authority for a given thruster configuration, thruster controller and thruster limit. Among the various controllers he studied, the simplest is the minimum power controller which is based on the pseudoinverse and is a linear controller when applied to two-sided thrusters capable of generating both positive and negative forces. Since one-sided thrusters are constrained to have non-negative outputs, the linear pseudoinverse controller will not work with one-sided thrusters.

This paper, describes two types of controllers for one-sided thruster systems. Techniques to calculate the margin of safety of a thruster system are developed and then incorporated into a technique to determine an optimal thruster configuration maximizing the margin of safety. Finally, these techniques are applied to the Quick STEP spacecraft to determine an optimal thruster configuration.

2. THRUSTER CONTROL METHODS

2.1 General Control Methods

The control methods are best illustrated by a simple 2-D three thruster example (Fig. 1) with the arrows indicating the positive direction of each thruster.

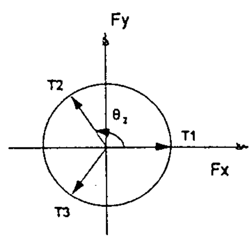

Fig. 1. 2-D three thruster system.

The force output and thruster force relation can be expressed as

$$\mathbf{F} = \mathbf{AT} \tag{2.1}$$

where $\mathbf{F}$ is a 2×1 force vector composed of F_x and F_y, respectively. $\mathbf{T}$ is the thruster force magnitude vector, composed of T1, T2 and T3, and $\mathbf{A}$ is 2×3 configuration matrix. The elements of $\mathbf{A}$ are the influence coefficients defining how each thruster affects each component of the force vector, $\mathbf{F}$. In general, A is $n\times m$ matrix, where n is the number of degrees of the object, and m is the total number of thrusters. For this example, (2.1) can be expressed as

$$\begin{bmatrix} Fx \\ Fy \end{bmatrix} = \mathbf{AT} = \begin{bmatrix} \cos\theta_1 & \cos\theta_2 & \cos\theta_3 \\ \sin\theta_1 & \sin\theta_2 & \sin\theta_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \tag{2.2}$$

Since $\mathbf{A}$ has more columns than rows, which is true for general cases, there is an infinite number of combinations of $\mathbf{T}$, which can generate the desired force $\mathbf{F}$. However, by minimizing some norm of vector T, the solution can be made unique. The three most common types of controllers corresponding to three different vector norms are [Wiktor 94]

1) *Minimum Flow Rate Controller*: $\min\|\mathbf{T}\|_1$
2) *Minimum Power Controller*: $\min\|\mathbf{T}\|_2$
3) *Minimum Peak Force Controller*: $\min\|\mathbf{T}\|_\infty$.

Using the minimum flow rate controller will yield the greatest control authority for flow rate limited thruster systems. Similarly for the other two controllers. Among the three controllers, the minimum power controller is by far the simplest to implement since the actuator command $\mathbf{T}$ is simply a linear function of the desired force, $\mathbf{F}$:

$$\mathbf{T} = \mathbf{A}^{+}\mathbf{F} \tag{2.3}$$

the matrix $\mathbf{A}^{+} = \mathbf{A}^{T}(\mathbf{AA}^{T})^{-1}$ is the 'pseudoinverse' of $\mathbf{A}$. The other two types of controllers, 1) and 3), involve solving a linear program for $\mathbf{T}$. Due to its simplicity therefore, the linear pseudoinverse controller is often used even for conditions where it does not yield the greatest control authority like flow rate limited cases for example. The controllers developed in this paper are all based on the pseudoinverse controller and are all applied to thruster systems limited by total flow rate. The total flow rate limit can be defined as

$$\|\mathbf{T}\|_1 \le m_1 \tag{2.4}$$

The minimum control authority using the pseudoinverse controller for a flow rate limited thruster system is

$$\min\|\overline{\mathbf{F}}_{2,1}\|_2 = \frac{m_1}{\|q_m^T\overline{\mathbf{A}}^{+}\|_2} \tag{2.5}$$

where $q_m^T\overline{A}^{+}$ is the row with the largest 2-norm of matrix $Q\overline{A}^{+}$, and Q is the $2^n \times n$ matrix formed by counting in binary to 2^n with 0's replaced by +1 and the 1's replaced by -1 (see [Wiktor 94] for details).

The linear pseudoinverse controller (2.3) works directly for a two-sided thruster system. However, it must be modified for one one-sided thrusters since these must satisfy the nonlinear constraint, $\mathbf{T}\ge 0$. Two types of thruster controllers for a one-sided thruster configuration will be described next. One is linear biased pseudoinverse controller, and the other is the nonlinear iterative pseudoinverse controller.

2.2 Linear Biased Pseudoinverse Controller

The linear biased pseudoinverse controller is the simplest controller for a one-sided thruster system. Given the desired force **F**, the thruster force can be computed by biasing each thruster's output.

$$\mathbf{T} = \mathbf{A}^{+}\mathbf{F} + \mathbf{T}_{n} \qquad (2.6)$$

where $\mathbf{T}_n$ is the null space vector satisfying

$$\mathbf{A}\mathbf{T}_{n} = 0,\ \mathbf{T}_{n} > 0. \qquad (2.7)$$

The constraints on **T** are

$$\mathbf{T} \geq 0 \qquad (2.8)$$

$$\|\mathbf{T}\|_{1} \leq m_{1} \qquad (2.9)$$

$\mathbf{T} \geq 0$, can also be expressed in the form

$$\min\left[\mathbf{A}^{+}\mathbf{F} + \mathbf{T}_{n}\right] \geq 0 \qquad (2.10)$$

The ***control authority*** of a one-sided thruster configuration with thruster limit (2.7 and 2.8), using the linear biased pseudoinverse controller (1) is the set of forces $\mathbf{F}_a$ given by

$$\min\left[\mathbf{A}^{+}\mathbf{F}_{a} + \mathbf{T}_{n}\right] = 0 \qquad (2.11)$$

with constraint

$$\left\|\mathbf{A}^{+}\mathbf{F}_{a} + \mathbf{T}_{n}\right\|_{1} \leq m_{1} \qquad (2.12)$$

The optimal $\mathbf{T}_n$, which makes

$$\left\|\mathbf{A}^{+}\mathbf{F}_{a} + \mathbf{T}_{n}\right\|_{1} = m_{1},$$

is

$$\|\mathbf{T}_{n}\|_{1} = m_{1} \qquad (2.13)$$

The corresponding ***minimum control authority*** is

$$\min_{\mathbf{F}_a} \left\|\mathrm{F}_{a}\right\|_{2} \qquad (2.14)$$

where $\mathbf{F}_a$ is given in (2.11). The minimum control authority given in (2.14) has to be calculated numerically. With the Matlab Optimization Toolbox [Grace 1992], it is easy to incorporate equality constraints (2.11, 2.12) into the minimization of the cost function given in (2.14).

Fig. 3 shows the control authority plots for a simple 2-D configuration of six one-sided thrusters (three two-sided thrusters). The figure shows that the control authority using the biased pseudoinverse controller (2.6) for six one-sided thrusters is much smaller than that for the pseudoinverse controller (2.3) for three two-sided thrusters. Fig. 3 also shows the control authority plot for the nonlinear iterative pseudoinverse controller which increases the control authority and is discussed in the next section.

2.3 Nonlinear Iterative Pseudoinverse Controller

The main motivation for developing the nonlinear iterative pseudoinverse controller is to increase the system control authority while still maintaining the simple form of pseudoinverse controller. The nonlinear iterative controller can generate desired force $\mathbf{F}_d$ by

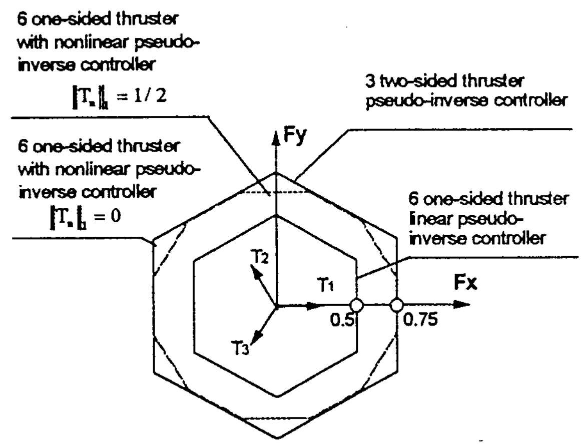

Fig. 3. Comparison of the control authorities for three pseudoinverse controllers.

$$\mathbf{T}_{pc} = \mathbf{A}^{+}\,\mathbf{F}_{c}(k) + \mathbf{T}_{n} \qquad (2.15)$$

$$\mathbf{T}_{c} = (\mathbf{T}_{pc} + |\mathbf{T}_{pc}|)/2 \qquad (2.16)$$

$$\mathbf{F}_{c}(k+1) = \mathbf{F}_{c}(k) + \left(\mathbf{F}_{d} - \mathbf{A}\ \mathbf{T}_{c}\right)\mathbf{K}_{i} \qquad (2.17)$$

where $\mathbf{F}_{c}(0) = \mathbf{F}_{d}$

The block diagram for implementing the above procedures is shown in Fig. 4. The algorithm converges very quickly: typically in only 3 iterations for the 2-D six one-sided thruster case. For Quick STEP satellite with 16 one-sided thrusters(shown later), it takes about 12 iterations, corresponding to 0.05 seconds when running (Matlab) with PC (Intel 486, 50 MHz).

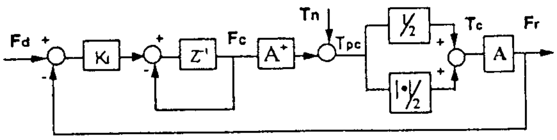

Fig. 4. Block diagram of the iterative nonlinear pseudoinverse controller for a one-sided thruster system.

The control authority for the nonlinear iterative pesudoinverse controller is defined as the set of forces

$$\mathbf{F_a} \equiv \mathbf{AT_c} \qquad (2.18)$$

so that $$\|\mathbf{T_c}\|_1 = \mathbf{m_1} \qquad (2.19)$$

where $\mathbf{T_c}$ is defined by

$$\mathbf{T_{pc}} = \mathbf{A^+ F_c} + \mathbf{T_n} \qquad (2.20)$$

$$\mathbf{T_c} = (\mathbf{T_{pc}} + |\mathbf{T_{pc}}|)/2 \qquad (2.21)$$

The ***minimum control authority*** is the shortest distance to the control authority surface

$$\min_{F_a} \|F_a\|_2 \qquad (2.22)$$

where $\mathbf{F_a}$ is defined in (2.18).
The control authority of a six, one-sided thrusters system controlled by the nonlinear pseudoinverse controller is shown in Fig. 3. The minimum control authority is greater by 50% relative to the linear biased pseudoinverse controller (2.6). With $\|\mathbf{T_n}\|_1 = 0$ (best choice), the control authority is the same as that of the three, two-sided thrusters system controlled by the standard linear pseudoinverse controller. The control authority with $\|\mathbf{T_n}\|_1 = 1/2$ is also shown in Fig. 3. The algorithm has been developed, and verified by 2-D 3-thrusters, 4-thrusters and 6-thrusters cases.

3. NORMALIZED MINIMUM CONTROL AUTHORITY AND MARGIN OF SAFETY, ms

3.1 Disturbance Boundary Model

A disturbance model is needed to both design and evaluate a thruster system. The most natural model for most spacecraft disturbance environments is the ellipsoid model,

$$\left(\frac{f_{dx}}{F_{dx}}\right)^2 + \left(\frac{f_{dy}}{F_{dy}}\right)^2 + \left(\frac{f_{dz}}{F_{dz}}\right)^2 + \left(\frac{m_{dx}}{M_{dx}}\right)^2 + \left(\frac{m_{dy}}{M_{dy}}\right)^2 + \left(\frac{m_{dz}}{M_{dz}}\right)^2 \le 1 \qquad (3.1)$$

where the upper case variables correspond to the peak disturbance levels and the lower case variables are the components of the disturbance model. A three dimensional disturbance model is shown in Fig. 5. The equivalent model in six dimensional space is impossible to visualize.

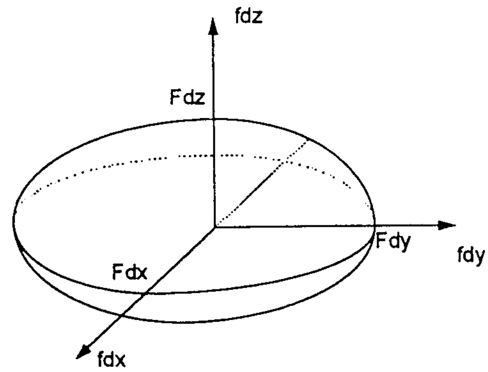

Fig. 5. Ellipsoid disturbance model.

If the control authority surface of a given thruster configuration completely contains the disturbance boundary then the thruster system has sufficient authority to overcome the worst case disturbances. This analysis is simplified by normalizing the components of the disturbance model by the worst case disturbance levels.

$$\frac{f_{dx}}{F_{dx}} = \bar{f}_{dx} \quad \frac{f_{dy}}{F_{dy}} = \bar{f}_{dy} \quad \frac{f_{dz}}{F_{dz}} = \bar{f}_{dz}$$
$$\frac{m_{dx}}{M_{dx}} = \overline{m}_{dx} \quad \frac{m_{dy}}{M_{dy}} = \overline{m}_{dy} \quad \frac{m_{dz}}{M_{dz}} = \overline{m}_{dz} \qquad (3.2)$$

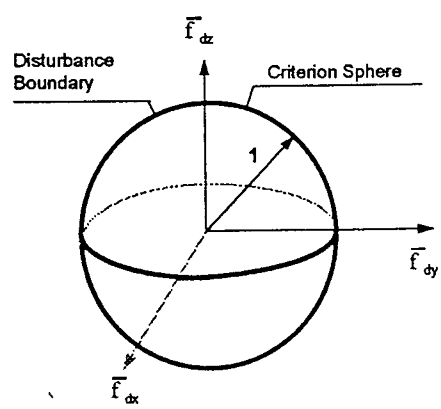

Fig. 6. Spherical disturbance boundary.

The disturbance boundary in terms of the normalized components is a sphere with radius 1 (Fig. 6).

3.2 Normalized Minimal Control Authority and Margin of Safety

In order to evaluate a thruster system relative to the normalized disturbance boundary (3.2), the output force of the thruster system must be normalized in the same way,

$$\bar{\mathbf{F}} = \begin{bmatrix} \bar{F}_x \\ \bar{F}_y \\ \bar{F}_z \\ \bar{M}_x \\ \bar{M}_y \\ \bar{M}_z \end{bmatrix} = \begin{bmatrix} F_x/F_{dx} \\ F_y/F_{dy} \\ F_z/F_{dz} \\ M_x/M_{dx} \\ M_y/M_{dy} \\ M_z/M_{dz} \end{bmatrix} = \begin{bmatrix} A(1,:)/F_{dx} \\ A(2,:)/F_{dy} \\ A(3,:)/F_{dz} \\ A(4,:)/(M_{dx} \\ A(5,:)/(M_{dy} \\ A(6,:)/M_{dz} \end{bmatrix} T = \bar{A}\, T \quad (3.3)$$

The minimum control authority of the normalized thruster system is found exactly as in Sect. 2, with A replaced by the normalized $\bar{A}$ (3.3). Now the minimum control authority is normalized. If the normalized control authority is bigger than 1, then the thruster configuration is able to handle the worst case disturbances.

Definition: Margin of Safety:

$$ms \equiv \min\|\bar{\mathbf{F}}\|_2 - 1 \qquad (3.4)$$

The margin of safety, ms, must be greater than zero in order to overcome all possible disturbances. An optimal thruster configuration is one which maximizes the margin of safety. The margin of safety, ms, is very useful in evaluating the redundancy of a given thruster system. If the margin of safety is greater than zero, ms>0, even with some thruster failures then the system is redundant.

4. OPTIMAL SEARCH METHOD AND SOME EXAMPLES

A procedure to automatically determine an optimal thruster configuration is developed based on Grace's Matlab Optimization Toolbox [Grace 1992]. The procedure involves varying the thruster configuration to maximize the normalized minimum control authority as given in (2.5), (2.14) and (2.22) with the appropriate constraints.

Two examples are presented to emphasize the importance of the margin of safety, ms, and to show how to maximize the normalized minimum control authority to obtain the optimal thruster configuration design. The two examples are 1) 2-D with three two-sided thrusters and 2) the QuickSTEP spacecraft with 6 degrees of freedom and 16 one-sided thrusters.

4.1 Example 1: 2-D with Three Two-Sided Thrusters

Assume a 2-D disturbance model, $F_{dx} = kF_{dy}$. For k=1, the optimal configuration has the three thrusters 120 degrees apart, a symmetrical configuration as expected from intuition.

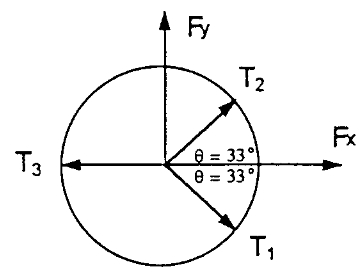

Fig. 9. The optimal configuration for k=4.

For k=4, $|F_{dy}| = 0.28$, $|F_{dx}| = 1.12$. the optimal configuration is shown in Fig. 9. The minimum control authority is 1.02. which is smaller than $|F_{dx}|$. No conclusion can be drawn about whether the system is good enough. However, the normalized control authority is 1.07. The margin of safety is ms=7.0%, from which one concludes that the system is able to overcome the worst disturbances with 7% margin of safety.

4.2 Example 2: Quick STEP Spacecraft

The Quick STEP spacecraft is a light satellite (Fig. 10) with strict constraints on its size and the placement of its thrusters. The ideal location for the thrusters is on a ring in the middle of the spacecraft. Since the thrusters must be very close to the spacecraft, only one-sided thrusters can be used. For ease of assembly, four thrusters are clustered together at one location with orientations, as shown in Fig. 10. Four clusters are evenly distributed around the ring, with identical α and β, Hence for this configuration design, there are only two variables to maximize the margin of safety. The angles, α and β are constrained to be bigger than 20 degrees to avoid plume impingement with the side of the spacecraft. For the optimal configuration design, the nonlinear pseudoinverse controller is used.

The optimal solution is $\alpha = 20°$ $\beta = 20°$ and ms = 101.77%. For the disturbance model 110%

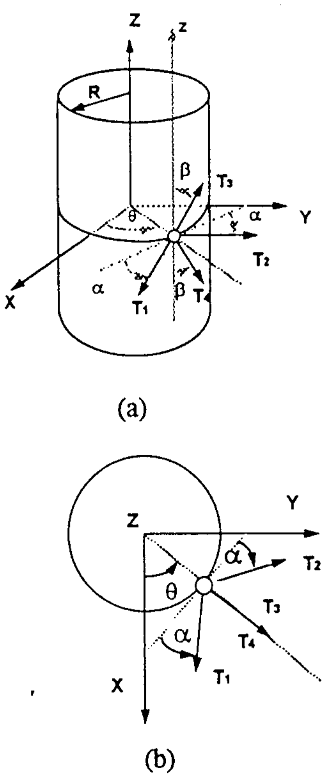

Fig. 10. Quick STEP spacecraft thruster configuration.

of the magnitudes of the nominal disturbances are used as worst disturbance model. For the same configuration, the linear biased pseudoinverse controller has ms = -0.15%, which means it can not satisfy the control requirements. For redundancy test, if one of the thrusters along z-axis fails the ms is 35%, while if one of the thrusters in x-y plane fails ms is 67%. Therefore, the above optimal configuration for Quick STEP satellite is a robust design for the nonlinear iterative pseudoinverse controller.

7. CONCLUSIONS

The normalized minimum control authority is a powerful tool for designing thruster systems. By maximizing this value an optimization algorithm can automatically determine the optimal thruster configuration. The margin of safety, ms, derived from the normalized minimum control authority indicates whether the thruster system has sufficient control authority to overcome disturbances. It also permits the redundancy of a thruster system to be evaluated. The nonlinear iterative pseudoinverse controller has been proven superior to the linear biased pseudoinverse controller for the Quick STEP spacecraft. For missions like GP-B, where the one-sided thrusters are formed in pairs and treated as two-sided thrusters, the nonlinear controller could be used when some thrusters fail making the normal linear pseudoinverse controller invalid.

8. ACKNOWLEDGMENT

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