GRAVITATION, RELATIVITY AND PRECISE EXPERIMENTATION*

C. W. F. Everitt
W. W. Hansen Laboratories of Physics
Stanford University, Stanford, California 94305

ABSTRACT

Experimentalists working in gravitation and relativity are heirs to a noble tradition of precise measurement which began in 1798, when Henry Cavendish, at the age of 67, carried out Michell's torsion balance experiment to measure the gravitational constant and "weigh the Earth". The tradition was carried forward in the Michelson-Morley experiment, the Eötvös experiment and in C. V. Boys's repetition of the Cavendish experiment, begun in 1889, which was perhaps the earliest attempt to analyse the influence of the size of an apparatus on its accuracy. Boys found that the accuracy of the Cavendish experiment was improved by making it smaller.

During the past fifteen years two new areas of research, space technology and large scale cryogenics technology have opened the way for an important class of modern gravitational experiments which depend on measuring small angular or linear displacements of suspended bodies just as the Cavendish and Eötvös experiments did. This article describes four such experiments (1) the Stanford Gyro Relativity experiment, (2) a new cryogenic test of the equivalence of gravitational and inertial mass to be performed both on Earth and in space, (3) the Stanford-LSU-Rome gravitational wave antennas, (4) an experiment to measure the Lense-Thirring nodal drag on a satellite and also obtain new geophysical data by means of precise Doppler ranging measurements between two counter-orbiting satellites in polar orbit around the Earth.

The fundamental limits to precision of these and similar experiments are reviewed.

CONTENTS

---

*Supported in part by NASA Grant 05-020-019.

1. THE CLASSICAL EXPERIMENTS 1798 - 1922

Just as the gravitational interaction occupies a special place in theoretical physics, so does experimental research on gravitation and relativity in physics history. Progress here is over a longer time span than elsewhere and experiments are fewer. Although space technology and cryogenic technology have brought enough new ideas in the last fifteen years to revolutionize our field, a single person can still read through the account of every experiment done in it. Those who experiment on gravitation are more likely to be written off as crackpots than to face the threat of instant competition by which somany scientists are perplexed.

The noble tradition of precise measurement to which we are heirs began in 1798 with Henry Cavendish (1731-1810). The aristocratic recluse who did so much and published so little was 67 years old when he "weighed the Earth". In performing this wonderful experiment Cavendish drew on a lifetime of careful observation from the accurate measurements of gas densities that underlay his discoveries of hydrogen, carbon dioxide, and even, unknown to himself, of argon, to the beautifully ingenious unpublished experiment of 1773 by which he verified the inverse square law of electrostatics to one part in 50, an accuracy far higher than that reached ten years later by Coulomb. The fundamental idea for measuring the gravitational constant, however, came not from Cavendish but from another 18th century Englishman too little known to physicists, the Reverend John Michell (1724-1793). Michell, who is best remembered as the discoverer of binary stars and the first man to make realistic estimates of stellar distances, had invented the torsion balance around 1750 independently of Coulomb. He conceived the principle of measuring the gravitational force between two large masses and two smaller ones attached to a torsion balance, and then determining the elastic constant of the suspension wire from the period of the torsional oscillations. Michell built part of the apparatus. On his death it passed to F. J. H. Wollaston of Cambridge (1762 -1823),brother of the better-known W. H. Wollaston, and from him to Cavendish. Cavendish saw the crucial need to isolate the balance-arm from disturbing forces, especially forces of convection; he had the simple but profound idea of placing the apparatus in a thermally isolated room and operating it by remote control. This difference in manner of observing, said Cavendish,"rendered it necessary to make some alterations in Mr. Michell's apparatus; and as there were some parts of it which I thought were not so convenient as could be wished, I chose to make the greater part of it anew" [1]. So the first modern physics experiment was born.

Figure 1, reproduced from Cavendish's paper, is a longitudinal section through the instrument and the room in which it was placed. Two lead balls, two inches in diameter, were hung from a six foot deal rod braced by a silver wire, a design for the balance arm fixed on by Cavendish as being strong and light, but at the

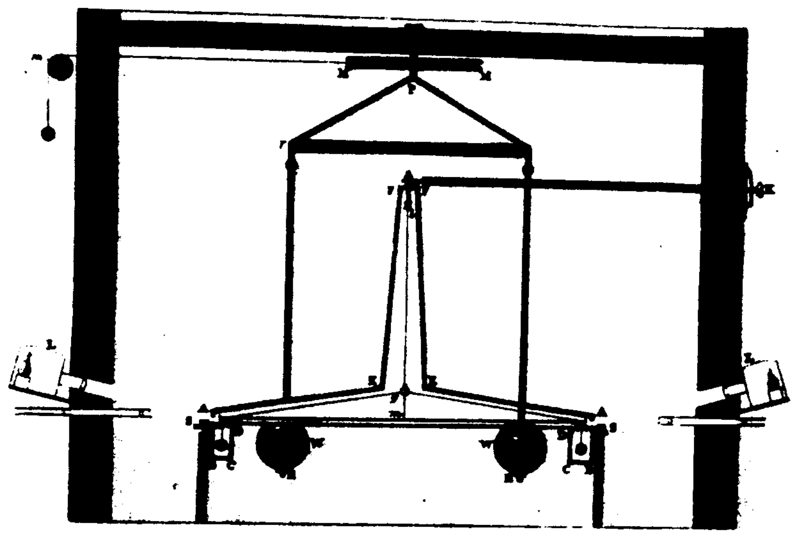


FIGURE 1

THE CAVENDISH EXPERIMENT

same time meeting with very little air resistance and having a form simple enough to compute easily its attraction to the external masses. The balance-arm was suspended inside a close mahogany box "to defend it from the wind", by means of a silvered copper wire which had a zero adjustment worked by remote control through the worm gear and rod FK. The box stood on piers and was levelled by four adjustment screws SS. The external masses were lead spheres, one foot in diameter, suspended first by iron rods, and afterwards by copper ones, from a beam which could be swung between two positions on either side of the mahogany case by a pulley and weights outside the building. The lead spheres were brought up within 0.2 inches of the case by coming against wooden stops fastened to the walls of the building: "I find" (wrote Cavendish) "that the weights may strike against [the stops] with considerable force without sensibly shaking the instrument". The final critical detail was the method of reading out the angular position of the balance arm. Slips of ivory engraved with scales to 1/20 inch were placed inside the instrument case at each end, very close to but not quite touching the balance arm, and corresponding vernier scales subdividing the main divisions into five parts were mounted on the arm. The scales were illuminated by collimating lanterns and observed through telescopes let into the end walls of the room. In this way the position of the arm was "observed with ease to 100 ths of an inch and . . . estimated to less" which corresponds to an angular resolution of about ±20 arc-seconds over the six foot balance arm.

Cavendish, at the accuracy he aimed for, encountered most of the disturbing forces that trouble modern experimenters engaged in observations of this kind. Two that he escaped were Brownian motion of the balance arm and seismic noise. For a system with 15 minute period (the longest he used) the Brownian motion is about 8×10^{-3} arc-seconds, three orders of magnitude below his limit of observation. Seismic noise might have contributed 0.1 arc-seconds of motion, also well below his limit. On the other hand Cavendish's study of magnetic disturbances is all too familiar. He decided in advance to make the balance of non-magnetic materials but cautiously checked for residual effects from the iron rods on which the large masses were hung. There was a displacement amounting to 3% of the gravitational deflection which disappeared when the iron was replaced by copper. Later Cavendish noticed a drift in the apparatus which he thought might come from the lead weights picking up magnetization in the earth's field. He hung the weights on pivots and reversed them each morning but saw no improvement. He then exercised the grand principles of experimental physics: if you are uncertain whether a disturbing effect is small try making it bigger and see how bad it is. He replaced the lead balls with ten inch magnets and reversed them instead. The displacement was negligible; a discovery which cast doubt on the earlier interpretation of the disturbance from the iron rods and left Cavendish very puzzled. He next investigated whether the drift in the torsion balance might be from elastic after-working in the suspension wire -- a phenomenon he accurately described though its discovery is usually credited to Kohlrausch in 1863. Having disposed of this possiblity (again by deliberately exaggerating the effect) Cavendish in a <u>tour de force</u> of experimental detective work traced the drifts to convection currents generated by the different thermal time constants in the system. The large lead masses did not cool overnight as much as the mahogany case surrounding the balance arm; their radiation induced temperature gradients in the case and hence convection in the air inside. The Bernoulli pressures then set up transverse forces driving the suspended masses towards the sides of the case. Cavendish was able to keep track of this effect by sealing a small thermometer in each of the large masses and hanging another thermometer next to the instrument case, reading the thermometers through the same telescope with which he observed the positions of the balance.

With these methods, having also made other admirable calculations, including a correction for the lengthening of the period of the torsion pendulum by air drag and an elaborate analysis of the attraction of the mahogany case on the balance arm, Cavendish achieved a measure of the transverse masses to 2×10^{-10} g and fixed the mean density of the earth as 5.48 ± 0.10 [2]. The modern value is 5.57. It is pleasing that this "heroic experiment" [3] should have had so brilliant a result, proving as it did that the Earth's core has a density much greater than the typical densities of 2.5 to 3.0 for surface rocks.

The Cavendish experiment was repeated three or four times up to 1880, but not until C. V. Boys (1855 - 1944) renewed the attack on it in 1889 did Cavendish find a worthy successor. Boys introduced two practical advances on Cavendish's apparatus, added one profoundly important new experimental idea, and made innumerable improvements in detail. His first innovation, the one that started him thinking about the experiment, was his invention in 1887 of a method of drawing fine quartz fibres for the torsion suspension. Quartz fibres are still unrivalled in strength and elastic qualities. The second advance on Cavendish was less original but equally important: this was to measure the angular displacement by an optical lever, by observing telescopically the reflection of a scale from a mirror mounted on the balance arm. The optical lever had been invented in 1826 by J. C. Poggendorf and been applied to many instruments by Gauss, Weber, William Thomson and others: Thomson's mirror galvanometer had even been the subject of a poem by Maxwell*: but it was this simple device in combination with the quartz fibre that made possible Boys's third and greatest innovation: his analysis of the influence of the size of the apparatus on its sensitivity. Boys was, I suspect, influenced by William Froude's researches during the 1850's on the scaling laws for ship models in towing tanks, as well as the wider extension of the dimensional analysis around that time: be that as it may he made the remarkable discovery that the accuracy of Cavendish's experiment was improved by making it smaller. Boys's balance arm was $\frac{5}{8}$ inch long rather than 6 feet. The advantages of a smaller apparatus is first that

* The lamplight falls on blackened walls
And streams through narrow perforations
The long beam trails o'er pasteboard scales
With slow decaying oscillations
Flow,current, flow, set the quick light-spot flying
Flow current, answer light-spot, flashing, quivering, dying.

in parody of Tennyson's "Bugle Song".

whereas the angular sensitivity remains constant if all the parts are scaled down, the external masses can be relatively larger. The attacting spheres used by Boys were $4\frac{1}{4}$ inches in diameter. To get an equivalent effect with Cavendish's apparatus their diameter would have had to be 27 feet. In order to realize this advantage with a short balance arm Boys separated the masses vertically as shown in Figure 2. A second advantage of smallness was the reduction of thermal

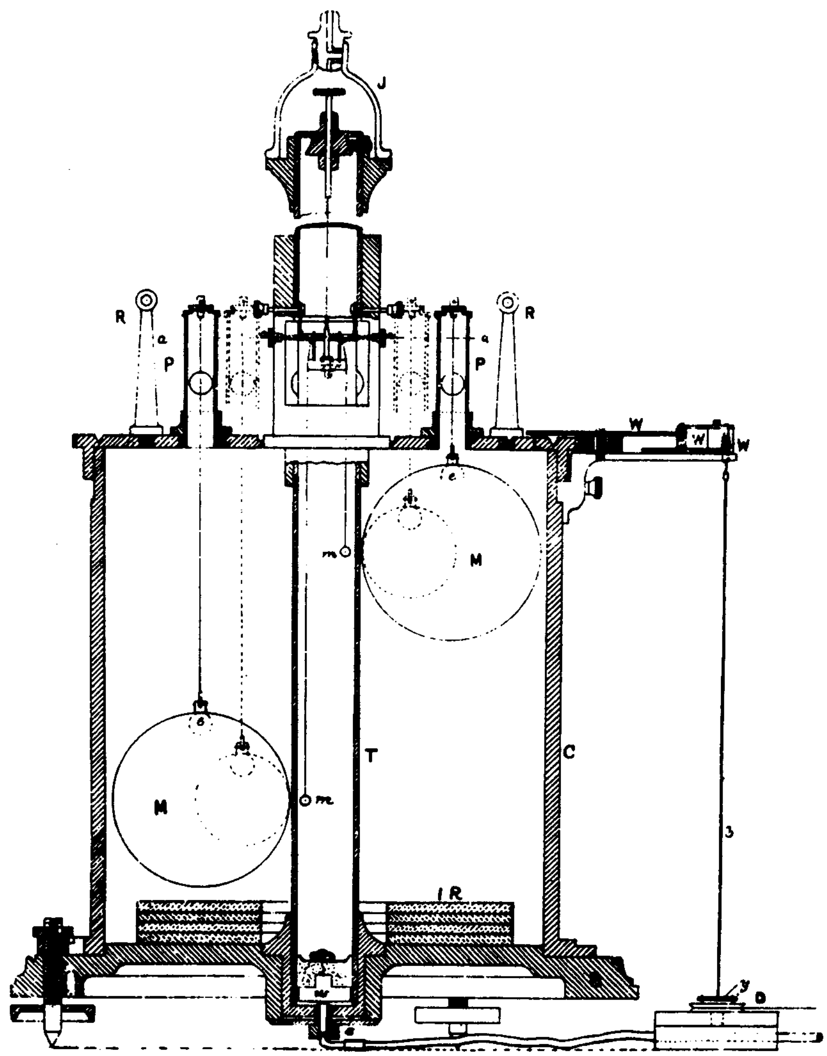


FIGURE 2: THE BOYS EXPERIMENT

time constants and temperature gradients: the convection currents which plagued Cavendish were eliminated. Boys considered evacuating his apparatus, but the lowest pressures then attainable were around 10^{-3} torr, a highly undesirable regime to work in because of the radiometer effect. The viscosity of air was a limiting factor in Boys' experiment. Unlike Cavendish Boys did reach the level of seismic disturbance. He worked at nights and weekends to avoid vibrations from passing trains; he saw the shock from an earthquake with epicenter in Romania, and the shaking from high winds in trees near his laboratory. All of these are problems painfully familiar to modern workers in our field.

Boys's optical lever was itself a triumph. Poggendorf had achieved a angular resolution of 5 arc-seconds, a little better than Cavendish's measurement. Boys could measure angular displacement of 0.7 arc-seconds (well below the 10 arc-second diffraction limit of the mirror) in accurately calibrated steps over a total scale width of 33,600 arc-seconds, that is very nearly to 16 bit accuracy --a precision few modern instruments reach. Consideration of this point leads to another view of Boys's achievement. He had separated the problems of optimizing his detector (the suspended bodies) and optimizing his position monitor (the optical lever). This is a recurrent theme in experimental gravitation.

Questions of mechanical stability were central in another classic investigation from the same era as Boys: the Michelson-Morley experiment. I have sketched the background elsewhere [4] . Michelson undertook the experiment in 1881 in response to a remark of Maxwell's about the change in velocity of light to be expected if the Earth is moving through a fixed aether of the kind advocated by Fresnel. Any earth-based experiment to see this effect requires a round-trip measurement and therefore depends on the second order quantity $(v/c)^2$ which Maxwell thought too small to detect. The alternative to Fresnel's theory was Stokes's theory of a convected aether. Michelson's interferometer compared the velocities of two light beams split and reunited in two orthogonal paths. Figure 3 illustrates the original apparatus with mirrors cantilevered on long steel arms. Although the,structure was sensitive to vibrations and thermal distortions, the result was convincingly negative, and therefore in Michelson's view, decisive in support of Stokes's aether. Afterwards Lorentz pointed out an error in Michelson's analysis: he had not seen that a light ray moving transversely to the earth's motion would also suffer a second order time delay half that of the longitudinal ray, so the expected shift in the interference pattern would be half Michelson's prediction. Lorentz also discovered a difficulty in Stokes's aether. It is commonly said that Lorentz's observation made the error in Michelson's first experiment so large as to discredit it, but this is an

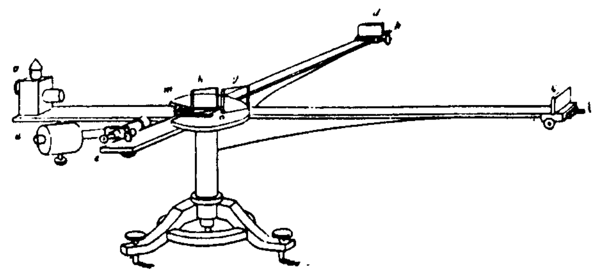


FIGURE 3 MICHELSON'S APPARATUS (1881)

exaggeration. Still a measurement so fundamental needed repeating. At the 1884 Baltimore Lectures Sir William Thomson (Lord Kelvin) used his magnetic persuasiveness to convice E. W. Morley that it must be done again. Figure 4 illustrates the famous apparatus of 1887 with mirrors mounted on a granite slab floating on mercury and path lengths increased by multiple reflection to eight times that of the original apparatus. The measurement was accurate to one-twentieth of an interference fringe or about 1% of the hypothetical displacement.

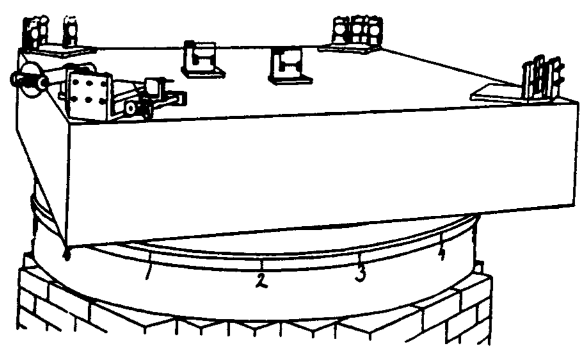


FIGURE 4

THE MICHELSON-MORELY EXPERIMENT (1887)

The many, still far from satisfactory, discussions of the historical and scientific significance of this experiment and its offshoots need not delay us here. The most searching theoretical analysis is still probably the one given by H. E. Ives in 1937. To the experimentalist an equal interest attaches to the change from Figure 3 to Figure 4, which speaks of one thing: mechanical stability. A measurement to a twentieth of a fringe corresponds to a 130 Å displacement of any mirror. A steel I-Beam 100 cm long and 4 cm deep deflects under its own weight by 1 arc-second, which means that a 2 cm mirror resting on it is tilted back at its upper edge by 2000 Å through beam flexure. Low frequency seismic vibrations of amplitude 10^{-3} g then cause motions of 2 Å and vibrations near the resonant frequency of the beam are much larger. Thermal distortions are even more serious. A temperature difference of 0.02° across the beam warps it by 1 arc-second. Such were the effects Michelson and Morley had to contend with. When, as in the Stanford Gyro Relativity experiment, one aims for measurements of 0.001 arc-second accuracy, questions of mechanical stability shape the whole experimental approach.

The famous experiment on the equivalence of gravitational and inertial mass, done first in 1890 by Baron Roland von Eötvös (1848-1919), and repeated over the years up to 1922 by Eötvös and his two colleagues Desiderius Pekár and Eugen Fekete, is notable more for the originality of its idea than for novelty in measurement technique. The apparatus was a gravity gradiometer invented by Eötvös for geophysical work, whose sensitivity to field gradients made it in some ways rather ill-adaped to an equivalence principle measurement. As a piece of instrumentation it fell far short of Boys's apparatus. But nothing can detract from the originality of the idea. Rarely does anyone think up an experiment based on a slight modification of a standard instrument that advances the accuracy of a fundamental measurement by four orders of magnitude. When H. H. Potter repeated Newton's pendulum measurement of equivalence in 1923 his accuracy was only one part in 10^5, three orders of magnitude short of Eötvös's. Figure 5 illustrates Eötvös's apparatus [5]. The experiment consisted in suspending two masses of different composition from a torsion balance, rotating the torsion head through 180° and checking whether the balance arm turns through exactly the same angle. Since both masses are simultaneously subject to gravity and to centrifugal acceleration from the Earth's rotation, any departure from equivalence causes a torque

$$\Gamma^E = \tfrac{1}{2}\eta\, MDf \sin\alpha \qquad (1)$$

where M is the sum of the masses, D the length of the balance arm, f the driving acceleration (the centrifugal acceleration of 1.4 cm/sec²), α the angle between the balance arm and north, and η the Eötvös ratio

$$\eta = \frac{2\left[\left(\frac{M}{m}\right)_A - \left(\frac{M}{m}\right)_B\right]}{\left(\frac{M}{m}\right)_A + \left(\frac{M}{m}\right)_B} \qquad (2)$$

where m and M are the inertial and passive gravitational masses of the two materials A and B. The quantity ηf may conveniently be called the Eötvös acceleration.

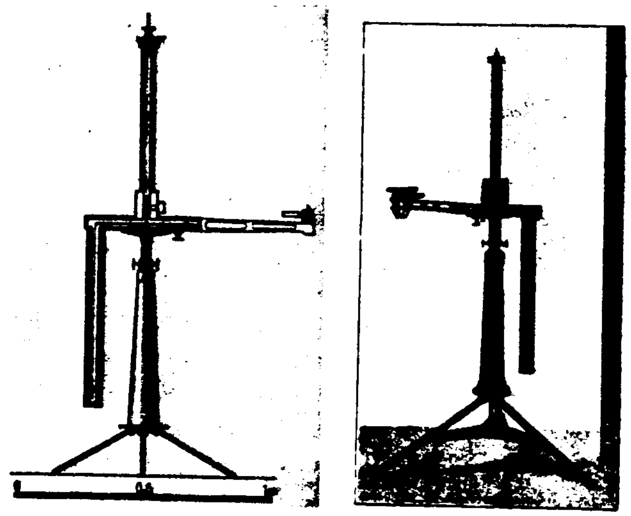


FIGURE 5 EÖTVÖS' APPARATUS

In appearance Eötvös' apparatus resembles Boys' in that the masses are hung at different levels, but this feature was conceived independently of Boys, and for a different reason, namely, to make the balance sensitive to vertical gravity gradients. The chief limitation of the apparatus aside from its sensitivity to gradients, was elastic hysteresis in the suspension. Hysteresis errors can be avoided, as Eötvös and his colleagues recognized, by keeping the apparatus fixed and looking for a daily torque from the sun's acceleration. This was the approach taken by Roll, Krotkov and Dicke [6] and by Braginsky and Panov [7] in their repetitions of the Eötvös experiment. Using the sun as source is not without its disadvantages. The driving acceleration is only 0.6 cm sec^{-2}, as compared with 1.4 cm sec^{-2} for the Earth's centrifugal acceleration, and the apparatus becomes susceptible to other diurnal disturbances -- for example Earth tides and temperature effects driven by the day-night cycle.

2. MORE RECENT EXPERIMENTS AT ROOM TEMPERATURE AND THE APPROACH TO FUNDAMENTAL LIMITS

In 1925 W. J. H. Moll and H. C. Burger invented a device to read an optical lever automatically from the differential heating of the light spot on a split thermopile [8]. The angular resolution of this "thermoelectric relay" was about 1/20 arc-second. Applying it to a very stable galvanometer they noticed a small jittering motion in the output, of amplitude about 0.5 arc second, which they attributed to microseismic disturbance, but which G. Ising [9] in "one of the most vital papers in the history of physical observations" [10] identified as the Brownian motion of the galvanometer. Ising had discussed this instrumental limit as early as 1917, following Smoluchowski and de Haas-Lorentz, but his paper of 1926 -- one hundred years after Poggendorf's invention of the optical lever -- was its first recognition in an actual instrument. The corresponding statistical noise limit in an electrical circuit, the Johnson noise, was discovered by J. B. Johnson and interpreted by H. Nyquist one year later, in 1927.

A fundamental measurement limit exists for the optical lever as well as for the suspended body. The detector measures the location of the diffraction pattern from the mirror by a finite number of photons; its output is disturbed by random fluctuations in intensity of the two halves of the beam. Optical levers and autocollimators that reach photon noise limits have been widely built; the best resolution so far being in R. V. Jones's [10] split-grating optical lever which can detect angular changes of a 2 mm x 2 mm mirror to about 10^{-5} arc-sec in a 10Hz bandwidth. Formally the limiting angular resolution with diffraction limited optics is

$$\delta\Theta \sim \frac{1}{D^2}\sqrt{\frac{hc\bar{\lambda}}{\phi\varepsilon}\Delta\nu} \tag{3}$$

where h is Planck's constant, c the velocity of light, D the dimension of the mirror, $\Delta\nu$ the bandwidth, ϕ the light flux on the mirror, $\bar{\lambda}$ the colour temperature of the source, and ε the overall photoelectric efficiency, that is, the ratio of the number of photoelectrons excited in the detector to the number of photons impinging on the mirror, or, in other words, the product of the optical efficiency of the light system with the quantum efficiency of the photodectector.

For a body of inertia I suspended from a torsion fibre, and lightly damped, having a damping coefficient β and a natural period τ_0, the Brownian motion occurs as oscillations of period τ_0 coherent over times of order $Q\tau_0$, where $Q = \pi/\beta\tau_0$ is the quality factor of the system. The amplitude $\langle\Delta\Theta\rangle$ of the oscillations is

$$\langle \Delta\Theta \rangle \sim \frac{\tau_0}{2\pi} \sqrt{\frac{kT}{I}} \qquad (4)$$

where k is Boltzmann's constant and T the absolute temperature. The magnitude of $\langle \Delta\Theta \rangle$ for a body of mass 10 gm and diameter 1 cm, hung at room temperature with a period of 10 sec, is 0.6 arc-sec. Defining a discrimination factor $\partial_B \equiv \frac{\delta\Theta}{\langle \Delta\Theta \rangle}\sqrt{\frac{\nu_0}{\Delta\nu}}$ which gives the precision in $\delta\Theta/\langle \Delta\Theta \rangle$ in a given fraction of the natural oscillation period, and writing the moment of inertia of the suspended body proportional to ρD^5, where ρ is the density and D the characteristic dimension we have

$$\partial_B = C \left(\frac{hc}{k}\right)^{\frac{1}{2}} \left[\frac{\rho D}{\tau_0{}^3 T}\right]^{\frac{1}{2}} \left[\frac{\bar{\lambda}}{\varepsilon\phi}\right]^{\frac{1}{2}} \qquad (5)$$

where the terms in the first square bracket characterize the suspended body and those in the second square bracket characterize the optical lever. The quantity hc/k has the value 1.4 cm^{-1} $°K^{-1}$. The quantity C depends on the shapes and relative size of the mirror and suspended body: for a sphere surmounted by a weightless circular mirror of equal diameter C is 0.063. A criterion for resolving the Brownian motion is that ∂_B be less than unity, though greater resolution may be useful. R. V. Jones and C. W. McCombie [11] used a discrimination factor of about 5×10^{-4} in their experiments on the statistical mechanical limits on galvanometers. Equation (5) tells the light flux needed for a given discrimination; alternatively it tells the dimensions of an apparatus that can usefully be operated with a given optical lever. The discrimination requirement becomes more severe as the size and density of the body are increased, the period shortened and the temperature lowered.

The first measurement in our field that approached a Brownian limit was the negative experiment on magnetism and the Earth's rotation started in 1949 by P. M. S. Blackett. Blackett had become interested in the hypothesis due originally to Arthur Schuster (1851 - 1934) that the primary magnetic fields of the Earth, the sun and other massive rotating bodies are a fundamental property of their rotation to be understood eventually in terms of some unified field theory. As formulated by Blackett the hypothesis may be written as an equation between magnetic field and a virtual current density vector

$$\text{curl}\ \underline{H} = 4\pi\beta \frac{G^{\frac{1}{2}}}{c} \rho\ a\ \underline{\omega} \qquad (6)$$

where G is the gravitational constant, c the velocity of light, $\underline{\omega}$ the absolute angular velocity, ρ the matter density at distance a from the rotation axis, and β is a numerical constant with value estimated from the known fields of the Earth, the sun and certain stars as about 0.3. A gold cylinder of dimension 10 cm X 10 cm rotating with the Earth generates on this hypothesis a magnetic field of 3.4×10^{-8} gauss at 5 cm above its surface. Blackett [12] developed an extremely sensitive astatic magnetometer, applying design principles that has been worked out in 1926 for the Paschen galvanometer by H. A. Daynes [13] and A. V. Hill [14] . Hill, who later achieved fame as a physiologist, had extended Ising's work to show that with a given optical lever there is an upper limit to the useful suspension period τ_0 of the galvanometer and hence a lower limit on current sensitivity. Blackett found that the astatic magnetometer, when optimized with respect to the choice and shape of magnet, had a minimum detectable field H_0 proportional to $1/\tau_0 D^{\frac{1}{2}}$ set by the Brownian motion; but there was a practical upper limit to τ_0 of about one minute set by the second order changes d^2H/dt^2 in the ambient magnetic field, and practical upper limits on the dimension D of the magnets set by the optical design and the reaction field of the magnets on the specimen to be measured. Blackett's most sensitive instrument was capable of detecting a field of 3×10^{-10} gauss in a single observation of 30 seconds duration, an improvement of three orders of magnitude over previous magnetometers. Its moment of inertia was 0.165 gm cm^2 which made the amplitude of Brownian motion with a 30 sec. period 0.7 arc-sec. The experiment showed that no rotational magnetic field existed greater than $[4.2 \pm 2.6]\%$ of the prediction of equation (6). As an explanation of the Earth's magnetic field the hypothesis was disproved. Interest in the magnetic fields of massive rotating bodies continues, however. The extreme Kerr-Newman solution for an isolated rotating black hole yields of gyromagnetic ratio in physical units of $\sqrt{G/c}$, corresponding to Blackett's formula with a β of unity, while Ruffini and Treves [15] have shown that even in the classical limit a magnetized sphere spinning <u>in vacuo</u> acquires a surface charge and has a gyromagnetic ratio proportional to the same quantities.

The equivalence principle experiment of Roll, Krotkov and Dicke [6] also called for a study of fundamental limits with results that are in instructive contrast to Blackett's. In Blackett's magnetometer, with its relatively small suspended body and short observation time, the dominant limitation was Brownian motion, and since the amplitude of the motion was only one order of magnitude less than the diffraction width of the mirror Blackett was able to ignore photon noise and made do with an old-fashioned visual optical lever simpler even than the one used by Boys. Very different was the situation of Roll, Krotkov and Dicke. Their balance had a moment of inertia of 270 gm cm^2 , three orders of magnitude larger than Blackett's, a natural period of four minutes and a Brownian motion of 0.09 arc-sec amplitude. With the design goal of measuring the Eötvös ratio to 1 part in 10^{11}, the detector had to be capable of resolving a signal of 4.5×10^{-4} arc-sec with 24 hour period. This, though a factor of 200 less than the Brownian motion was relatively easy to separate from it because the period was so long. On the other hand, the resolution, being four orders of magnitude less than the diffraction width of the mirror, called for a sophisticated optical lever. Not only did the resolution have to approach photon noise limits: the apparatus had to be stable, optically and mechanically, to 10^{-3} arc-sec a day. For mechanical stability Roll, Krotkov and Dicke established close temperature equilibrium throughout the apparatus. Drifts in the optical lever were minimized by using a single photomultiplier with a vibrating wire at the image plane, oscillating across the diffraction image with frequency ν (3kHz). With the image exactly centered on a detector of this kind the photomultiplier sees only even harmonics of ν, but as the balance turns and shifts the pattern off-centre the fundamental frequency begins to appear in the output. The phase relative to the driving oscillator tells the sign of the displacement; for small displacements the amplitude is proportional to the rotation angle. Vibrating wire or vibrating slit detectors have been used in many instruments. They have two merits. Changes in sensitivity of the photomultiplier do not affect the null point, and the angular sensitivity can be calibrated internally from the ratio of the fundamental frequency to the first even harmonic, independent of the light intensity and photomultiplier gain. The chief question is the stability of the centre of vibration of the wire.

The principal limitations on Roll, Krotkov and Dicke's experiment were (1) seismic disturbances (2) temperature effects, (3) gravity gradient torques. Seismic noise coupled from pendulum to torsional modes of the balance through non-linearities in the system. Large shifts were observed from construction activity near the instrument site as well as quarry blasting five miles away; like Boys the experimenters were driven to working mostly at weekends. Temperature changes caused distortions of the optical lever, torques on the balance and drifts in the electronics. Roll, Krotkov and Dicke considered it hopeless to account for all the effects individually; instead they controlled the temperatures as best they could and correlated the results with the measured temperature coefficients -- exactly the procedure Cavendish had followed 170 years earlier.

The gravity gradient torque deserves special comment because of its importance in later experiments. Consider a balance with unequal moments of inertia I_1, I_3 situated at a distance R from a point mass M. It will experience a torque

$$\Gamma^g = \frac{3}{2}(I_3 - I_1)\frac{GM}{R^3}\sin 2\alpha \tag{7}$$

where α is the angle of the principal axis of the balance to the vector $\underline{R}$. Since Γ^g depends on $\sin 2\alpha$ it is a maximum when the principal axis is at 45° to $\underline{R}$; furthermore the sun's gravity gradient exerts a torque with a period of 12 hours in contrast to the 24 hour period of the Eötvös torque.

The quantity $(I_3 - I_1)$ may be written $J_2 m l^2$, where m is the mass of the balance, l the radius of gyration and J_2 the quadrupole coefficient. For a balance with two masses made in half rings of diameter D the ratio of amplitudes of the gravity gradient and Eötvös torques is

$$\frac{\Gamma^g}{\Gamma^E} = \frac{3\pi}{8}\frac{J_2 D}{\eta f}\frac{GM}{R^3} \tag{8}$$

or in the particular case when M is the source of Eötvös acceleration

$$\left(\frac{\Gamma^g}{\Gamma^E}\right)_{\text{source}} = \frac{3\pi}{8}\frac{J_2 D}{\eta R} \tag{9}$$

where R is the distance to the source. For the sun R is 1.5×10^{13} cm, so to make the gravity gradient torque less than the Eötvös torque J_2D has to be less than 140: a modest requirement. It is a different story, however, if one tries to improve the measurement by using a torsion balance in a satellite with the earth as source. Then R is 7×10^8 cm and to make Γ^g/Γ^E unity J_2D must be less than 8×10^{-3} cm for an experiment to measure η at the 10^{-11} level and less than 8×10^{-9} cm for an experiment at the 10^{-17} level. Roll, Krotkov and Dicke had a J_2D of 3×10^{-2} cm; a practical manufacturing limit in a balance of reasonable size is 3×10^{-4} cm, five orders of magnitude larger than one would like in a 10^{-17} experiment. The torsion balance in a satellite is not a good idea.

It is useful to transpose (8) into a condition on the distance r which a body of density ρ and diameter d may be allowed to approach the balance before causing a disturbance comparable to the Eötvös torque

$$\frac{r}{d} > 0.85\left(\frac{GJ_2D}{\eta f}\right)^{1/3}\rho^{1/3} \tag{10}$$

For Roll, Krotkov and Dicke's apparatus with $J_2D \sim 3 \times 10^{-2}$ and $\eta f \sim 6 \times 10^{-12}$ r/d had to be less than $6.6\,\rho^{1/3}$. A 100 kg man could not approach closer than

4m or a 10-ton truck closer than 18 m without disturbing the balance. With more ambitious design goals the restrictions become still more severe.

The foregoing arguments, due originally to Roll, Krotkov and Dicke, and in their present form to P. W. Worden, Jr., lead to a discussion of the size of the apparatus that would have appealed to Boys. From equation (1) the Eötvös torque increases as D^4 as the apparatus is scaled up. This being so it is tempting to argue, as J. Faller has done, that Earth-based equivalence principle experiments can be made enormously more sensitive by using a bigger apparatus. The catch is that Γ^g/Γ^E gets worse. Consider an attempt to improve the measurement of η by four orders of magnitude by means of a factor of ten increase in the size of the apparatus. The lowest plausible J_2 in the larger size is about 10^{-3}. Then from equation (10) no man can be allowed within 90 m of the apparatus; no truck within 400 m. Or consider atmospheric pressure effects. Taking ρ for air as 10^{-3} gm cm^{-3}, one finds that pressure differences in the nearby atmosphere as small as 0.2 mm can upset the balance, while the minimum r/d for a hurricane 200 km in diameter with a 20% pressure drop is 10. No hurricane can be allowed within 2000 km of the apparatus.

Although the Brownian motion in Roll, Krotkov and Dicke's experiment was 200 times the designed sensitivity, it was, as we have seen, characterized by the four-minute period of the balance and could be averaged over the 24 hour period of the measurement. It is important to grasp how far averaging can go. According to the mechanical counterpart of Nyquist's formula the mean square fluctuation torque on a body of inertia I during an observation time S is

$$\langle\Gamma^2\rangle = 2I\beta kT/S \tag{11}$$

where β is the damping coefficient. For a balance of total mass M with the two bodies formed in half-rings of diameter D, the mean value of the Eötvös torque through each half cycle is $2MD\eta f/\pi^2$. The moment of inertia is $MD^2/4$. The root mean square error in determining η in a time long compared with 12 hours is

$$\langle\eta\rangle \sim \frac{3.4}{f}\sqrt{\frac{\beta kT}{MS}} \tag{12}$$

where the numerical factor 3.4 is $\pi^2/2\sqrt{2}$. The damping coefficient β may be replaced by $\pi/\tau_0 Q$, where Q is the quality factor and τ_0 the period of the balance. Roll, Krotkov and Dicke had $f = 0.6$ cm sec^{-2}, $M = 90$gm, $\tau_0 = 230$sec; according to the discussion on page 458 of their paper the natural Q of the balance was 10^5. The limit from (12) on determining η in one daily cycle is 1.5×10^{-13}: two orders of magnitude below the observed experimental limit of

3×10^{-11}.

The Nyquist limit on equivalence principle experiments was first discussed by V. B. Braginsky [16]. Equation (12) is due to P. W. Worden, Jr. and C. W. F. Everitt [17] who based their analysis on the discussion by C. W. McCombie [18]. It is worth emphasizing how fundamental equation (12) is. Except for slight differences in numerical factor every equivalence principle experiment is subject to it. No elaborations to the apparatus, however useful they may be on other grounds can change the fluctuations associated with natural damping. The addition of a controller with negative damping, for example, may give an effective Q higher than the natural damping: it does not reduce thermal noise. Reference to this point brings out one of the few mistakes in Roll, Krotkov and Dicke's paper. They added a controller with positive damping to kill off the thermal oscillations of the balance, making it in effect critically damped, and said that the noise was thereby reduced through a reduction in the apparent temperature of the system. This statement is plainly wrong. An increase in natural damping would increase the noise; an addition of feedback damping should ideally leave the noise unchanged. The only sense in which feedback might be said to reduce the noise is that for a given Q the noise is less if damping is provided dynamically rather than dissipatively, in which case the feedback could be thought of artificially as lowering the temperature. Possibly this was the meaning intended by Roll, Krotkov and Dicke.

Although Roll, Krotkov and Dicke's result was two orders of magnitude from the limit set by equation (12), Braginsky [7] seems to have thought it was limited by Nyquist fluctuations. Perhaps he assumed the feedback generated noise. For critical damping, with a Q of unity instead of 10^5, equation (12) yields a limit of 3×10^{-11} on η, very near the observed limit. Of course if the feedback servo has significant internal losses it can add noise, just as the external circuit of a galvanometer does. However, since Roll, Krotkov and Dicke adjusted the feedback gain to minimize the observed noise [19] the electronic damping must have been primarily dynamic not dissipative, so the experiment was most unlikely to have been limited by Nyquist fluctuations. Be that as it may in 1971 Braginsky and Panov repeated the experiment with an undamped system. Their apparatus had several differences in design from Roll, Krotkov and Dicke's. The balance was somewhat larger; it was made with eight bodies rather than three to reduce gravity gradient torques; and the period was much longer -- six hours instead of four minutes. The longer period increased the angular deflection for a given Eötvös signal, the predicted deflection for an η of 10^{-12} being 2×10^{-2} arc-sec as compared with 5×10^{-5} arc-sec in the Roll-Krotkov-Dicke apparatus. The Brownian motion was also larger, the characteristic amplitude, as calculated from (4), being 15 arc-sec instead of 0.09 arc-sec. The damping coefficient β was not much different from Roll, Krotkov and Dicke's: the natural damping time for a balance of moment of inertia 400 gm cm^2 was about two years as compared with one year for a 270 gm cm^2 body. Again the fundamental limit on η from equation (12) was 1.5×10^{-13}. The estimated limit from measurement was 0.9×10^{-12}.

Null experiments are always difficult to assess. The published accounts of Braginsky and Panov's experiment are much less complete than Roll, Krotkov and Dicke's. The chief advantage seems to have been a quieter seismic environment; the chief shortcoming the method of reading the orientation of the balance. A laser beam was reflected over a distance of 12 m to a photographic film on a rotating drum, the drum being inclined to the light path at an angle of 0.2 rad to increase the effective path length to 50m. The spot was 0.3 cm wide; its position was estimated microscopically to 5×10^{-4} cm. Such precision seems surprising in visual examination of so wide a diffraction image, especially since the spot was swinging with 6 hour period over a range of ±0.7 cm (the observed displacement, which corresponds to a balance swing of ±15 arc-sec, quite close to the calculated Brownian motion). Qualms on that point were partly offset, however, by averaging data from three observers. Still more surprising to anyone who has strugged with long optical paths is the absence of errors from atmospheric refraction. According to the mirage formula a beam of light passing a distance ℓ through a gas of refractive index n is deflected under a transverse temperature gradient dT/dx through a distance x_n

$$x_n = \frac{n-1}{2n} \frac{\ell^2}{T} \frac{dT}{dx} \tag{13}$$

If the light source stands next to the detector x_n has to be doubled. With $\ell = 12$ m, $x_n/\ell < 10^{-7}$ rad, and n for air at STP = 1.00029 the transverse temperature gradient cannot exceed 7×10^{-5} °C cm^{-1}. The daily temperature cycle of the room was 2×10^{-2} °C [7] . The stability seems hard to achieve. A comparable problem is thermal distortion of the apparatus, including the building in which it was housed. A body of length ℓ and expansion coefficient α under a transverse temperature gradient dT/dx is deflected through a distance x_α equal to $\frac{1}{2}\alpha\ell^2 dT/dx$, which also has to be doubled if the source stands next to the detector. For masonry $\alpha = 7 \times 10^{-6}$ °C^{-1}, so to make $x_\alpha/\ell < 10^{-7}$ the transverse temperature gradient must not exceed 1.1×10^{-5} °C cm^{-1}. Again this is an extraordinarily small gradient.

The thermal distortion formula is sometimes conveniently written in terms of the thermal conductivity K and transverse heat flux Φ for the apparatus, where $\Phi = KdT/dx$. Then

$$x_\alpha = \tfrac{1}{2}\left(\frac{\alpha}{K}\right) \ell^2 \Phi \qquad (14)$$

and any apparatus used in precise angular measurements should be constructed of materials with low α/K.

Irrespective of the final assessment of Braginsky and Panov's experiment equations (13) and (14) show how undesirable long optical paths are. The angular errors x_n/ℓ and x_α/ℓ both increase with ℓ. Whatever success Braginsky and Panov had depended on the enormous period of the balance, which, though it did not alter the noise limit from equation (12) did increase the signal amplitude for a given Eötvös ratio.

It is interesting to return to Hill's argument that there is an upper limit to the useful period of a torsion balance set by competition between the Brownian motion and the resolution of the optical lever. In its original form the argument is not rigorous but it may be modernized using equation (3) and (11). Equation (11) gives the fundamental limit on torque measurement from the Nyquist criterion. The question is whether the optical lever has enough resolution to reach that limit. Assume the torque is either constant or sinusoidal with a period much longer than the natural period τ_0 of the balance. The deflection Θ due to a torque Γ is Γ/κ, and κ, the elastic constant of the suspension, equals $4\pi^2 I/\tau_0{}^2$. The minimum torque the optical lever can resolve is therefore

$$\delta\Gamma = \frac{4\pi^2 I}{\tau_0{}^2}\,\delta\Theta \qquad (16)$$

where $\delta\Theta$ is the quantity from equation (3). To reach the fundamental limit, $\delta\Gamma$ must be less than $\langle\Gamma^2\rangle^{\frac{1}{2}}$ from equation (11). The ratio $\delta\Gamma/\langle\Gamma^2\rangle^{\frac{1}{2}}$ is identical with the discrimination equation (5) except that the term in the first square bracket of (5) is multiplied by $2\pi\,\tau_d/\tau_0$, where $\tau_d = Q\tau_0$ is the characteristic damping time of the system. Transposing to a limit on τ_0

$$\frac{\tau_0{}^4}{\tau_d} > C'\left(\frac{hc}{k}\right)\left[\frac{\rho D}{T}\right]\left[\frac{\lambda}{\varepsilon\phi\partial^2{}_N}\right] \qquad (17)$$

where C' is $2\pi\, C^2$ from (5) and ∂_N is a factor similar to ∂_B defined from the ratio of sensor noise to Nyquist limit rather than sensor noise to Brownian amplitude. Again ∂_N less than unity is the criterion for a good measurement. For both Dicke's and Braginsky's experiments, the combination of quantities $C'\rho D$ was about 10^{-5}, τ_d about 10^8, $\epsilon\phi$ about 10^2 erg/cm², $\overline{\lambda}$ about 6×10^{-5} cm, making the shortest allowable suspension period τ_0 in each case about 10 sec.

Further discussion hinges on practical considerations. If one is interested as Hill was, in making multiple observations with a galvanometer, any suspension period longer than that from equation (17) is undesirable in theory and probably pointless in practice. For equivalence principle experiments on the other hand, with a fixed driving frequency, there is no intrinsic reason for sticking to short periods. Provided the optical lever reaches photon noise limits the practical bounds are set by null shifts and non-linearities. Null shifts in the readout become progressively less troublesome as the signal amplitude is increased by increasing τ_0, but if τ_0 is too long Brownian motion, or the signal itself, may cause errors through optical or mechanical non-linearities in the system. Alternatively, as in Blackett's magnetometer, there may be an upper limit on τ_0 from non-linearities in the time rate of change of an external disturbing torque. Such questions can only be resolved by detailed study of the performance limitations on each individual experiment.

An apparatus of a quite different kind where Brownian motion is critical is the much discussed gravitational wave antenna developed by J. Weber from 1959 onwards, and copied by several other groups since 1970. An aluminum bar of length 1.55 m and diameter 0.66 m was hung from steel cables inside a vacuum chamber, piezoelectric strain gauges cemented to the bar sensed displacements down to 10^{-15} cm in 150 mS sampling time. The resonant frequency was 1660 Hz and the mechanical Q about 10^5. The bar responds to gravitational waves much as a Hertzian dipole responds to radio waves. For a favourably polarized source whose line of sight is perpendicular to the long axis of the bar the energy absorbed in the n^{th} longitudinal mode from an incoming gravitational wave pulse of energy spectral density $F(\nu)$ is

$$E_s = \frac{8}{\pi} \left(\frac{G}{c}\right) \frac{M}{n^2} \frac{v_n^2}{c^2} F(\nu_n) \tag{18}$$

where M is the mass of the bar, ν_n the resonant frequency of the mode, and v_n the velocity of sound at ν_n. In 1967 Weber began to detect pulses not characteristically seismic in origin, and in 1970, after coincidence checks between bars 1600 km apart in Maryland and Argonne, announced statistically significant events which he identified as gravity waves from a source near the centre of the galaxy.

It is convenient to defer the main discussion of noise limits on gravitational wave detectors until Section 5. Controversy over the results apart, Weber's apparatus is interesting in an instrumental view for its analogy to those just described on torsionally suspended bodies. One is looking for a gravitational impulse on a massive detector (the bar) subject to Brownian motion using a sensor (the piezoelectric crystal) which is also subject to noise. With some assumptions about noise in the bar and the crystal, Gibbons and Hawking [20] found an optimum sampling time τ_s for Weber's apparatus. In times shorter than τ_s the sensitivity is limited by Johnson noise in the crystal; in longer times it is limited by the task of resolving the ringing signal in the bar from Brownian motion. Good coupling is critical. Under optimum conditions gravitational pulses can be detected if they impart to the bar an energy

$$E_{min} > 0.8\ kT_B \sqrt{\frac{\tan\,\delta}{\beta Q_B}} \tag{19}$$

where T_B and Q_B are the temperature and quality factor of the bar, $\tan\,\delta$ the loss tangent of the crystal, and β the coupling coefficient, that is, the proportional of elastic energy in the bar that can be extracted electrically from the crystal in one cycle. For Weber's bar with a β of 5×10^{-6} the optimum τ_s was 0.4 sec, and a single measurement could detect a gravitational pulse with energy approaching $kT_B/12$.

Experiments on gravitation may be divided into two classes depending whether the test object is a massive body, as in the Cavendish experiment or an electromagnetic wave, as in the Michelson-Morley experiment. Apart from the experiments just described, some improved Cavendish experiments, and laser ranging measurements on the Earth-moon system, most of the new experiments completed since 1960, have been on electromagnetic radiation. Examples are the Mössbauer measurements on gravitational redshift by Pound, Rebka and Snider [21] ; the radar-ranging measurements of the time delay of signals passing the sun suggested by Shapiro in 1963; the measurements by Dicke and Goldenberg [22] and by Hill [23] and his colleagues on the sun's visual oblateness; and Hill's daylight star detector for the Arizona solar telescope. The Dicke-Goldenberg experiment is based on a rotating slit detector; the daylight star detector on a vibrating slit counterpart to the vibrating wire for Roll, Krotkov and Dicke's optical lever. Ultimately all experiments on electromagnetic waves are limited by photon noise; the practical limits are complex. In optical telescopes the limitations are atmospheric seeing and mechanical stability. In radar and laser ranging the disturbances are many and large; they have to be elaborately modelled and taken out in data analysis.

The next sections describe four gravitational experiments with massive bodies currently being developed at Stanford University and elsewhere. Three -- the Gyro Relativity experiment, the Stanford-LSU-Rome gravitational wave experiment and a new equivalence principle experiment -- depend on measuring small angular and linear displacements of laboratory-sized objects, just as the Cavendish and Eötvös experiments did. Ultimately they too are limited by Brownian motion and measurement noise. Since these are lowered by lowering the temperature a *prima facie* case exists for applying cryogenic techniques to the experiments, and in fact all three are done at low temperatures. However the immediate reasons for low temperature operation are not noise, but other more technical considerations: improved mechanical properties of materials, reduction of disturbances from residual gas and black body radiation, and the use of superconducting magnetic shields to stabilize the field or reduce it to very low values. Critical to all three experiments is the use of Josephson junction devices as position detectors. With so much in common one might think the experiments could be reduced to common design principles. This is not so. Each is subject to its own constraints which make for variety; plenty of room exists for specific ingenuity.

In the fourth experiment the test bodies are a pair of drag-free satellites in polar orbit around the Earth.

3. THE GYRO RELATIVITY EXPERIMENT

The idea of testing General Relativity by gyroscopes was discussed soon after Einstein's theory appeared by deSitter [24] , Schouten [25] , Fokker [26] and Eddington [27] . Several of these authors suggested looking for the small geodetic precession of the Earth due to its motion about the sun, but since the change of the Earth's polar axis from this cause is only 8×10^{-3} arc-sec/year, it is a factor of five smaller than the uncertainty in the Chandler wobble. During the 1930's Blackett [28] examined the prospects for a laboratory experiment but concluded that with then-existing technology the task was hopeless. In 1959, two years after the launch of Sputnik, and following also the improvement in gyroscope technology since World War II, L. I. Schiff [29] and G. E. Pugh [30] independently proposed experiments with gyroscopes in space.

3.1 Description of the Experiment

Schiff's formula for the relativistic precession-rate of an ideal torque-free gyroscope in free fall about a rotating massive sphere is

$$\underline{\Omega} = \frac{3GM}{2c^2R^3}(\underline{R} \wedge \underline{v}) + \frac{GI}{c^2R^3}\left[\frac{3\underline{R}}{R^2}(\underline{\omega}_e \cdot \underline{R}) - \underline{\omega}_e\right] \tag{20}$$

where $\underline{R}$ and $\underline{v}$ are the coordinate and velocity of the gyroscope, and M, I and ω_e are the mass, moment of inertia and angular velocity of the central body. The first term gives the spin-orbit, or geodetic, precession Ω^G due to motion of the gyroscope through the gravitational field; the second gives the spin-spin, or motional, precession Ω^M due to rotation of the central body. Integrated around an elliptic orbit and expressed as the rate of change $\dot{\underline{n}}_s$ of the unit vector $\underline{n}_s$ defining the gyro spin axis ($\dot{\underline{n}}_s = \Omega \wedge \underline{n}_s$) the geodetic term is

$$\dot{\underline{n}}_s^G = \frac{3(GM)^{3/2}}{2c^2a^{5/2}(1-e^2)^{3/2}}\left[\underline{n}_0 \wedge \underline{n}_s\right] \tag{21}$$

where a and e are the semi-major axis and eccentricity and $\underline{n}$ is the unit orbit-normal. The motional term is

$$\dot{\underline{n}}_s^M = \frac{I\omega_e}{2c^2a^3(1-e^2)^{3/2}}\left[\underline{n}_e \wedge \underline{n}_s + 3(\underline{n}_0 \wedge \underline{n}_e)(\underline{n}_0 \wedge \underline{n}_s)\right] \tag{22}$$

where $\underline{n}_e$ is the unit vector defining the Earth's axis. The geodetic term therefor scales as $a^{-5/2}$ and the motional term as a^{-3}. The geodetic precession lies in the orbit plane and in a 500 km circular orbit around the Earth amounts to 6.9 arc-sec/year. The motional precession depends on the orbit inclination c'. For a gyro perpendicular to the Earth's axis its magnitude in a 500 km polar orbit is + 0.05 arc-sec/year where the positive sign implies a precession in the same sense as the Earth's rotation; in an equatorial orbit it is -0.10 arc-sec/year; in an orbit where $\cos 2i = -\,^1/_3$ ($i = 54°16'$) it vanishes. The goal of the experiment is to make a gyroscope with residual drift-rate 10^{-16} rad/sec (6×10^{-4} arc-sec/year). If achieved this would measure Ω^G to 1 part in 10,000 and Ω^M in polar orbit to 1 part in 70.

The Schiff motional effect is sometimes miscalled the Lense-Thirring effect. The Lense-Thirring effect is a nodal drag of the orbit plane of a moon in orbit around a massive rotating body. It is measured by the twin-satellite experiment described in Section 6 and has different magnitude and different vector form from the Schiff effect.

The gyroscope precessions are measured in the framework of the fixed stars. An experiment to measure them requires one or more gyros and a reference telescope pointed at an appropriate star. In addition to the principal terms there are three smaller relativistic effects measurable by a gyro with 6×10^{-4} arc-sec/

drift-rate: (1) the geodetic precession the Earth's motion about the sun (0.021 arc-sec/year), (2) the higher order geodetic term calculated by Barker and O'Connell [31] and by Wilkins [32] from the Earth's quadrupole mass-moment (0.010 arc-sec/year in a 50 km polar orbit), (3) deflection by the sun of the starlight signal for the reference telescope. During the time of year when the line of sight approaches the sun the starlight deflection superimposes on the gyro drifts an apparent motion away from the sun which reaches a maximum at closest approach. It can be extracted from the data by in effect turning the experiment around and using the gyros as reference for the telescope. For Rigel, which is 30° from the ecliptic plane the maximum deflection is 0.016 arc-sec. There are also much larger non-relativistic deflections of the apparent position of the star due to aberration: ±20 arc-sec in the ecliptic plane from the Earth's motion about the sun, and ±5 arc-sec in the orbit plane from the satellite's motion about the Earth. The aberrations are calculated very exactly from the known ratios of the orbit-velocities to the velocity of light. They supply handy scaling checks of the gyro and telescope readouts, since the relativistic effects can be expressed as ratios of the known aberrations.

Various possibilites exist for the choice of orbit and the configurations of the gyroscopes. The simplest is an ideal polar orbit with two gyro pairs, one parallel and antiparallel to the Earth's axis and sensitive only to Ω^G, the other parallel and antiparallel to the orbit-normal and sensitive only to Ω^M. The telescope is then pointed at a bright star on the celestial equator orthogonal to both gyro axes. In reality no star is in the right place and no orbit is exactly polar. The Newtonian regression of the orbit-plane from the Earth's quadrupole mass-moment causes a mixing of terms, as a result of which some people have argued that the experiment cannot distinguish Ω^G and Ω^M unless the orbit is within a few arc-minutes of the poles. This opinion is mistaken; the nodal regression actually makes inclined orbits richer in relativity information than polar orbits. The information that can be extracted from different orbits depends on practical considerations reviewed elsewhere [33].

A better configuration is a spacecraft that rolls slowly around the line of sight to the star, containing two gyroscopes with axes parallel to the boresight or the telescope and two at right angles to the telescope and approximately parallel and perpendicular to the Earth's axis. As before one of the perpendicular gyros primarily sees Ω^G and the other Ω^M; both serve also as accurate roll references. With a star lying on the celestial equator and an ideal polar orbit the two gyros parallel to the boresight see a signal periodic in the roll-rate, of amplitude $t\sqrt{\Omega^{G^2} + \Omega^{M^2}}$ and phase $\tan^{-1} \Omega^M/\Omega^G$. I have shown elsewhere [33] how the separation of terms is carried through in inclined orbits with real stars. The advantage is that torques on the gyros, and drifts in the gyro and telescope readouts are strongly averaged by roll. A typical roll-period is 25 minutes.

Figure 6 is a general view of the experiment. The telescope, four gyroscopes and the proof mass for a drag-free control system from a single package mounted inside a superinsulated dewar vessel containing 800 liters of liquid helium and designed to maintain cryogenic temperatures for two years. Boil-off of helium

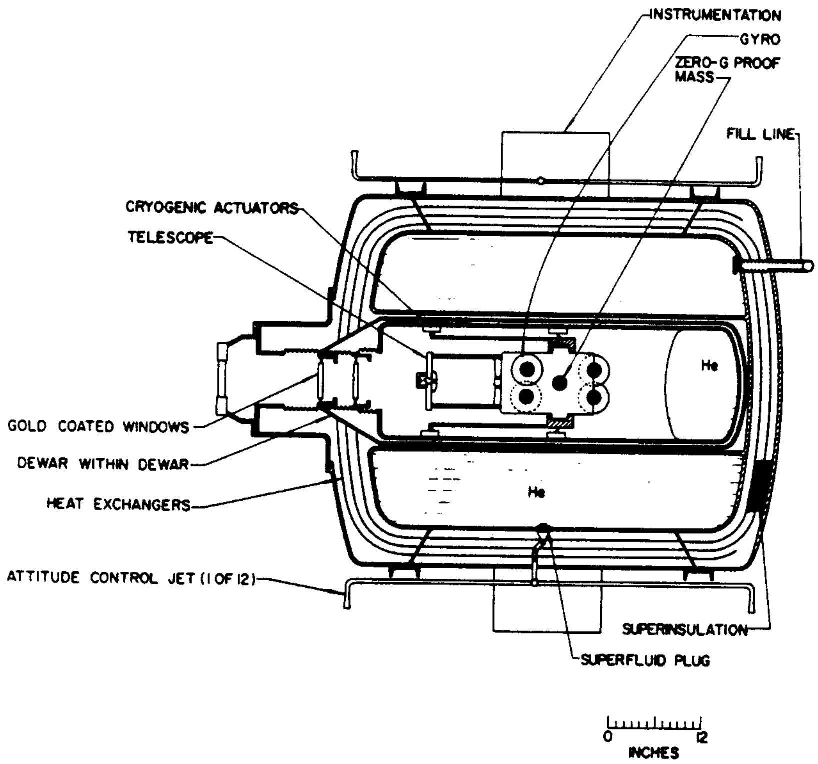


FIGURE 6: GENERAL VIEW OF GYRO RELATIVITY SATELLITE

in the zero-g environment is controlled by a porous plug device invented by Selzer, Fairbank and Everitt [34] . The gyroscopes are quartz spheres, coated with superconductor and suspended electrically in a quartz housing attached to the telescope. Each gyro is surrounded by a spherical superconducting magnetic shield. The telescope is a folded Schmidt-Cassegrainian system of 5½ inch aperture and 150 inch focal length, also made entirely of fused quartz.

For mechanical stability no cements are used in the structure: the gyro and telescope parts are held together in molecular adhesion by optical contacting.

Pointing control of the spacecraft is based on signals from the telescope, switched automatically to the gyroscopes during the portion of each orbit when the star is occulted. Thrust is obtained from the helium boil off from the dewar, which is copious enough to mechanize in a very smooth proportional control system. Drag-free control is mechanized through the same thrustors referenced to the internal proof-mass. Making the satellite drag-free helps in two ways: it improves averaging of residual accelerations on the gyroscopes and it reduces errors in the orbit determinations needed in analysing relativity data. The first drag-free satellite, with the DISCOS control system developed by members of the Stanford Department of Aeronautics and Astronautics, was successfully launched by the U. S. Navy in July 1972 [35] . The performance level was 5×10^{-12} g.

The superconducting gyroscope has to operate in a magnetic field below 10^{-7} gauss. To obtain this the experimental package is held in a separate cylindrical dewar vessel within the main dewar, having a diameter between 12 and 16 inches, built entirely from non-magnetic materials. The inner dewar is first cooled in an independent ultra-low field facility developed at Stanford by B. Cabrera [36] ; after trapping the low field in the gyro shields it is removed from the facility and slid into place in the main dewar.

3.2 Principles of Gyro Design.

A gyroscope may be a spinning body, a nucleus, a superconducting current or a circulating light beam. Present laser gyros are orders of magnitude from the performance needed. Nuclei and currents have the shortcoming of being highly susceptible to magnetic torques; their gyromagnetic ratios are up to 10^{14} times those of ordinary bodies; a free precession He^3 gyroscope would have to be in a field below 10^{-20} gauss to do the experiment [37] . The only horse in the race is a supported spinning body, and no elaborate thought is needed to see that the most torque-free body is a very round, very homogeneous sphere. The problems come down to four: the size of the sphere, and how it is to be supported, spun up and read out.

Size turns out not to be critical. The drift-rate $\dot{\underline{n}}_s^r$ of the gyro spin vector $\underline{n}_s$ due to some extraneous non-relativistic torque $\underline{\Gamma}^r$ is $\underline{\Gamma}^r \wedge \underline{n}_s / I\omega_s$ where I is the moment of inertia and ω_s the spin angular velocity. Substituting

$(8\pi/15)\,\rho r^5$ for I and replacing ω_s by v_s/r, where v_s is the peripheral velocity, we have

$$\dot{\underline{n}}_s^r = \left(\frac{15}{8\pi}\right) \frac{\underline{\Gamma}^r \wedge \underline{n}_s}{\rho r^4 v_s} \tag{23}$$

with a limit on v_s from elastic distortion under centrifugal forces given by

$$(v_s)_{max} = 1.88 \left(\frac{\Delta r}{r}\right)_{max} \left[\left(\frac{E}{\rho}\right)^{\frac{1}{2}} (1 - 11\sigma/28)\right] \tag{24}$$

where E is the Young's modulus and σ the Poisson's ratio for the ball, and $\left(\frac{\Delta r}{r}\right)_{max}$ the maximum allowed difference between the polar and equatorial radii.

The torques may be divided into two categories: those related to the surface area of the ball and those related to its volume. Each surface dependent torque Γ^σ is proportional to (area) x (radius) x $\sigma(r)$, where $\sigma(r)$ is a function which in some instances is constant and in others depends on deviations of the ball from perfect sphericity. Each volume dependent torque is proportional to (density) x (volume) x (radius) x $\phi(r)$, where $\phi(r)$ is a function measuring deviations from perfect homogeneity. Over a fair range of radii $\sigma(r)$ may be taken proportional to r^s and $\phi(r)$ proportional to r^v where s and v each lie between 0 and 1. Thus Γ^σ varies as $r^{(3+s)}$ and Γ^ϕ as $r^{(4+v)}$ and from (23) the drift rates $\dot{\underline{n}}_s^\sigma$ and $\dot{\underline{n}}_s^\phi$ from all torques in these categories vary as $r^{(s-1)}$ and r^v. Thus some errors increase and some decrease with increasing rotor size; in neither case is there much advantage to a change of diameter.

The actual rotor is a ball 4 cm in diameter made from optically selected fused quartz, homogeneous in density to 1 part in 10^6 and spherical to a few parts in 10^7. It is coated with superconducting niobium and supported inside a quartz or ceramic housing by voltages applied to three mutually perpendicular pairs of electrodes. Figure 7 illustrates piece parts of a ceramic housing manufactured by Honeywell for experiments at Stanford. The electrodes are 2 cm diameter circular pads 4×10^{-3} cm from the ball. The suspension system used in most of the work was designed by the late J. R. Nikirk [38] . It holds the ball against accelerations up to 5 g by 20 kHz signals of amplitude between 2 and 3 kV rms. The ball position is sensed by a 1 MHz 2 V signal. The suspension servo has a bandwidth of 600 Hz and long term centering stability good to 10^{-5} cm. In space the support voltage is about 0.5 V.

The gyro is spun to 200 Hz in 30 minutes by passing gas at approximately 16 torr pressure through circumferential channels around the ball, after which the gas is pumped out and the ball runs freely in a 10^{-9} torr vacuum. The configuration of the channels is seen in Figure 7. To prevent electrical breakdown and reduce

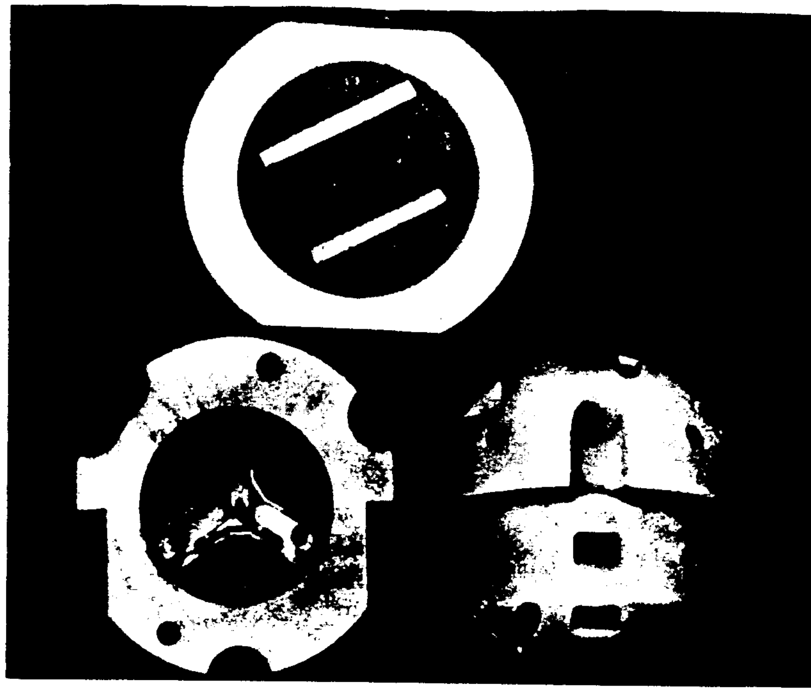


FIGURE 7 : CERAMIC GYRO HOUSING

gas drag in the cavity the channels are surrounded by raised lands 5×10^{-4} cm from the ball, and differential pumping is provided by ports in the housing. About 95% of the gas exits at high pressure through the main channels; the rest seeps over the lands and is exhausted through a separate low pressure system. Optimization of the design is due to Bracken and Everitt [39] . In space, where the support voltages are low, differential pumping might seem unnecessary, but the pressure needed to spin without it is much higher (about 0.5 atmosphere) and the ball is therefore subject to large gas forces which cannot readily be overcome except by increasing the voltages. So it is best to stick to the same design.

The gyro is read out by magnetic observations of the London moment in the spinning superconductor. According to the London equations of superconductivity there is in a rotating superconductor a magnetic moment aligned with the instantaneous spin axis which reduces in spherical geometry to a dipole of magnitude $M_L = \frac{mc}{2e} r^3\omega_s = 5 \times 10^{-8} r^3\omega_s$ gauss cm^3. Figure 8 illustrates the principles of the readout. The ball is surrounded by a superconducting loop connected to a sensitive Josephson junction magnetometer. Any change in

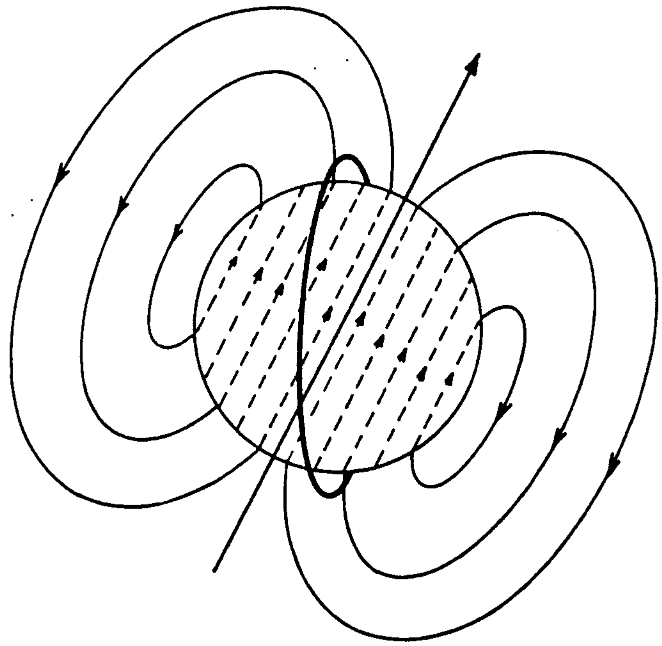


FIGURE 8

PRINCIPLES OF THE LONDON-MOMENT READOUT

orientation of M_L changes the flux through the loop and can therefore be measured by the magnetometer. Three mutually perpendicular loops give a three axis readout.

The following are some of the advantages of the London moment readout:
(1) The angular resolution in a 100 radian bandwidth with realistic coupling to commercially available shake-tested SQUIDs is 1 arc-sec, or 0.001 arc-sec after 10^4 sec integration. Better performance is likely with more advanced magnetometers. Existing optical readouts give 15 arc-sec resolution in a 100 radian bandwidth. (2) application to homogeneous rotors. Other readouts for mechani-

cal gyroscopes depend on observations tied to the body axes of the rotor and require a rotor with unequal moments of inertia. The London moment is aligned with the instantaneous spin axis, which cones about the angular momentum vector at a rate ω_p in inertial space nearly equal to ω_s ($\omega_p < \omega_s \pm \Delta I/I$) and with an amplitude Θ_s moving slowly between limits of order $\Theta_I \; \Delta I_{13}/I$ and $\Theta_I \; \Delta I_{12}/I$, where Θ_I's are maximum and minimum values of the body cone, and ΔI_{12}, ΔI_{13} are the residual inertia differences. With $\Delta I/I \sim 10^{-6}$ the coning of the spin axis is never more than a fraction of an arc-second and averages to much less: the London moment readout works regardless of polhoding. Other readouts do not. This is a point of crucial significance since a homogeneous rotor is essential to the experiment. (3) The gyro drift from reaction currents in the readout is completely negligible -- below 10^{-5} arc-sec/year. (4) The centering errors due to a displacement t of the gyro are negligible with the London moment readout since they vary as $(t/r)^2$, whereas in standard optical readouts they vary as (t/r) and may cause null drifts up to 2-3 arc-sec. (5) The readout has the high intrinsic linearity and null stability of superconducting circuits and shields. (6) mechanization as an all angle readout. A Josephson junction magnetometer has a saw-tooth response quantized in fixed steps of hc/2e, each of which can be resolved with great precision. By combining the normal feedback measurement with flux counting techniques, an all angle readout may be developed having a resolution of twenty or more bits per quadrant, or 0.1 arc-sec throught the range [40]. The best conventional angle encoders have resolutions of 17 bits per quadrant.

The all angle readout is not needed in the Gyro Relativity experiment, but would be in applying the gyroscope in inertial references for an astronomical telescope.

The choice of magnetic readout, electrical suspension and gas spin up neatly separates problems in the three areas. The separation is less complete on Earth than in space because the large 20 kHz suspension signals generate pickup in the readout. In fact suspension interference was an awkward problem in the first stages of the laboratory experiment. It may be reduced by reducing the support frequency. J. A. Lipa has shown that a lower support frequency also eases other problems. One cannot on Earth conveniently go below 2 kHz since the ball drops too far during each half-cycle. A d.c. suspension is possible, but hard to mechanize.

Assuming a London moment readout one might ask if other alternatives are worth considering for suspension and spin up. The best claim another suspension might have would be that it exerts a smaller torque on the ball, particularly if the torque were low enough to do a relativity experiment on Earth. Various support schemes have been proposed for spherical gyro rotors -- gas bearings, superconducting magnets, flotation in superfluid helium, and so on -- and amazing claims are sometimes heard about torque levels. No universal judgement can be offered; the following argument shows where the heart of the problem lies.

Any scheme for supporting a massive body against gravity depends on creating pressure differences across the surface. If the body is nearly but not quite spherical a change in orientation results in work being done against these pressures; conversely the pressures exert a torque that drives the body towards an energy mimimum. The torque evidently vanishes for a true sphere; it depends on the size and shape of deviations from perfect sphericity. Discounting extraneous effects, such as the interaction of a magnetic suspension with the London moment or turbine torques in a gas bearing, we may expect different suspensions to exert similar torques on bodies of the same form, for the pressure that has to be applied over a given area to balance a given acceleration is always the same. More accurately we may think of two extreme suspension mechanizations between which all others fall. One which may be called the plain man's suspension generates pressure simply to counteract gravity. To fix ideas think of a light sphere floating on mercury. If the total deviation from sphericity is Δr the difference between maximum and minimum energies is $Mf\Delta r$ where f is the residual acceleration. Upper bounds on the torques are found by expanding the shape of the spinning body in Legendre polynomials and identifying Δr with each polynomial in turn. The drift-rates are

$$\dot{\underline{n}}_r^m = \frac{5m\chi}{2v_s} \left(\frac{\Delta r}{r}\right) \underline{f} \wedge \underline{n}_s \tag{25}$$

where m is the order of the polynomial and χ is a quantity between 0 and 1 depending on m and the size of the support pads. If the diameter of the pad is d then $m_\chi \rightarrow 0$ as $m \gg \pi r/d$, and satisfactory limits are got by considering the first few even and odd harmonics. Taking $\frac{\Delta r}{r}$ as 3×10^{-7} and f in space as 10^{-9} g the expected drift rate with a plain man's suspension and $\underline{f}$ perpendicular to $\underline{n}_s$ is around 10^{-15} rad/sec, a factor of ten higher than the design goal for the experiment. Further improvements depend on the extent to which f averages through the orbit or the plain man's suspension can be improved on.

To do better the energy put in at one point must be taken out elsewhere. The simple picture is not a sphere on mercury but a neutral density body immersed in an incompressible fluid. The pressure extends over the whole surface and the torque vanishes. Remembering Archimedes we may call this the eureka suspension. An electrical suspension can be arranged to mimic an eureka suspension by applying voltages to all six electrodes at once. It is then said to be preloaded. In the most common mechanization the voltages V_2, V_4 on opposite electrodes are adjusted to keep $(V_2 + V_4)$ constant and the preload acceleration h is the acceleration along a support axis required to send the voltage on one plate to zero. The most critical shape is the oblate spheroid, since it determines the gyro drift due to centrifugal distortion of the ball, and for a ball spinning at 200 Hz the centrifugal $\Delta r'/r$ is 3×10^{-6}, a factor of ten larger than the polishing errors. Defining a preload compensation factor ζ = $(hx - hy)/hz$ etc., the torque on an oblate spheroid inclined to the electrode axes turns out to be proportional not to f as in the plain man's suspension but to $\left[\frac{f}{h} + \zeta\frac{h}{f}\right] f$. There is therefore an optimum preload $h = f/\sqrt{\zeta}$; and if the preloads are matched to 1%, as is reasonable, this torque is an order of magnitude less than with the plain man's suspension (actually nearer a factor of 40 less since the numerical coefficient is smaller).

Two other, more elaborate, mechanizations deserve mention. One known as "sum of the energies" control has voltages continually adjusted to hold $\sum C_i V_i^2$ constant where C_i and V_i are the capacitance and voltage of the i^{th} electrode. The energy is independent of orientation; this is a true eureka suspension. The second is "sum of the squares" control, for which the voltages on the three axes fulfil the condition $V_2^2 + V_4^2 = V_1^2 + V_6^2 = V_3^2 + V_5^2$. This leaves the higher order terms but makes the torque on an ellipsoid vanish. Defining a sum of squares control factor ξ analogous to the preload compensation factor , the ellipsoid torque is proportional to $\left[\frac{h}{f} + \zeta\frac{f}{h}\right] f\xi$, still further down from the plain man's suspension.

Other suspension techniques such as a superconducting bearing might also outperform the plain man's suspension. The trouble always is that eventually the analytic arguments break down through secondary effects like polishing errors in the housing. The eureka suspension is a will o' the wisp. In fact in Honeywell's studies of electrical suspensions sum-of-the-squares control really helps, sum-of-the-energies does not. One may conjecture that with comparable work all suspension techniques will beat the plain man's suspension in about the same degree.

Consider now an attempt to do a Gyro Relativity experiment on Earth, say in an observatory near the equator. The combined relativistic precession $(\dot{\underline{n}}_s^G + \dot{\underline{n}}_s^M)$ is 0.4 arc-sec/year. Suspension errors may be reduced either by approximating a eureka suspension or by averaging. If the spin axis $\underline{n}_s$ lies in the equatorial plane the quantity $\underline{f} \wedge \underline{n}_s$ in equation (24) averages to $g\bar{\lambda}$, where $\bar{\lambda}$ is the average uncertainty in $\underline{f} \wedge \underline{n}_s$ from fluctuations in the local vertical, say 10^{-5}

rad . Then with $\frac{\Delta r}{r}$ for a spinning ball equal to 3×10^{-6}, the uncertainty in gyro drift with a plain man's suspension is about 100 arc-sec/year. Better things might be hoped from a preloaded suspension. Alas not! The residual torques, instead of being parallel to the local vertical are in an unknown direction in the housing: experience with live gyros suggests a limiting drift-rate nearer 1000 arc-sec/year -- worse than straight averaging with the plain man's suspension.

Conside another torque: mass-unbalance from inhomogeneities in the rotor. If u is the distance from center of mass to center of geometry the torque is Mfu and the drift-rate

$$\dot{\underline{n}}_s^u = \frac{5}{2}\left(\frac{u}{r}\right) \frac{\underline{f} \wedge \underline{n}_s}{v_s} \tag{25}$$

which with $(u/r) \sim 10^{-6}$ and $\overline{\underline{f} \wedge \underline{n}_s}$ - 10^{-5}g averages to 60 arc-sec/year, comparable to the suspension torques. In space with $f \sim 10^{-9}$ g, the drift-rate is below 0.001 arc-sec/year. The mass-balance might be improved by evaporating material on the surface of the ball and checking its pendulum period before spin up. If Ω_0 is the design goal and $_s$ the gyro spin-rate the pendulum period T must exceed $2\pi \sqrt{\sin\lambda/\Omega_0\, \omega_s}$, and with $\lambda \sim 10^{-5}$ the period for an 0.1 arc-sec experiment is four hours -- perhaps a factor of 20 beyond the limits of feasibility. An Earth based experiment is hopeless.

In space with a rolling drag-free spacecraft $\underline{f} \wedge \underline{n}_s$ and higher terms average extremely well. The estimates in my earlier papers were unnecessarily conservative; I am in process of revising them. In the final experiment support dependend drifts should go below 10^{-4} arc-sec/year. Drifts from other sources, calculated in earlier papers, should be below 10^{-3} arc-sec/year.

The choice of spin system turns on two questions. To spin a gyro a torque Γ^s has to be applied for time t_s, after which Γ^s is switched to its residual value Γ^r. A simple calculation establishes that Γ^r/Γ^s must be below $\Omega_0 t_s$, which for an 0.001 arc-sec gyro spun up in 30 minutes is 10^{-12}. Gas spin up, with its large change in operating pressure and favourable averaging provides this enormous torque-switching ratio; other methods do not. But there is a further consideration. A problem with gas is pumping down afterwards, especially through the pressure range 10^{-7} to 10^{-9} torr. Suppose we try another spin system. It will have losses which generate heat; the heat has to be removed to stop the ball warming above its superconducting transition; at low temperatures the only way is by exchange gas, and with any realistic efficiency the pressure has to be well above 10^{-7} torr [41] . One might as well stick to gas spin up.

The arguments for choosing a space experiment with electrical suspension, London moment readout and gas spin up, are persuasive. Ingenuity should never be discounted but in the Gyro Relativity experiment the issues ingenuity has to address are severely constrained.

3.3 Present Status

Research on the Gyro Relativity experiment has been going on at Stanford jointly between the Hansen Laboratories of Physics and the Department of Aeronautics and Astronautics with NASA support since 1963.* Areas of progress include: (1) the gyroscope (2) the telescope (3) ultra-low magnetic field technology (4) long hold-time dewars for space (5) the porous plug for controlling helium in space (6) attitude and translational control with helium thrustors (7) the relativity data instrumentation system (8) analysis. Two Mission Definition Studies and a Phase A study of the flight program have been performed for NASA by Ball Brothers Research Corporation. An opportunity for the first flight has been identified on Shuttle 10 in 1980. Complementing the work at Stanford has been research at NASA Marshall Center on precision manufacturing of quartz gyro rotors and piece parts, engineering development of the porous plug device, and low noise readout work. A rocket flight of the porous plug by NASA Marshall Center in cooperation with JPL and the Los Alamos Scientific Laboratory was successfully performed in June 1976.

The main thrust at Stanford has been to develop a laboratory model of the flight experiment with a gyro telescope package mounted in a long hold time helium dewar, tilted and aligned parallel to the Earth's axis with an artificial star reference. The dewar is a 100 litre superinsulated vessel made entirely from non-magnetic materials: aluminum, copper, titanium and low thermal conductivity plastic . It was fabricated to Stanford design by AGS Incorporated and delivered in 1969, since when it has been greatly modified and improved. Gyro designs were completed in 1967 in cooperation with Honeywell Incorporated and after experiments with quartz ang ceramic parts the ceramic housing (Figure 7) was delivered in 1971. An electrical suspension system had been purchased from Honeywell earlier. Besides gas spin up the gyro had several novel features for cryogenic operation, one of which was electrodes of relatively thin sputtered titanium with a gold overcoating rather than the electroless nickel electrodes used in standard Honeywell gyros. Although earlier test pieces had shown good electrical breakdown characteristics the housing itself suffered at first from severe arcing

* Present members of the research team are J. T. Anderson, B. Cabrera, R. R. Clappier, D. B. DeBra, C. W. F. Everitt, W. M. Fairbank, J. A. Lipa, B. Nesbit, F. J. van Kann, R. A. Van Patten.

under high voltage. Experiments by J. A. Lipa and J. R. Nikirk progressively overcome this and other difficulties and in January 1973 the gyro was spun for the first time at room temperature. Low temperature spin with a superconducting ball was achieved in June 1973. Between then and August 1975 about 1000 hours of cryogenic operations were logged at spin speeds up to 40 Hz. We have not tried to go faster, except once by mistake, because residual problems in the ceramic housing impart an element of risk to high speed spin. Higher speeds are expected in a new apparatus.

Readout magnetometers were developed by J. T. Anderson and R. R. Clappier. Elaborate shielding and filtering against suspension interference was needed in applying them to the gyro. Simultaneously with the readout work J. R. Nikirk developed a new suspension system, having better servo response and centering capability, with sensing bridges operating at a single 1 MHz frequency of 2V rms amplitude rather than the three 30 V signals at independent frequencies used by Honeywell. This reduced suspension interference. The hardest problem was trapped magnetic flux in the gyro rotor. The apparatus has two Mu-metal shields to bring the ambient field to 10^{-5} gauss, but initial trapped fields were commonly nearer 3×10^{-4} gauss, because of residual magnetism and thermo-electric currents in the experimental chamber. The London moment as 35 Hz corresponds to 2.1×10^{-5} gauss. Laborious procedures with field coils around the gyro brought the trapped fields down to 10^{-5} gauss, after which London moment data was obtained by processing signals from a three-axis readout. Figure 9 from a paper by Lipa, Nikirk, Anderson and Clappier [42] compares theory and experiment for the London moment as a function of spin speed.

A precise readout needs a much lower field: around 10^{-7} gauss. During the past six years B. Cabrera of Stanford has developed methods of creating superconducting shields up to 8 inches in diameter with fields below 10^{-7} gauss over a 30 inch length. The technique depends on two special properties of superconductors. One is that the magnetic flux through a closed superconducting surface is conserved. A tightly compressed lead shield is cooled in a low field and then expanded. Since flux is equal to field times area, the increase in area causes a corresponding reduction in field. The second method of field reduction is by heat flushing. If a temperature gradient is applied along a superconductor as it is cooled one end will be superconducting and the other normal, with a transition zone somewhere in between. Further cooling makes the boundary move steadily forward and in suitable circumstances the magnetic field is progressively pushed out of the enclosed volume. Flushing is complicated by the magnetic fields from thermoelectric currents, but by flushing and expanding a series

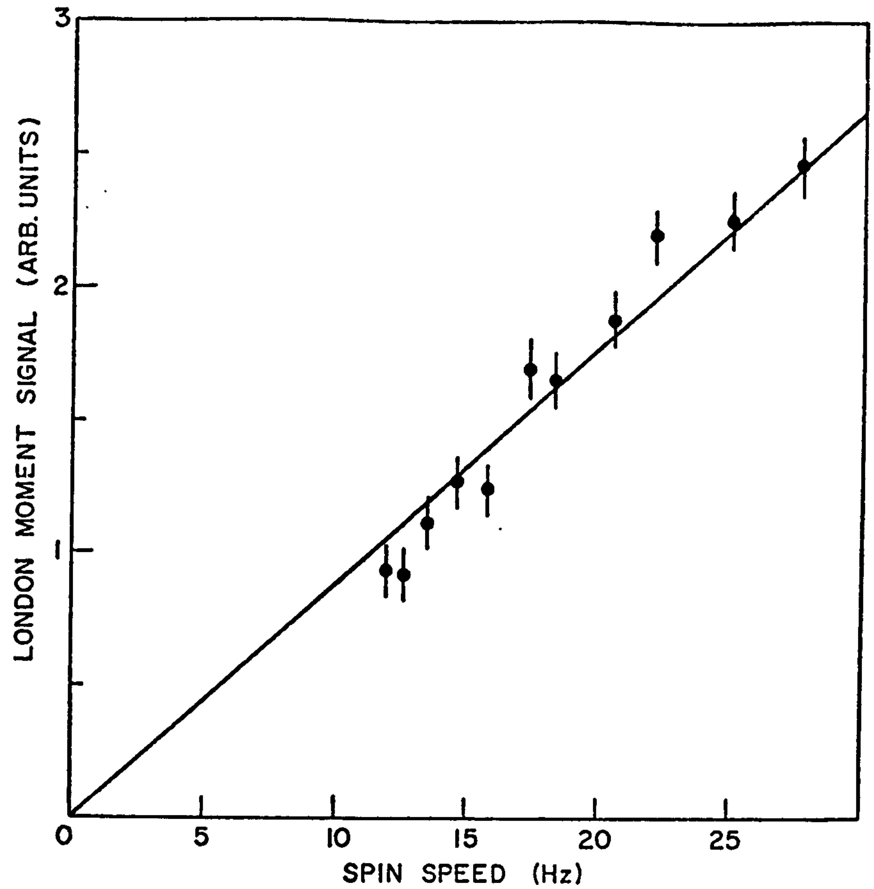


FIGURE 9

LONDON MOMENT DATA FROM A SPINNING GYROSCOPE

of balloons one inside the next, a 10^{-7} gauss region can be produced.

F. J. van Kann and B. Cabrera have developed a new test apparatus, shown in Figure 10, in which the gyro is held in an 8 inch ultra-low field shield. The gyro probe assembly is suspended from a rigid frame which stands on a concrete pad and the dewar containing the shield is slowly raised around it on a hydraulic piston, using an airlock to prevent condensation of solid air. The gyro probe can be transferred to a laminar flow clean bench nearby for assembly and dis-assembly under ideal conditions. Gyro operations in a low field should begin in August 1976.

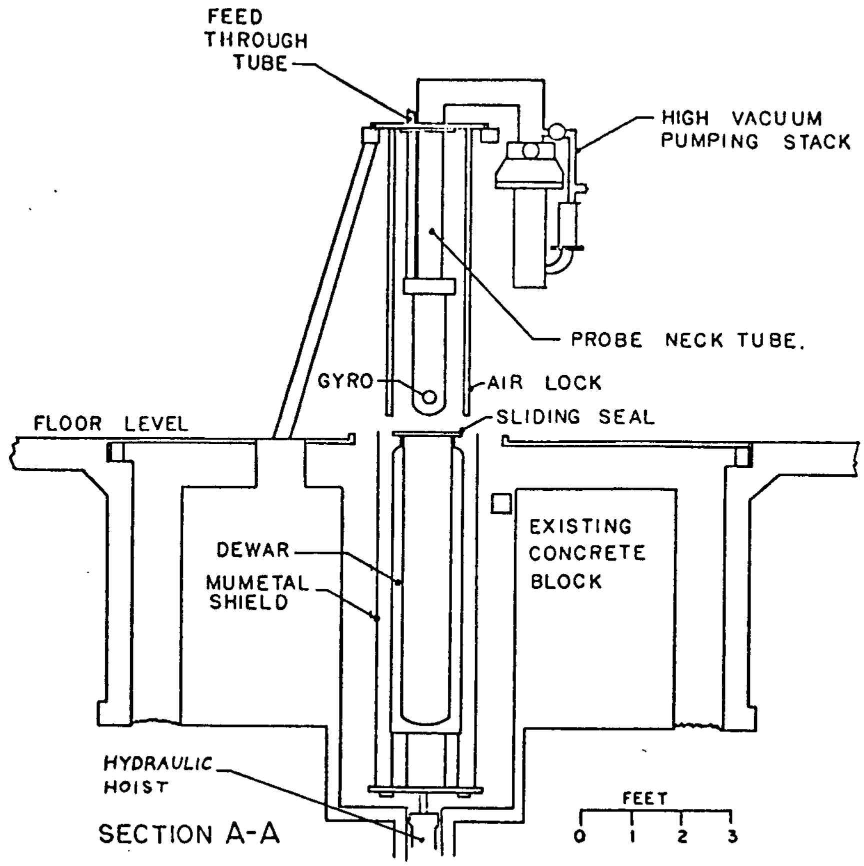


FIGURE 10
APPARATUS FOR OPERATING GYROSCOPE IN ULTRA-LOW MAGNETIC FIELDS

Work on the telescope has gone on in parallel. Figure 11 is a view of the telescope designed in conjunction with D. E. Davidson, which is a folded Schmidt-Cassegrainian system of 150 inch focal length and 5.5 inch aperture, fabricated entirely of fused quartz. The light is divided by a beam splitter near the focal plane to give two star images, one for each readout axis. Each image falls on the sharp edge of a roof prism where it is again subdivided and passed to a light-chopper and photodetector at room temperature. The limitations are the sharpness of the prism edge and photon noise. A simple analysis [43] shows that the prism edge should have no nicks greater than 3500 Å. The prisms were made by polishing two optical flats, contacting them together, lapping a second

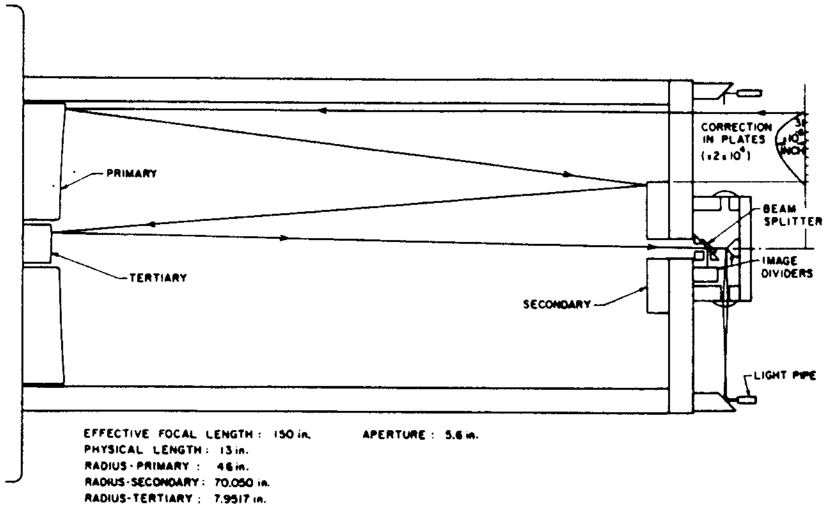


FIGURE 11: DAVIDSON TELESCOPE

surface at right angles, and separating the two halves. Each edge protected the other. The final prisms have no nicks greater than 1500 Å.

Much attention has been paid to mechanical and optical stability. The space telescope, being in zero g and looking through the vacuum of space, is free from effects of sag or atmospheric turbulence. Low temperature operation is equally important. As equation (14) shows, the thermal distortion depends on the transverse heat flux and the ratio (α/K) of the expansion coefficient to thermal conductivity. At ambient temperature the radiation falling on the spacecraft would have to be cancelled to 1 part in 10^5 to avoid 10^{-3} arc-sec distortions. At low temperatures (α/K) is orders of magnitude smaller; the heat flux can be a factor of 100 higher than the total heat input into the dewar. Problems of stress-relaxation in the quartz, ultraviolet darkening, tarnishing of the mirrors and aging of the light pipes are discussed elsewhere [44] .

The photon noise is given by an equation similar to (3). Experiments by R. A. Van Patten, H. Gey and myself on a single axis model telescope and artifical star give results for slightly defocused images (Figure 12) in good agreement with theory. A star equal to Procyon was resolved to 0.02 arc-sec in 0.3 sec. With diffraction limited optics, Rigel would be resolved to 0.01 arc-sec in 0.1 sec.

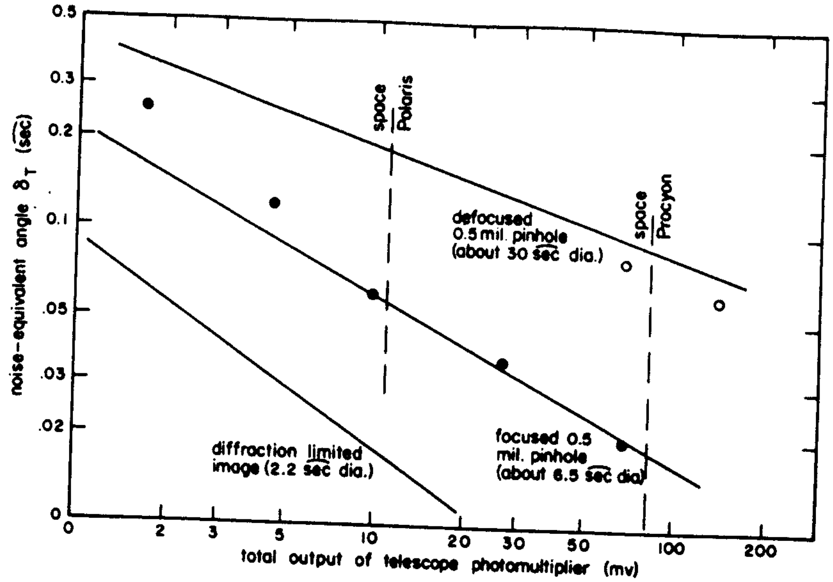


FIGURE 12: COMPARISON OF OBSERVED DATA WITH CALCULATED PHOTON NOISE FOR A 2.5 INCH APERTURE MODEL TELESCOPE WITH ROOF PRISM IMAGE-DIVIDERS

Recently we took delivery of the North Star simulator designed by D. E. Davidson that will be used in testing the real telescope. The device has a 200 inch focal length 8 inch aperture off-axis parabola mirror, support optics and tipping and dither plates for exploring the star image, all enclosed in an evacuated chamber. The telescope is expected to be linear to 0.001 arc-sec over a range of ±0.05 arc-sec.

Other laboratory research at Stanford is described in papers by T. Dan Bracken [39] (gas spin up), J. Bull [45] (attitude control), D. Klinger [46] (fixed base simulation), J. R. Nikirk [38] (suspension system), P. M. Selzer [34] (porous plug for controlling helium in space), R. A.Van Patten [47] (relativity data instrumentation system), D. C. Wilkins [32] (relativity effects in perturbed orbits). More about the dewar appears in articles by Lipa, Fairbank and Everitt [48] .

4. TESTS OF THE EQUIVALENCE OF GRAVITATIONAL AND INERTIAL MASS BASED ON CRYOGENIC TECHNIQUES

The possibility of testing the equivalence of gravitational and inertial mass in Earth orbit has been discussed several times. An orbital experiment has two advantages. The disturbances are smaller than on Earth and the driving acceleration, instead of being derived from the sun or the Earth's rotation, may be the full gravitational attraction of the Earth, which at 300 nautical miles is 950 cm sec^{-2}, three orders of magnitude larger than either of the sources available in ground-based experiments. During the past four years P. W. Worden, Jr. and I, with other colleagues at Stanford, have been designing an orbital experiment using cryogenic techniques to measure the Eötvös ratio η to 1 part in 10^{17}. A preliminary laboratory experiment now in progress should reach an accuracy approaching 1 part in 10^{13}.

The difficulty in space is gravity gradients. For a torsion balance the ratio Γ^g/Γ^E of gravity gradient to Eötvös torques from a common source is proportional (equation (9)) to $J_2D/\eta R$ where D is the diameter of the balance and R the distance to the source. The ratio is five orders of magnitude higher with the Earth as source than with the sun. For an orbital experiment with J_2D around 10^{-3}, Γ^g exceeds Γ^E when η is less than 2×10^{-12}. The gravity gradient disturbance, being doubly periodic with the orbit, can be partially suppressed by resonating the balance with the orbit-period, but even if the system were given three months to come to equilibrium (a startling thought!) the maximum allowable Q would be 1000. At that level the attenuation of the second harmonic is only 0.0016, and the gravity gradient signal in a 10^{-17} experiment is still 400 times the Eötvös signal. Resolution is difficult in the presence of such a large disturbance and non-linearities may generate suharmonics of Γ^g that masquerade as Eötvös signals.

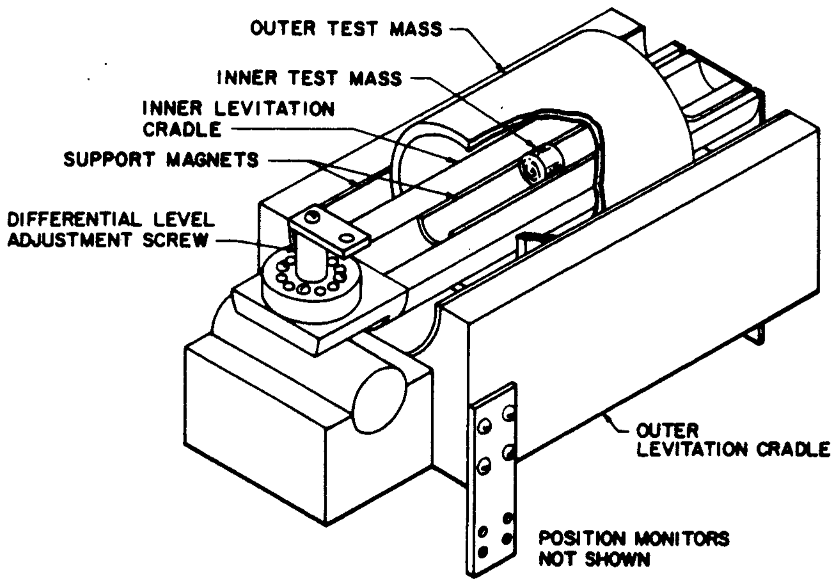
The difficulties are removed by abandoning the time-honoured torsion balance, for which the measured quantity is an angular displacement, in favour of an experiment to measure the relative linear displacement of two nearly coincident freely falling masses, for example two coaxial cylinders. The ratio a^g/a^E of gravity gradient to Eotvos accelerations is then $2\ \Delta R/R\eta$ where ΔR is the displacement between the mass-centres of the cylinders. Again a^g is doubly periodic with the orbit; however in an elliptic orbit of eccentricity e it has a singly periodic subharmonic $eg'\ \Delta R/R$. No such term occurs in the gravity gradient torque Γ^g on a torsion balance. If the cylinder axes are aligned with the semi-major axis of the orbit the subharmonic masquerades as an Eötvös signal. The error is eliminated by centering the masses, driving the twice orbital signal a^g to null with a centering servo. To make a^g/a^E unity $\Delta R \sim R\eta/2 \sim 3.5 \times 10^{-8}$ cm., which for an η of 10^{-17} requires coincidence in the average position of the mass-centers to about 0.35 Å. This is an unnecessarily restrictive centering criterion, but one that can be achieved.

Recently P. W. Worden, Jr. has extended the comparison of the free-fall and torsion balance experiments by forming expressions for the higher order gravity gradient terms on the free-fall cylinders and comparing the ratios (S/N) of a given Eötvös signal S to gravity gradient disturbances N for the two experiments. Provided the cylinders are centered enough to make the first order term negligible, the ratio of the two (S/N)s is

$$\frac{(S/N)_{\text{free fall}}}{(S/N)_{\text{torsion balance}}} \sim \frac{\varepsilon\xi^2}{J_2 DR} \qquad (26)$$

where R is the distance to the disturbing mass, ξ the radius of gyration of the outer cylinder in the free fall experiment, and ε the quantity which when multiplied by the principal moment of the cylinder gives the difference between its largest and smallest principal moments. For given manufacturing procedures the quantities J_2 and ε are comparable, as are the characteristic dimensions ξ and D for the two apparatuses. Hence the ratio of the two (S/N)s is of order ξ/R. Take R as the radius of the Earth: the free fall experiment is eight orders of magnitude less sensitive to the Earth's gradient than a torsion balance. Next consider gradient disturbance within the spacecraft, say from sloshing motions of the liquid helium. With R about 100 cm the free fall experiment is in this respect one order of magnitude quieter than the torsion balance. The centering requirement to make the first order term in (26) negligible is $\Delta R \ll \varepsilon\xi^2/R$. With $\varepsilon \sim 10^{-3}$ and $\xi \sim 3$ cm, R has to be below 10,000 Å to remove the term for the helium tides, but nearer 0.001 Å to remove it for the Earth's gradient. In reality R is more likely to be about 0.1 Å, so the free fall experiment is six rather than eight orders of magnitude better than the torsion balance. That is enough.

The problem in any improved determination of η is the minuteness of the displacements. Consider the free fall experiment. If the masses are essentially free floating, a differential acceleration ηg with period T will cause a relative displacement $T^2\,\eta g/4\pi^2$ which implies a displacement of 0.6 Å for a satellite period of 90 minutes and an η of 10^{-17}. If, as is likely, the masses are restrained in a harmonic oscillator with natural period shorter than the orbital period the amplitude is correspondingly smaller. Large advances in sensitivity and stability over existing apparatus are needed.



EQUIVALENCE PRINCIPLE ACCELEROMETER

FIGURE 13

TEST MASSES AND ADJUSTABLE SUPERCONDUCTING SUPPORT CRADLES FOR THE GROUND-BASED VERSION OF THE CRYOGENIC EQUIVALENCE EXPERIMENT

(In the present apparatus the inner test mass is made of niobium and the outer test mass is of niobium coated aluminum.)

Figure 13 illustrates the arrangement of test masses for the laboratory experiment now under development at Stanford. They consist of a rod of superconducting niobium approximately 1 cm x 1 cm and a concentric hollow cylinder of niobium coated aluminum, approximately 5 cm x 5 cm. Later other materials will be tried. The two bodies are independently supported by two superconducting magnetic bearings and are free to move along their common axis. The experiment consists in measuring the relative accelerations of the masses throughout the day with the sun as source. The position of each mass is controlled by a servo-mechanism which in effect tilts the support plane; the difference in control efforts to centre the two masses contains the signal. Data is recorded digitally and computer analysed to detect any 24 hour component in the difference output.

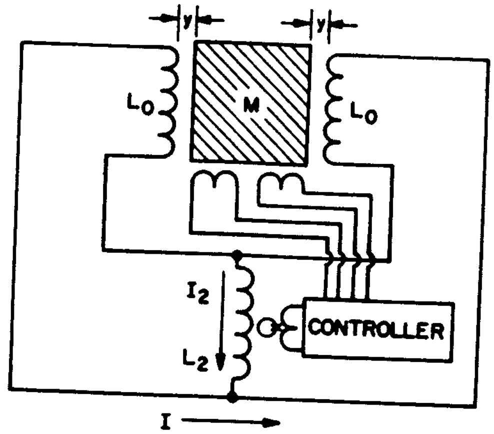


FIGURE 14: POSITION DETECTOR FOR EQUIVALENCE PRINCIPLE EXPERIMENT

Figure 14 illustrates the principle of the position readout. Each superconducting test body is placed between two coils of inductance L_0 joined in a continuous superconducting loop in which a magnetic field is trapped. A third inductance L_2 is connected in parallel. The test body being a perfect diamagnet, its motion causes a redistribution of field in the coils. If the current in L_2 is initially zero and the current trapped in the main loop is I, a motion Δx of the test body generates a current in L_2

$$\Delta I_2 = 2I \frac{L_0}{L_0 + 2L_2} \frac{\Delta x}{y} \tag{27}$$

where y is the distance from the coils L to the test mass. The current ΔI_2 is read by a Josephson junction magnetometer, which then drives the centering servo.

For a quantum interference detector capable of resolving in τ sec an amount of magnetic flux equal to a small fraction $\varepsilon(\tau)$ of the flux quantum ϕ_0 the minimum detectable current change is $\delta I_2 = \varepsilon\phi_0/L_2$ and the minimum detectable position change is

$$\delta x = \frac{\gamma\phi_0}{I}\left(\frac{L_0 + 2L_2}{2L_0L_2}\right) \varepsilon\ (\tau) \qquad (28)$$

and in the ideal case where resolution is limited solely by measurement noise $\varepsilon = \gamma^{\frac{1}{2}}\tau^{-\frac{1}{2}}$, with $\gamma^{\frac{1}{2}}$ being typically of order 10^{-4} $Hz^{-\frac{1}{2}}$. Following the approach of equation (5) we may define a discrimination factor ∂_B giving the resolution of the Brownian motion $<\Delta x>$, where $<\Delta x>$ for an oscillator of natural period is again $\frac{\tau_0}{2\pi}\sqrt{\frac{kT}{M}}$. Putting $\phi_0 = hc/2e$

$$\partial_B = C\left(\frac{hc}{ek^{\frac{1}{2}}}\right) \left[\frac{1}{\tau_0 T}\right]^{\frac{1}{2}} \left[\frac{\gamma(1 + 2L_2/L_0)}{L_0}\right]^{\frac{1}{2}} \qquad (29)$$

where in analogy with (5) the terms in the first square bracket characterize the suspended body and those in the second square bracket characterize the readout.

The quantities determining τ_0 are different in space and on Earth. On Earth the masses tend to be pendulums, rocking back and forth in local depressions of their support cradles and τ_0 is set by machining tolerances in the cradles. In space the support is small and the largest restoring force is the reaction of the readout. In the ideal case of a single body the system becomes a simple harmonic oscillator whose period is $\tau_0 = \pi\gamma \sqrt{2M(L_0 + 2L_2)} / IL_0$ and the characteristics of the readout and suspension are no longer separable. An interesting consequence is that the sensitivity (i. e. the ratio of the Eötvös signal $|\Delta x|$ to δx) is increased by decreasing the readout current I. In fact to make τ_0 resonant with the orbit, which gives maximum sensitivity, I may be only a few μA.

In all circumstances the discrimination is excellent; the readout noise is typically some four orders of magnitude less than the Brownian motion.

The fundamental limit is given by an equation identical with (12) except for a smaller numerical factor

$$<\eta> \sim \frac{1.4}{f}\sqrt{\frac{\beta kT}{MS}} \qquad (30)$$

With $M \sim 100g$, $\beta \sim 10^{-5}$ sec^{-1}, $T \sim 2K$, $f \sim 950$ cm sec^{-2}, η can be resolved in a space experiment to 10^{-17} in 4 hours and 10^{-18} in 16 days.

The practical limit in a space experiment is set by tidal slosh in the liquid helium. Taking ℓ as the length of an optimized outer cylinder and z as the mean distance to the helium, the mass amplitude M_H of the tide must satisfy

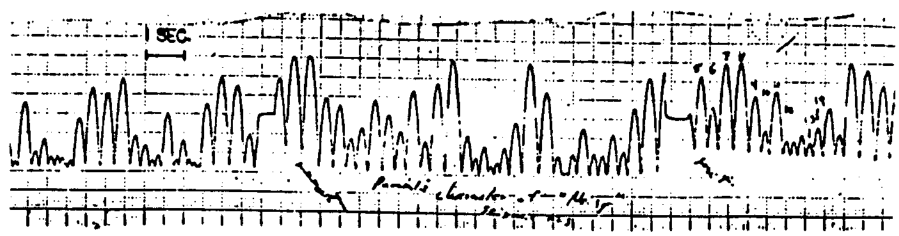
$$M_H < \frac{fz^6}{G\,\ell^4}\,\eta_0 \qquad (31)$$

to avoid gravity gradient disturbances greater than η_0 . With ℓ around 10 cm, M_H must be less than 20 gm for a 10^{-17} experiment. The corresponding tidal amplitude is 1 mm. Several techniques are available to control slosh. Since M_H scales as ℓ^{-4} there is great advantage in making the apparatus smaller.

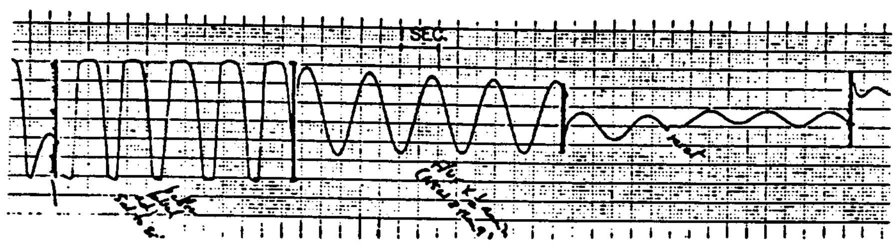
Other limits to the experiment, including effects of residual gas, residual electric charge, trapped magnetic fields, and black body radiation in the experimental chamber, are discussed elsewhere [17] .

The copper support block for the laboratory test masses (Figure 13) comprises two hemicylindrical cradles joined by a flat spring hinge with a differential screw adjustment to tilt the inner cradle with respect to the outer one. The levitation coils are 5 mil niobium-titanium wires wound back and forth parallel to the axes of the cradles. They were made by winding the wires on a precisely machined aluminum mandrel, epoxying the structure into the cradle and then etching out the mandrel with sodium hydroxide. The inner coil has no deviations from an ideal cylinder greater than 2.5×10^{-4} cm. The positions of the test masses are read out by two circuits of the kind already described, with the outputs being fed back to subsidiary tilt coils on each cradle. Any differential acceleration between the two bodies is found by comparing and subtracting the control efforts needed to keep them centered. The control gains have to be very closely matched; otherwise spurious signals appear in the output if the apparatus is tilted back and forth through a small angle once per day, as it may be for example through ground motions. The gain match is done in two stages: (1) by matching superconducting control transformers inside the dewar to 1 part in 10^3 or better, (2) by sending the control currents in opposite directions through a common external resistance R_{diff} and adjusting a precision decade box in one half of the system to remove the residual mismatch. The electronic components all have precision of 1 part in 10^6. The combined matching should be better than 1 part in 10^8. Data from the experiment appears in the form of three 16-bit binary words once per second, corresponding to the three signals A,B, (A-B) where A and B are the two control efforts. It is fed to a computer for analysis and gain matching.

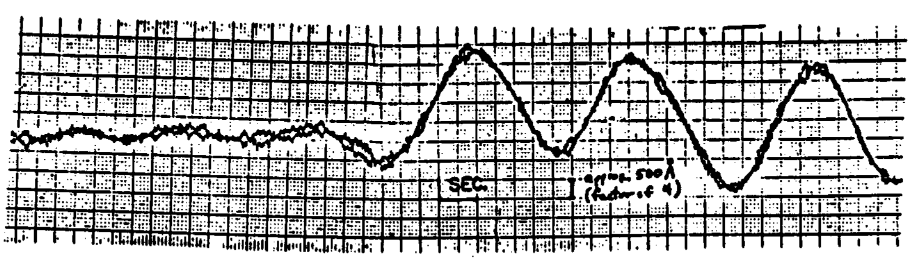
Low temperature operations began in late 1974. The inner test mass has been tried in several runs. The free period varies with the location of the mass in the cradle: The maximum useful working sensitivity of the position readout is at present 5 Å , although 5×10^{-3} Å resolution has been reached in special circumstances. The 5 Å limit is set by the need to handle seismic noise with the controller off. Figure 15 shows a sequence of events during levitation of the inner mass. In the top record the mass is floating in the bearing but rests initially at one end because the cradle is tilted. Seismic noise kicks the mass away from the end, after which it follows a parabolic trajectory, strikes and bounces away again. The middle chart record shows the mass floating freely after levelling. The third record shows excitation of the test mass by seismic noise.



TEST MASS EXCITED BY SEISMIC NOISE
AND BOUNCING AGAINST STOP
$1' < \theta < 9'$ $1 < X < 10$ MICRONS



FREE OSCILLATIONS OF TEST MASS AFTER LEVEL ADJUSTMENT, SHOWING EFFECT OF RFDUCING FLUX IN POSITION MONITOR



TYPICAL SEISMIC NOISE EXCITING THE SYSTEM

The outer test mass has not been levitated. Various levitation attempts early in 1975 failed because it was jammed in the cradle, which had been damaged. The cradle has been repaired. Although the failure has delayed the main experiment, Dr. Worden has tried an interesting subsidiary experiment with the Earth and the inner test mass as proof bodies, by searching for evidence of a differential acceleration between the test mass and the Earth when the apparatus was run over-night. The best of several such runs verified the equivalence principle for niobium versus the Earth to 1 part in 10^5. With the cradle repaired and the controller operating, the main experiment should reach 1 part in 10^{12} or 10^{13}. The limiting factors are gravity gradient disturbances and seismic noise.

Analysis of the flight experiment is continuing.

5. THE STANFORD-LSU-ROME GRAVITATIONAL WAVE ANTENNAS

The observation by Joseph Weber in 1969 of coincident pulses in widely separated gravitational wave antennas started a flood of theoretical and experimental research. Six groups at least have operated gravity antennas, and more are on the way but none has confirmed the original observations. Theorists have balked at finding sources for Weber's pulses, which require the annihilation of 100 solar masses per year in our galaxy. However substantial quantities of energy should be released through the collapse of stars and the capture of stellar material by black holes or neutron stars. The ideal would be to correlate gravitational wave observations with the visible supernovae following stellar collapse. For 408 galaxies (including the Virgo cluster) within 22 Mpc of the Earth the estimated rate of visible supernovae over a sample period corresponding to 32 years is 3.2 events per year. The predicted energies range up to 10^{53} ergs, or 0.04 solar masses, released in a few millisec. To see this at 22 Mpc the detector needs six orders of magnitude more energy sensitivity than Weber's bar.

Various ideas, more or less exotic, have been mooted for better detectors. For Weber's detector equation (18) shows that the bar should be massive and made from a material with high velocity of sound: in practice, that is, the largest possible hunk of aluminum. An improvement discussed by Aplin, Gibbons and Hawking [20] and implemented in a room temperature bar by Drever [49] is to increase the coupling coefficient β by sandwiching piezoelectric crystals between the two halves of the bar. Drever and his colleagues reached a sensitivity of $kT_B/100$ in a bar smaller than Weber's. Other possibilities include operating at cryogenic temperatures and increasing the mechanical Q of the bar. The Gibbons-Hawking equation (19) suggests that the sensitivity might be increased almost indefinitely by increasing Q_B, and Braginsky [50] has shown that Q's as high as 10^9 -- four orders of magnitude higher than Weber's -- are attainable in large single crystals.

To gain the advantage of high Q one must have the right detector. Recently R. P. Giffard [51] in a very penetrating paper has shown that the Gibbons-Hawking argument is incomplete. In general the input noise of the amplifier for the electrical signal is more significant than the transducer noise. The critical problem is matching the bar to this amplifier, in which process current noise must be taken into account as well as the Johnson or voltage noise. With perfect matching by an ideal transducer to a noiseless parametric amplifier of gain G feeding an amplifier of noise temperature T_N the smallest excitation detectable in the n^{th} mode of the bar has equivalent energy

$$E_{min} = 2k \left[\frac{\pi(\nu_n \tau_s) T_B}{Q_B} + \frac{T_N}{G} \right] \ln (1/\dot{N}\tau_s) \qquad (32)$$

where $\dot{N}$ is the "accidentals rate", that is the maximum rate of occurence of spurious pulses acceptable to the experimenter. Equation (32) fixes $\tilde{Q}_B$, the maximum useful Q of the bar, for a particular amplifier and sampling time: $\tilde{Q}_B \sim \pi (\nu_n \tau_s) G T_B/T_N$. With the best available FET amplifier ($T_N \sim 0.2$ K) and a sampling time of 10 mS, $\tilde{Q}_B$ is 1.5×10^5 for a room temperature bar and 10^4 for a 2 K bar. Further progress therefore requires at least as much attention to developing an adequate parametric amplifier as it does to increasing Q_B or lowering the temperature of the bar.

Giffard's formula, unlike that of Gibbons and Hawking, does not yield an optimum sampling time. This is because the amplifier, having current as well as voltage noise, can be characterized by a noise temperature T_N and in measuring an energy pulse (in contrast to a continuous signal from a power source) there is no improvement in resolution with time. Theoretically τ_s should be as short as possible, subject to the constraints that it should be longer than the time B_t^{-1} reciprocal to the bandwidth of the transducer and that both times should be longer than τ_p the duration of the gravitational pulse. In practice good matching to the amplifier may require a transducer of narrow bandwidth and a compromise may have to be struck between good matching and the condition $B_t^{-1} > \tau_p$. The conclusion should be contrasted with the earlier discussion comparing Brownian motion and photon noise in a torsion balance with optical readout. Equation (17), modernizing Hill's argument described above, shows that readout noise is dominant in a torque measurement unless the suspension period of the balance, and hence the observation time, exceeds a certain minimum value. Equation (32) shows that the sampling time in a pulse measurement should be short. The difference is attributable partly to the difference between a steady torque and an energy pulse and partly to the different characters of readout noise.

During the past five years research teams at Stanford, Louisiana State University and the University of Rome have cooperated in applying cryogenic techniques to gravitational wave antennae. The obvious advantage of low temperature operation is the reduction in thermal noise -- by a factor of 10^5 if the bar is cooled to 3 mK -- but this, as just remarked, has to be qualified in the light of equation (32). The real improvement comes through applying superconducting techniques to develop low noise high gain parametric amplifiers and match them to the cold bar. Another advantage of low temperatures is the improved mechanical and electromagnetic isolation obtained with superconducting magnetic shields and superconducting supports.

Similar large antennas are being built at Stanford, LSU and Rome, each 3 m long and 0.9 m in diameter. The Stanford team* has also built the small prototype illustrated in Figure 16, which has been run at liquid helium temperatures since early 1975. The dewar is a modification of the unit for the Stanford superconducting accelerator, with the helium tank enclosed by two helium vapour cooled shields. The boil-off rate is about 1 ℓ/hour, which necessitates transfer from a storage dewar every two weeks. The antenna is an aluminum cylinder 2 m long and 0.4 m in diameter, clad with a layer of niobium-titanium foil 0.38 mm thick. Its resonant frequency is 1312 Hz. The bar is levitated magnetically with coils on the helium tank, wound from 0.6 x 1.3 mm rectangular superconducting wire. The support field is 2400 gauss. The apparatus has been run with the bar up for as long as 900 hours. The resonant frequency of the magnetic support is 5 Hz; its attenuation at bar frequency is 57 db. The helium tank is suspended by acoustic isolation stacks constructed from alternate layers of rubber and iron, mechanically independent of the rest of the cryostat. With the bar levitated in exchange gas at 10^{-4} torr, the longest decay time for the 1312 Hz mode was 40 sec, which corresponds to a Q of 3.3×10^5.

The position detector conceived by Paik, Opfer and Fairbank [52] for the Stanford bar has elements in common with the detector for the free fall equivalence principle experiment. The two systems, developed at the same time in the same institution, were independently conceived. The principle is illustrated in Figure 17. Attached to the end of the bar is a superconducting diaphragm

* Present members include S. P. Boughn, W. M. Fairbank, R. P. Giffard, M. W. McAshan, H. J. Paik and R. C. Taber.

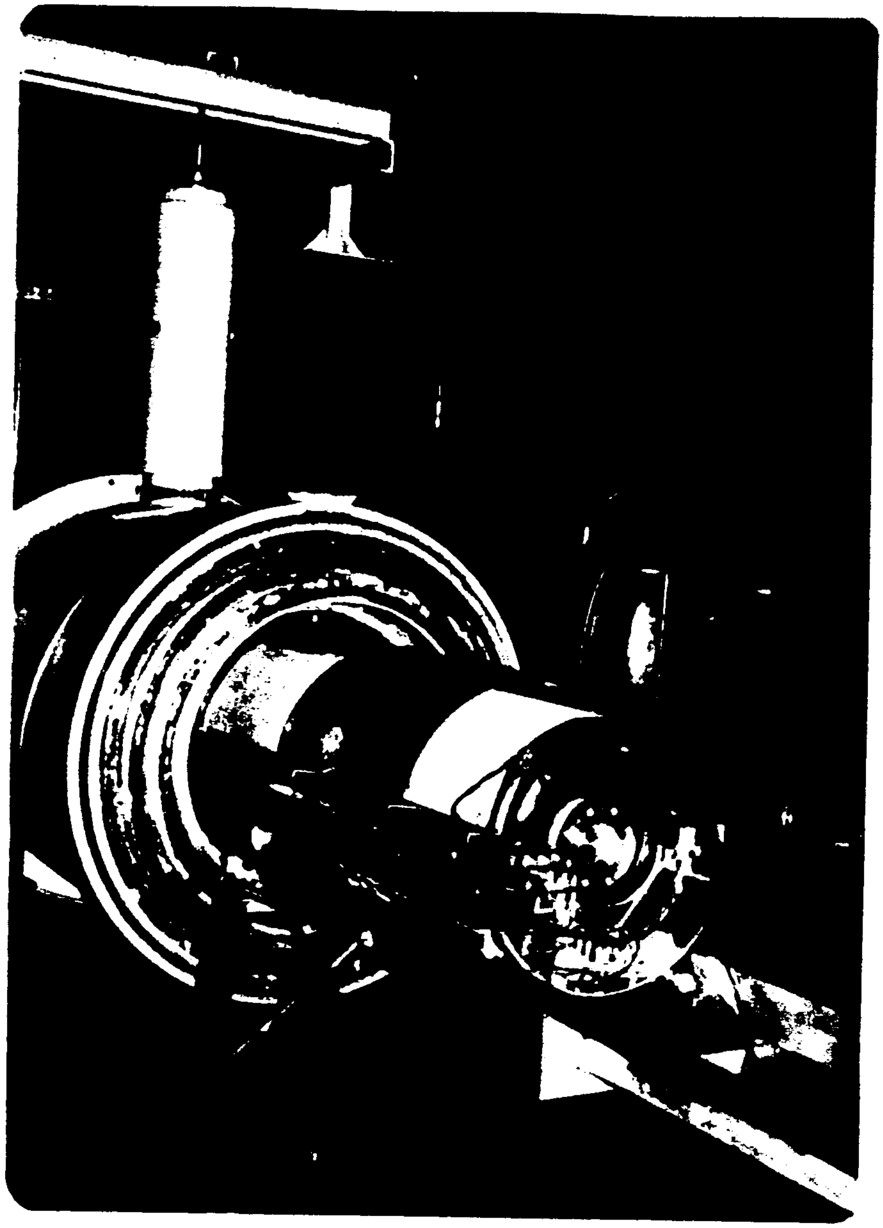


FIGURE 16: ASSEMBLY OF THE 2 M LONG STANFORD CRYOGENIC GRAVITY WAVE ANTENNA resonant at antenna frequency. Two flat coils, wound with 2 mil niobium-titanium wire, are placed 0.1 mm from the opposite faces of the diaphragm. They and the circular edge of the diaphragm are rigidly clamped to the bar. A large

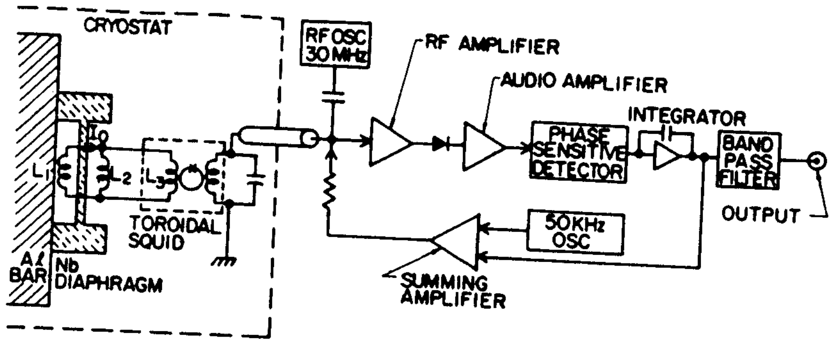


FIGURE 17: SUPERCONDUCTING TRANSDUCER AND READOUT FOR CRYOGENIC GRAVITY WAVE ANTENNA

persistent current of about 5 A is stored in the double loop formed by L_1, L_2. As the diaphragm oscillates it modulates the inductance L_1 , L_2 and a net alternating current appears in L_3 , where it is measured by a SQUID magnetometer as in the similar three loop circuit for the equivalence principle experiment. The large persistent current in L_1, L_2 acts as a magnetic spring which centers the diaphragm between the coils and also changes its resonant frequency. The frequency is raised in one version from about 300 Hz to 900 Hz or 1312 Hz and the diaphragm is tuned to the bar by adjusting the current.

In both the orbiting equivalence principle experiment and the gravity wave apparatus the trapped flux in the position detector serves as a spring and maximum sensitivity is reached when the oscillator resonates with the driving signal. However since the Eötvös acceleration is at 1.8×10^{-4} Hz (90 minute period) while the antenna frequency is 900 Hz or 1312 Hz the optimization conditions are very different. The trapped flux needed to resonate the equivalence principle masses is a few μA as compared with 5 A for the resonant diaphragm. The equivalence principle detector has, as already indicated, ample sensitivity even when used off-resonance.

One way of looking at the resonant diaphragm detector is as a mechanical transformer, which increases the amplitude of the bar motion enough to be seen by the loop detector. The amplitudes of diaphragm motion to bar motion are in the ratio $\sqrt{M/m}$ where M and m are the effective masses of the bar and the diaphragm. More formally, the diaphragm serves as a linear network connecting the bar to the magnetometer, whose operation can be described by a complex transformation matrix. The transformation coefficients are varied by changing the mass of the diaphragm, the spacing of the coils, and the trapped magnetic field. Ideal conditions are when the input reactance of the magnetometer is tuned out by the diaphragm, and the mechanical impedance of the antenna is transformed into the optimum noise matched source impedance for the magnetometer. The diaphragm used at present is 10 cm in diameter. A larger diaphragm and coils would give better matching, but is hard to make.

The transducer and magnetometer have been operated on the antenna. The output signal was processed in the following way. The signal was fed to a two-phase lock-in amplifier whose reference was driven at antenna frequency by means of a frequency synthesizer. The two output channels were then smoothed by R. C. filters, digitized and stored and processed in a PDPH-45 minicomputer. The output noise was Gaussian. To investigate its properties the two channels of recorded data were squared and summed to yield an output proportional to the signal power from the magnetometer. The upper curve in Figure 18 is a sample pulse height analysis with lnP(E) plotted against energy. The slope implies a noise temperature of 5.9 ± 1.5 K. Since the observed noise was primarily narrow band a differencing technique reduces the effective noise temperature for pulse detection. The lower curve illustrates the effect of differencing the signal in increments of 0.2 sec and integrating via two R-C filters each with 0.1 sec time constant. The observed noise temperature was 0.22 K. Of this 80% was from wideband transducer noise and only 20% from narrowband noise in the bar. After various corrections the final effective noise corresponds to 0.7 K energy in the bar.

Like all gravitational wave antennas the bar is sensitive to acoustic noise. One source was the violent boiling of nitrogen in the liquid nitrogen shield originally used in the dewar. It was replaced by a helium vapour cooled shield. A sound insulating room has also been put around the cryostat. External noise has been much less in recent runs, but occasional bad pulses still occur, probably from motions between the outer heat shield and the dewar.

Giffard's analysis shows the need for using a high gain parametric amplifier with the bar. If the amplifier receives its input signal at the bar frequency ω_b and delivers its output in the form of sidebands on a higher pump frequency ω_p the input noise temperature T_N cannot be less than $T_A\ (\omega_b/\omega_p)$, where T_A is the noise temperature of the amplifier detecting the sideband power. For best performance the amplifier should be cooled to reduce dissipation in the parametric element itself; with superconducting elements the dissipation can be minute. Recently Josephson junction devices have been demonstrated with self-pumping frequencies of order 500 MHz and loud noise at 4.2 K. If they function

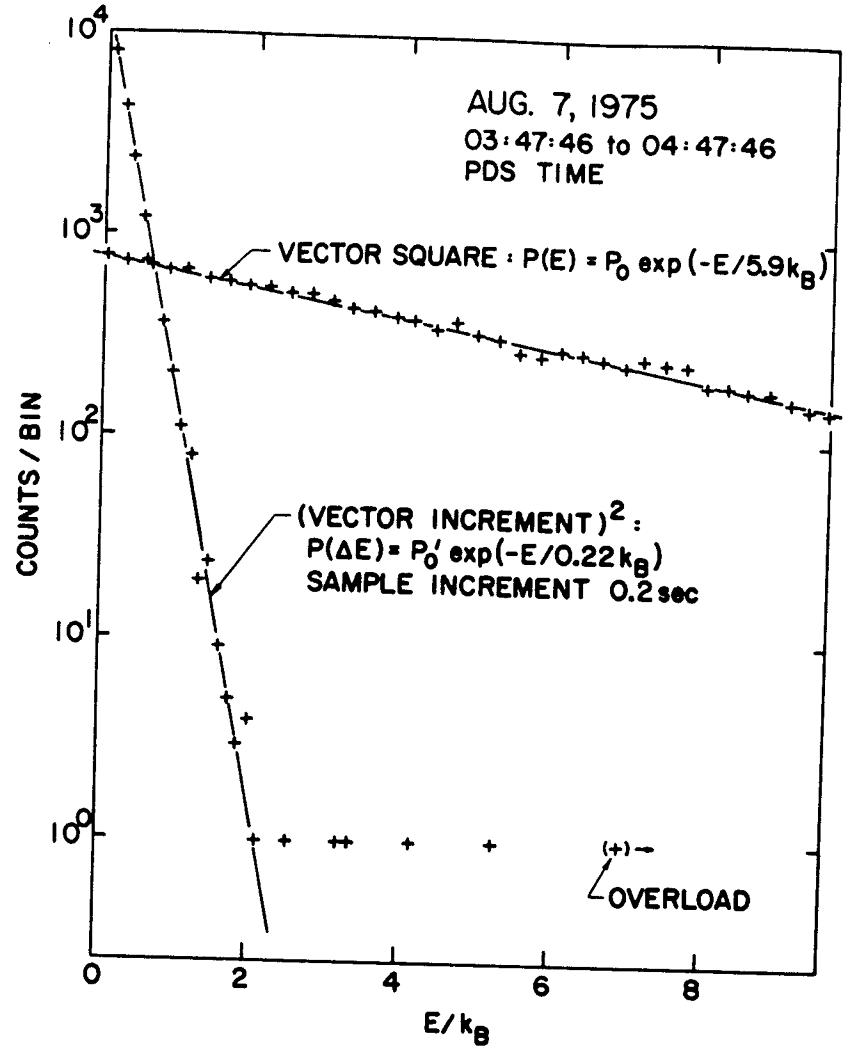


FIGURE 18: OUTPUT STATISTICS OBTAINED IN 8.64×10^6 SEC FROM PROTOTYPE GRAVITATIONAL WAVE ANTENNA

as parametric amplifiers the effective noise temperature could be as low as 10 μK. All such devices have electrical inputs and require transducers such as the resonant diaphragm. Until the optimum source impedance of the following amplifier is determined it is impossible to say how close the resonant diaphragm is to an ideal transformer.

An alternative to the detector of Figure 17 is the one being developed by the LSU team*, which is a resonant microwave cavity modulated by a diaphragm attached to the bar. It too is a parametric amplifier. A calculated minimum noise temperature is 0.1 μK. For a 5 X 10^6 kg bar the antenna noise becomes negligible when T_B/Q_B is below 3 x 10^{-8} K. With a Q of 4 X 10^6 the bar should be cooled to 0.1 K. The minimum detectable energy flux is 80 erg cm^{-2}, which corresponds to detecting an isotropic mass conversion of 4 x 10^{-7} solar masses at the centre of the galaxy or 1.6 solar masses at 20 Mpc. Since the energy converted to gravitational radiation in a typical supernova explosion is calculated to be less than 0.04 solar masses there is small chance of detecting pulses from visible events in the Virgo cluster unless some device is made not subject to the limitations of conventional amplifiers. The gravitational wave astronomer will not lack challenges for many years to come.

6. THE TWIN SATELLITE EXPERIMENT

A spacecraft orbiting a massive body like the Earth may be regarded as a gyroscope. The angular momentum of such a global gyroscope, being determined by the orbit radius and period and the mass of the spacecraft, is about fourteen orders of magnitude larger than that of the spinning spheres in the Gyro Relativity experiment. If the satellite is constrained to follow an internal shielded proof mass by means of drag-free controllers and thrustors its orbit is vey little disturbed by air drag, solar radiation pressure or other non-gravitational forces. The question arises whether it too might detect small relativistic precessions like those investigated by Schiff for an ordinary gyroscope.

Two principal relativistic effects influence a satellite's motion about the Earth. In track there is a perigee precession similar to the perihelion advance of Mercury about the sun, equal in an 800 km orbit to 12.9 arc-sec/year or 360 m displacement along the path. Cross-track the Earth's rotation causes a nodal dragging of the orbit plane, first calculated by Lense and Thirring [53] in 1918, giving a rate of advance of the right ascension of the ascending node

$$\underline{\dot{\Omega}}_{LT} = \frac{2I\omega_e}{c^2a^3(1 - e^2)^{3/2}} \underline{n}_e \tag{33}$$

equal to 0.18 arc-sec/year in an 800 km orbit.

* T. Bernat, W. O. Hamilton, W. Oelske, J. M. Reynolds

The DISCOS controller of the TRIAD satellite was good to about 5×10^{-12}g. With any drag free satellite the residual error, due chiefly to self-gravitation, tends to stay fixed in spacecraft coordinates. Consider an Earth-oriented satellite. In track the displacement builds up continuously according to Hill's equation $S = (3/2)ft^2$. After one year the error from a constant 5×10^{-12} g acceleration is 60 km, a factor of 200 times the relativistic perigee advance, but this could be much reduced by spinning the satellite or periodically turning it through 180° about the local vertical. Cross-track a constant 5×10^{-12}g acceleration simply displaces the orbit plane sideways through a fraction of a mm , but with DISCOS residual errors may have caused a cumulative orbit plane drift of about 2cm/year, or in angular measure 10^{-16} rad sec^{-1} . Thus the drift-rate of the global gyroscope from non-gravitational sources is comparable with that expected from the 4 cm diameter electrically suspended gyroscope and only about 0.3% of the Lense-Thirring nodal drag.

Equation (33) should be compared with Schiff's equation (23) for the motional precession $\dot{\underline{n}}_s^M$ of the spin axis of an orbiting gyroscope. The two effects have the same functional dependence on I, ω_e, a and e, but whereas the gyro precession $\dot{\underline{n}}_s^M$ depends on the orbit inclination i and reverse sign when i is 54° 16', the nodal advance $\dot{\underline{\Omega}}_{LT}$ is independent of i. This surprising result occurs because the perturbing function and the transverse component of orbital angular momentum both depend on sin i, which cancels from the final expression. In identical polar orbits $\underline{\Omega}_{LT} = 4\dot{\underline{n}}_s^M$. D. C. Wilkins has pointed out that both effects originate in the off-diagonal terms g_{14}, g_{24} of the metric for the rotating Earth, but the centre of mass of the satellite follows a geodesic and has time-like four velocity, while the gyro spin vector obeys Fermi-Walker transport and is space-like.

In addition to the Lense-Thirring drag the global gyroscope undergoes a geodetic precession due to the Earth's motion about the sun, just as the electrically suspended gyro does. The motion in the plane of the Earth's equator is 0.020 arc-sec/year; there is also a change of 0.0084 arc-sec/year in orbit inclination. The total relativistic displacement with respect to inertial space is $(\Omega_{LT} + \Omega_G)$ which amounts after 2.5 years to a lateral shift of (13.9 + 1.51) m referred to the Earth's surface. Since the Earth and moon also undergo geodetic precessions in their motion about the sun, the nodal advance of the orbit-plane with respect to the Earth-moon system is Ω_{LT} rather than $(\Omega_{LT} + \Omega_G)$.

In April 1973 R. A. Van Patten conceived the idea of looking for the Lense-Thirring drag in tracking data from TRIAD. The U. S. Navy's TRANET Döppler tracking network can maintain a running average measurement of the cross-track position of a satellite from each ground station to about 1 m over any 15 day arc, with integration to lower values over longer times. Thus both the non-gravitational drift of the global gyroscope and the tracking errors in determining its nodal position with respect to a ground station are well below the cumulative value of $(\Omega_{LT} + \Omega_G)$ after 2.5 years. The idea of performing a relativity experiment with a polar-orbiting drag-free satellite seems promising.

The limitation to a measurement of $(\Omega_{LT} + \Omega_G)$ with a single satellite comes in estimating the non-relativistic nodal regression Ω_Q due principally to the Earth's oblateness

$$\underline{\dot{\Omega}}_Q = -\frac{3}{2}\,\bar{\omega}_0\,J_2'\left[\frac{R_e}{a(1-e^2)}\right]^2 \cos i\;\underline{n}_e \qquad (34)$$

where $\bar{\omega}_0$ is the mean motion, R_e the Earth's equatorial radius and J_2' is a regression coefficient formed from the Earth's quadrupole mass-moment J_2 with corrections for higher terms. The quantities J_2' and R_e are well-known, and $\bar{\omega}_0$, a, and e can be accurately determined, but appreciable uncertainties remain in the time history of the inclination angle i from both tracking errors and uncertainties in the Earth's polar position, which is known only to about 0.3 m or 10^{-2} arc-sec. Small as the uncertainties are, they are enough to cause errors in determining the time history of $\dot{\Omega}_Q$ with a single satellite about six times the relativistic modal shift $(\Omega_{LT} + \Omega_G)$.

In January 1974 R. A. Van Patten and I conceived an experiment which has the remarkable property of measuring $(\Omega_{LT} + \Omega_G)$ and simultaneously yielding information about perturbations of the orbit plane and altitude some three orders of magnitude more accurate than any previously obtained. The experiment depends on two counter-orbiting drag-free satellites in polar orbits, initially adjusted by in-flight corrections to be very nearly equal and opposite but planned to avoid collisions. The orbits are chosen so that encounters occur close to the north and south poles. At each polar passing the distance between the spacecraft is measured to about 1 cm. by satellite to satellite Döppler ranging, while at lower latitudes each satellite is tracked independently from existing ground stations. The satellite to satellite data yields a measurement of the angle 2α which is the sum of the coinclinations $(i_1' + i_2')$ for the two orbits. Since orbit inclinations are defined from the ascending node, counterorbiting satellites in the same plane have opposite coinclinations and the nodal rates are

$$\dot{\Omega}_{Q_1} + \Delta\dot{\Omega}_Q = KJ_2'\,(i_1' + \Delta i') \qquad (35)$$

$$\dot{\Omega}_{Q_2} - \Delta\dot{\Omega}_Q = KJ_2' (i_2' - \Delta i') \tag{36}$$

where $\Delta i'$ is the angle from the true pole to an estimated pole location and $\Delta\Omega_Q$ is the error in nodal rate associated with $\Delta i'$. When (35) and (36) are combined with the definition of α and integrated over the life of the experiment the calculated sum of the two nodal regressions is

$$\left[\Omega_{Q_1} + \Omega_{Q_2} \right]_{calc} = 2KJ_2' \int_{t_1}^{t_2} \alpha \, dt \tag{37}$$

independent of the uncertainty in orbit inclinations. The regression sum calculated from (37) contains the Newtonian perturbations from J_2', but the measured regressions $\Omega_{1_{meas}}$ and $\Omega_{2_{meas}}$ of the two satellites, individually tracked from ground stations at lower latitudes, each also contain the relativistic term $(\Omega_{LT} + \Omega_G)$

$$2(\Omega_{LT} + \Omega_G) = \Omega_{1_{meas}} + \Omega_{2_{meas}} - \left[\Omega_{Q_1} + \Omega_{Q_2} \right]_{calc} \tag{38}$$

Thus the combination of polar ranging and ground-tracking data yields $(\Omega_{LT} + \Omega_G)$.

The experiment dictates the use of closely matched very nearly polar orbits obtained by in-flight corrections during the first phase of the mission. The mission lifetime is 2.5 years. Occasional (i.e. every 3 month) orbit adjustments are needed to assure collision avoidance. The resultant errors may be kept small by restricting the adjustments to impulses applied at equatorial crossings.

The point that the Lense-Thirring effect drags counter-orbiting satellites in the same sense, while Newtonian perturbations are opposite, was recognized by Shapiro, Miller and Jaffé [54] for a suggested experiment in polar orbit around the sun, and by R. W. Davies [55] for a suggested experiment in equatorial orbit around the Earth. However the idea of eliminating the error in the calculated nodal regression by satellite to satellite ranging between polar orbiting spacecraft is new, and crucial to this experiment.

The satellite to satellite Döppler ranging measurement picks up more than just a mean plane separation. The orbits are perturbed by solar and lunar gravity gradients and by the higher order mass-moments of the Earth. The gravity perturbations may be calculated in various ways: one nice method is to utilize the concept of the satellite as a global gyroscope and calculate the drift rate $\dot{\underline{n}}_s^g$ of the gyroscope. This leads to an interesting comparison given below between gravity gradient effects in the twin satellite and Gyro Relativity experiments. The formula is the same as that derived by Laplace in 1789 for the precession of the equinoxes; for a gyroscope of inertia ratio $\Delta I/I$ spinning with angular velocity ω_s at a distance R from a point mass M

$$\dot{\underline{n}}_s^g = \frac{3}{2}\frac{\Delta I}{I}\frac{GM}{\omega_s R^3}(\underline{n}_s \cdot \underline{n}_g)(\underline{n}_s \wedge \underline{n}_g) \qquad (39)$$

where $\underline{n}_g$ is the unit vector defining the gradient direction. Taken around an elliptic orbit (39) yields periodic terms and a secular drift $\bar{\dot{\underline{n}}}_s^g$ towards or away from the orbit normal

$$\bar{\dot{\underline{n}}}_s^g = \frac{3}{4}\frac{\Delta I}{I}\frac{GM}{\omega_s a^3(1-e^2)^2}\sin 2i \qquad (40)$$

where i is the angle between $\underline{n}_s$ and the orbit normal [56] .

For a satellite ω_s is just the mean motion $\bar{\omega}_0$ and $\Delta I/I$ is equal to 0.5, the inertia ratio for a ring of matter about axes in and normal to its orbit-plane. With matched counter-orbiting satellites the secular and periodic disturbances from lunar and solar gradients are equal in magnitude but opposite in direction. Since the orbit-normal is tilted with respect to the ecliptic plane and the plane of the moon's orbit there are two secular terms. The lunar effect also has a long term variation since the moon's orbit vector moves along the eliptic vector in a circle of radius 5° with nineteen-year period. The combined secular and nineteen year terms can be very nearly balanced out for any launch date by choosing a suitable orbit node. After removal of the long term effects the remaining direct variations in $2a\alpha$ are a twice yearly term of amplitude ± 1100 m and a twice monthly term ± 180 m. Tides raised on the Earth's surface add components nearly but not quite in phase with the main solar and lunar perturbations, having amplitudes about 15% of the direct terms. All ten effects (secular and periodic) propagate into the nodal motions through the regression equation (34). There are in addition direct perturbations of the nodes from the lunar and solar gradients, having amplitudes 30% to 50% of the 1100 m and 180 m perturbations in $2a\alpha$. None of the nodal perturbations cause significant errors in $(\Omega_{LT} + \Omega_Q)$ since all have equal and opposite effects on the two satellite orbits.

The effects of the Earth's higher order mass-moments have been investigated by D. Schaechter and J. V. Breakwell [57] . Tesseral harmonics ($J_{\ell m}$) of the field with even ℓ and m produce fluctuations in the lateral separation at the poles in times per day, the amplitude of the largest term J_{22} being ± 550 m in a nearly circular 800 km orbit. Tesseral harmonics with ℓ and m odd produce vertical perturbations in times per day at the poles, the largest amplitude (J_{31}) being ± 174 m. Harmonics like J_{41} with even ℓ and odd m have relatively minor effect on the vertical separation but introduce a small difference between the daily histories of the vertical separations at the two poles. Harmonics like J_{32} with odd ℓ and even m have a similar small effect on the lateral separations. Thus the m times daily fluctuations (both lateral and vertical) are not quite the same at the two poles.

In addition to the foregoing effects slight orbital eccentricity differences cause altitude fluctuations from perigee regression, having a period of 15 weeks. The direct effect of the gradient of the lunar gravity gradient cause monthly relative altitude fluctuations of about 10 m. Finally Graziani and Breakwell [58] have shown that longitude differences in the elastic tidal response of the Earth will appear as lateral fluctuations at integral numbers of times per day, modulated by the solar and lunar periods, in other words, apparent seasonal fluctuations in tesseral harmonics. North-south elastic asymmetry of the Earth will cause twice yearly and twice monthly vertical perturbations.

The slant range distance r between the two spacecraft is determined by Döppler ranging with a standard deviation σ_r given theoretically by a formula supplied independently by J. D. Anderson and J. V. Breakwell

$$\frac{\sigma_r}{r} = \left(\frac{8}{\pi}\right)^{\frac{1}{2}} \frac{c}{V} \frac{\sigma_f}{f} \left(\frac{Vt_c}{c}\right)^{\frac{1}{2}} \qquad (41)$$

where V is the relative velocity, f the nominal frequency, σ_f the standard deviation of f, and t_c the correlation time for σ_f. Equation (41) assumes the spacecraft pass on parallel courses with data taken over all time (sight angles ϕ between $\pm\pi/2$). R. A. Van Patten has made preliminary studies of a practical system based on a transmitted frequency of 4 GHz (chosen to give a reasonable antenna size) in which data is taken for a few seconds between ϕ's of $\pm 11\pi/24$, i.e. $\pm 82\frac{1}{2}°$. This yields an amount of data reasonable to process with an on-board computer between passes: from 170 to 3400 15 bit words, depending on range. The σ_v is within $\sqrt{2}$ of the theoretical value from equation (41). At 1000 m r can be determined to 1 or 2 cm.

Figure 19 a illustrates the slant range data to be expected from the experiment during a typical six month period, with the lunisolar perturbations at the poles and the envelope of the geophysical perturbations. Each 90 day period yields about 2600 data points. A good mathematical model must include all parameters that perturb the orbit down to the Döppler limit of 1 cm. Based on theoretical

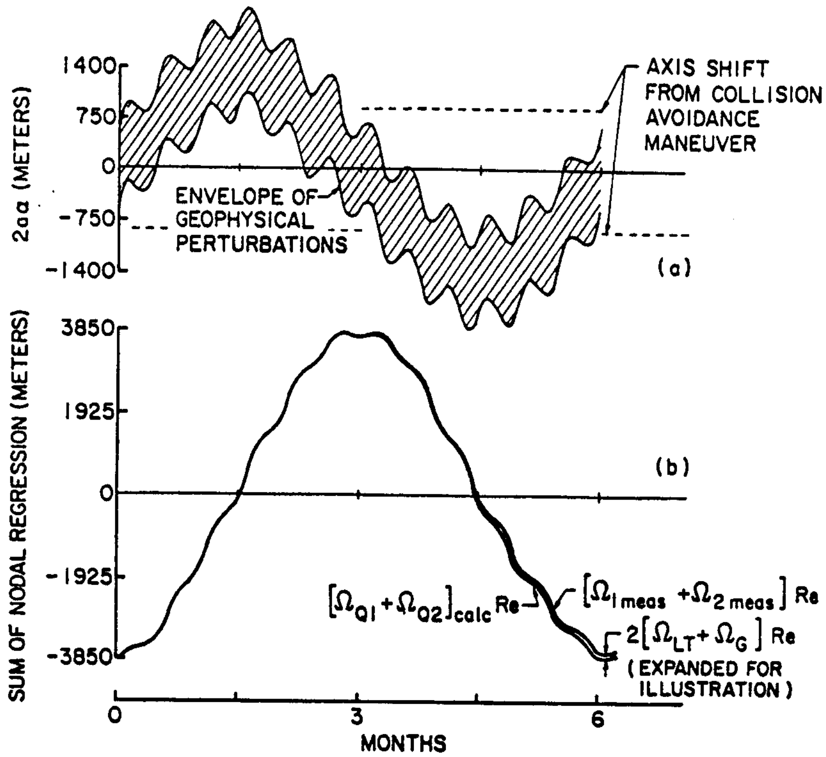


FIGURE 19: FORM OF DATA FROM TWIN SATELLITE EXPERIMENT
(a) lateral polar data (b) extraction of relativity information from nodal experiment

estimates by Allan of the magnitudes of $J_{\ell m}$, Breakwell and Schaechter have developed a model containing 202 parameters including the magnitudes and phases of tesseral harmonics up to m = 60 in the lateral perturbations and m = 59 in the vertical perturbations. Solar and lunar gradient and tidal effects were included in the lateral direction as well as the gradient of the lunar gravity gradient in the vertical direction, and frequencies corresponding to seasonally varying tesseral coefficients were included in the lateral direction through m = 8. Two plane separations and two eccentricities were also included. Although the slant-range data contains the odd-frequency differential altitude functions in quadratic combination with the lateral fluctuations, so that altitude perturbations cause twice daily fluctuations in slant range, the lateral and vertical perturbations can be separated in data processing because the slant-range sensitivity to vertical perturbations changes appreciably during the lunisolar cycle.

A covariance analysis of model data for a typical six month period at one pole has been performed by Schaechter and Breakwell. The covariances were very insensitive to different recent nominal descriptions of the Earth's gravity field. The accuracy of the Doppler determination of slant range was assumed to vary with the range to be measured in accordance with equation (41). A computer program employing a Householder transformation technique was used to sequentially update the covariance matrix, starting with a pessimistic initial covariance. The following results were obtained for a six month experiment:

1. All periodic lateral fluctuations (m even) are measurable to a statistical uncertainty of 0.5 mm.
2. All periodic vertical fluctuations (m odd) are measurable to 2.5 cm.
3. Mean plane separations are measureable to 0.2 cm.

Figure 19 b illustrates extraction of the relativity data when α is integrated over the experimental period. The high frequency geophysical terms are small, and the calculated regression $\left[\Omega_{Q_1} + \Omega_{Q_2}\right]_{calc}$ has the form illustrated in the lower curve of 19 b, dominated by solar and lunar terms. The quantity $\left[\Omega_{1_{meas}} + \Omega_{2_{meas}}\right]$ from tracking data is the upper curve of 19 b. The relativistic quantity $2(\Omega_{LT} + \Omega_G)$ is the secularly increasing difference between the two curves. Error sources in the measurement include the 0.2 cm uncertainty in mean plane separation of the orbits at the poles, uncertainties in the J_2 used to calculate the nodal regression, fluctuations in the self-gravitation of each satellite on its proof mass, nodal shifts from the collision avoidance manoeuvers, and errors in $\Omega_{1_{meas}}$ and $\Omega_{2_{meas}}$ from ground tracking. They are analysed in detail elsewhere [59] . The greatest present limitation is the uncertainty in time base for ground tracking. The quantities $\Omega_{1_{meas}}$ + $\Omega_{2_{meas}}$ have to be referred to inertial space, and the uncertainty in UT1(BIH) may be 0.5 to 1.0 m over the 2.5 year duration of the experiment, giving an error in $(\Omega_{LT} + \Omega_G)$ of 3 to 4 %. However there are good prospects for improving UT1 over the next few years by corrections from lunar laser ranging. With reasonable assumptions about the improvement the error should come down to 0.1 m, making the net error in $(\Omega_{LT} + \Omega_G)$ from all sources 1.1% for a 2.5 year experiment.

The satellite-to-satellite ranging data determines the orbit perturbations due to combinations of Earth harmonics $J_{\ell m}$ with the same m to accuracies some three orders of magnitude higher than they are now known. Thus the differences between vertical perturbations calculated [57] from two recent gravity models (SAO 1973 and GEM 1974) are as much as 50 m whereas the experiment determines the quantities to a few cm. New geophysical data is also obtained from the tidal perturbations. K_2 , the Love number associated with the Earth's elastic response to a second harmonic disturbing function, is measurable to 1 part in 10^5 , two orders of magnitude better than it is now known. K_3 , the Love number associated with the Earth's elastic response to a third harmonic disturbing function, is measuable to 1 part in 10 . No determination of K_3 has been made.

The relativistic perigee advance, large as it is, is not measurable in the twin satellite experiment. The difficulty is the 15 week non-relativistic perigee regression. Unfortunately both relativistic and non-relativistic terms reverse sign in a reversed orbit. The greatly improved geophysical data offers some help in calculating out the non-relativistic term, but not enough. The best hope would be to put another satellite in an orbit with an inclination near the value 64° 16' at which the non-relativistic perigee regression changes sign. If the regression were solely due to the Earth's oblateness, the combination of an orbit near 64° 16' and accurate calculations based on tracking data and the improved knowledge of J_2 from the twin satellite experiment, might succeed. However the perigee regression also depends on high harmonics in different combinations from those measured by the twin-satellite experiment. A measure of perigee advance does not seem to be feasible.

7. CONCLUSION

From the Cavendish experiment to the four new experiments on massive bodies described here runs a smooth arc , on which Poggendorf's invention of the optical lever, Boys's investigation of the optimum size of an experiment, Ising's study of the Brownian limit on galvanometers, and the more recent developments in cryogenic technology, serve as defining points. Comparisons among the four new experiments are useful, both to understand the experiments themselves and as background for other possible gravitational experiments.

Consider first experimental size. The equivalence principle experiment needs to be small in order not to be disturbed by gravity gradients. The gravitational wave antenna needs to be large to increase its cross-section to incoming waves. The cryogenic gyroscope is little affected by changes in size. It is indeed striking, and not wholly fortuitous, that the global gyroscope, with fourteen orders of magnitude higher angular momentum, has approximately the same residual drift-rate from non-gravitational forces as the 4 cm diameter spinning sphere.

Consider next the frequency range of the effects to be measured. The Gyro Relativity and twin satellite experiments measure quantities whose periods for a complete 360° rotation are from 2×10^5 to 3×10^7 years. The equivalence principle effect has characteristic periods of 90 minutes or 24 hours. Gravitational wave antennas have resonant frequencies around 1000 Hz, and look for pulses with charactersitic times of order 0.01 sec. These differences in frequency domain affect the experiment plan, even to the extent of determining whether a particular experiment should be done on Earth or in space. The gyroscope drift, being essentially a d. c. effect, cannot on Earth be separated from the d. c. component of drift due to suspension forces; the experiment must be done in space. An equivalence principle experiment is subject on Earth to tides and other diurnal disturbances in the same frequency domain as the effect to be measured; it too is better performed in space, even apart from the increase in driving acceleration. For a gravity wave experiment, however, there is little advantage in space operation, since seismic disturbances are easy to filter at 1000 Hz.

The arguments for cryogenic operation require thought. One line, dear to the heart of a certain kind of theoretical physicist, is the reduction in thermal noise. For gravity wave antennas this has some truth, but the relevant comparison is between the two ratios T_B/Q_B for the bar and T_N/G for the amplifier; the real advantage of low temperatures is in the combination of a high gain parametric amplifier cryogenically coupled to the bar. For the gyroscope thermal noise in the rotor has three principal effects (1) a jitter of the apparent spin axis due to exchange of energy between the lattice angular momentum and the molecular degrees of freedom, (2) a much higher frequency jitter due to thermal fluctuation currents in the superconductor, (3) random walk drifts of the gyro spin axis from the transfer of angular momentum to the ball through impacts of photons or gas molecules. I have shown elsewhere [60] that the random walk drifts are completely negligible even at room temperature. The current fluctuations correspond to displacements of several arc-sec in the readout but being at a frequency of about 1 GHz they are unobservable. The lattice exchange jitter is about 10^{-6} arc sec. Readout noise is therefore wholly dominant, or to put the same point in other words, several order of magnitude improvement in readout sensitivity are needed to achieve a discrimination factor ∂_B of unity. Thermal noise in the rotor has no bearing on the cryogenic operation of the Gyro Relativity experiment.

Nor is thermal noise a determining factor yet in equivalence principle experiments. The limiting $\langle\eta\rangle$ for both Dicke's and Braginsky's experiments was 10^{-13}

after a single day, well below the practical limit, while in the space experiment $\langle\eta\rangle$ is around 10^{-19}, again well below other limits. It is indeed a nice point, and one whose explanation lies but a short way below the surface, that Brownian motion has so far only been a limit for very small bodies like Blackett's magnetometer or very large bodies like the gravity wave antennas. For objects of intermediate size like the Cavendish, Eötvös and gyroscope experiments it does not yet determine a limit.

An Eötvös or Cavendish experiment is ultimately a measurement of an acceleration f on a suspended body. The limit from (11) on determining f in time S is

$$\langle f\rangle \sim \sqrt{\frac{2\beta kT}{MS}} , \qquad (42)$$

where β is the damping coefficient and M the mass of the suspended body. In nearly all experiments, particularly those done at low temperatures, the damping is predominantly that from the residual gas surrounding the balance which for a body of characteristic dimension D and gas at pressure p is given by

$$\beta = CD^{-1}\sqrt{\frac{2\pi m}{kT}}\ p , \qquad (43)$$

where for a spherical body C is $5/32\pi$. Substituting (43) in (42) and putting M proportional to ρD^3 we have

$$\langle f\rangle \sim \frac{A}{D^2}\left(\frac{1}{\rho S}\right)^{\frac{1}{2}} p^{\frac{1}{2}}\ T^{\frac{1}{4}} \qquad (44)$$

so at constant pressure, lowering the temperature yields only an improvement proportional to $T^{\frac{1}{4}}$: a meagre gain.

The reduction of noise with temperature becomes still more questionable if the experiment depends on an optical readout. As the temperature is lowered an optical readout faces new problems of three kinds, each of which limits the useful reduction of T. The first, of academic interest, applies even in a perfect vacuum. Since in lowering the temperature the Brownian motion decreases while the number of photons needed to discriminate the signal increases, there must be some temperature T_ϕ at which disturbances from the random impacts of photons exceed the ordinary Brownian motion. For a balance of period τ_0 observed by means of an optical lever of efficiency ε and discrimination factor ∂_B the temperature is

$$T_\phi = 6.9\ \frac{h}{k}\ \frac{1}{\varepsilon^{\frac{1}{2}}\ \tau_0\ \partial_B} \qquad (45)$$

from which for a ∂_B of 10^{-3}, τ_0 of 10 sec and ε of 1% the operating temperature has to be above 0.3 μK. The second limit is that the light beam, being a photon gas, introduces damping which again goes up as the temperature is reduced. Further related limits have been suggested by Braginsky [61] from effects of an imperfectly centered light beam; his assumptions appear unnecessary since the light can be centered on the mirror very accurately by means of servo driven tipping plates controlled by signals derived from a chopped light source.

The real limit to cryogenic operation with an optical lever is that when the light beam falls on the mirror some radiation is absorbed and must be got rid of. At room temperature heat can be removed either by radiation or exchange gas, but at low temperatures radiative transfer becomes ineffective. The addition of exchange gas increases the damping and hence the Nyquist limit. Since the amount of gas needed depends on the intensity of the photon beam, there is an optimum working pressure which is found by minimizing the sum of the squares of the Nyquist limit, expressed as the angular displacement corresponding to the limiting torque from equation (11), and the measurement noise (equation (3)). The result depends on the absorption coefficient Λ of the mirror and ΔT the maximum allowed temperature difference between the suspended body and its surroundings. For a body of diameter D surmounted by a mirror of diameter D'

$$\langle \Theta_N^2 \rangle + \delta\Theta^2 = \frac{1}{D^2}\left[\frac{M}{P} + Np\right]\frac{T^{\frac{1}{2}}}{S} , \qquad (46)$$

where for a spherical body

$$M = 1.24\, hc \left[\frac{m}{k}\right]^{\frac{1}{2}} \frac{\bar{\lambda}\,\Lambda}{\varepsilon\,\Delta T\, D'^2} \quad \text{and} \quad N = 4 \times 10^{-5}\,(mk)^{\frac{1}{2}}\,\frac{\tau_0^4}{\rho D^4} .$$

Thus in any experiment where exchange gas provides the only method of removing heat from the suspended body there is an optimum pressure equal to $\sqrt{M/N}$ independent of the working temperature T. For a reasonable Λ and ΔT (say 1% and 10 mK) the operating pressure has to be $1.9 \times 10^{-2} \sqrt{\rho}\, D^2/D'\tau_0^2$. For Dicke's balance with period 230 sec. the optimum pressure is 10^{-5} torr. From equation (44) the minimum detectable acceleration $\langle f \rangle$ scales as $p^{\frac{1}{2}}\, T^{\frac{1}{4}}$. Suppose one starts cooling the balance from room temperature at initial pressure 10^{-7} torr. First $\langle f \rangle$ goes down, but near 90 K radiation cannot provide adequate cooling and the pressure must be increased two orders of magnitude, so $\langle f \rangle$ increases a factor of 10. To recover the lost ground the balance must be cooled below 9 mK. The difficulty can only be avoided by using a balance whose suspension period τ_0 exceeds $0.14\, \rho^{\frac{1}{4}}\, D/D'\, p_{min}^{\frac{1}{2}}$, where p_{min} is the lowest operating pressure. In our example with p_{min} of 10^{-7} torr, τ_0 has to exceed one hour. Should one wish to work at lower pressures, the natural period of the balance has to be correspondingly longer.

Such difficulties are avoided with the linear and angular position readouts based on Josephson junction devices for which heat transfer to the suspended body is negligible. However the London moment readout for the gyroscope, superior as it is to standard gyro readouts, does not yet attain the resolution of the Jones optical lever. The optimization conditions of each experiment have to be found individually.

Besides specific ideas like the London moment readout, the chief assets of cryogenic technology are (1) the extraordinary improvements in mechanical stability, thermal stability and magnetic shielding at liquid helium temperatures, (2) the many applications of superconducting devices, (3) the reduction in disturbing forces on a suspended body, especially radiation pressure. Consider the equivalence principle experiment. At room temperature the unbalanced pressure from black body radiation at one end of a cylinder of density 10 causes an acceleration of 1.3×10^{-9} g, five orders of magnitude larger than the Eötvös acceleration for an η of 10^{-17}. Cyclically varying temperature differences of 0.001 C across the experimental chamber could masquerade as an Eötvös signal of 2×10^{-17}. Such effects, and corresponding effects of residual gas, become almost negligible at low temperatures.

One last instructive comparison is in the influence of gravity gradient torques on the Gyro Relativity, twin satellite and orbiting equivalence principle experiments. Both the 4 cm spinning sphere for the Gyro Relativity experiment and the global gyroscope of the twin satellite experiment are subject to periodic and secular drift terms derived from equations (39) and (40), but with quite different effects on experiment planning in the two cases. For the twin satellites there are two important secular terms, since the orbit vectors are inclined to both the ecliptic plane and the plane of the moon's orbit, but by choosing a suitable orbit node these can be balanced out. The critical terms for experiment planning are the periodic ones which cause excursion of the ascending node up to ± 3800 m (Figure 19b) or ± 120 arc-sec, two orders of magnitude larger than the expected relativity drift. The terms are separated in data analysis, in which, as already explained, the large lunisolar effects play an important part in separating the lateral and vertical geophysical perturbations. For the Gyro Relativity experiment solar and lunar gradients are negligible, but the gradient of the Earth's field is all important. As was remarked in the discussion of the orbiting equivalence principle experiment, the gravity gradient from the Earth on a body a few hundred km above its surface is five orders of magnitude larger than the sun's gradient at the same point. Hence, the secular drift term from equation (40) is critical and just as the equivalence principle masses must be well-centered, so the $\Delta I/I$ for the spinning sphere of the Gyro Relativity experiment must be kept small. A criterion derived from (40) is that density inhomogeneities should be below one part in 10^6, which is comparable to the criterion obtained from the mass-unbalance equation (26). The periodic gravity gradient terms, on the other hand, have negligible significance in the Gyro Relativity experiment. Their amplitude depends inversely on the frequency of the torque, and for a gyroscope orbiting the Earth the period is 45 minutes rather than the twice monthly and twice yearly periods characterizing the lunisolar torques on the counter-orbiting satellites. The maximum amplitude of the sine wave for a gyrosocpe with $\Delta I/I$ of 10^{-6} is 2×10^{-4} arc-sec. Procedures separating the gravity gradient torques from the relativity terms in inclined orbits are discussed elsewhere [33] .

To find so much variety among experiments having so much in common is one of the pleasures of working in experimental gravitation. The opportunities for ingenuity will not soon be exhausted.

ACKNOWLEDGEMENTS

I thank W. M. Fairbank and other colleagues named in the text for innumerable discussions, and R. P. Giffard, R. A. Van Patten and P. W. Worden, Jr. for criticizing portions of the manuscript.

REFERENCES

[1] H. Cavendish, Phil. Trans. Roy. Soc. 83 (1798) 470. Reprinted in the 1809 abridgement of the Transactions by C. Hutton, G. Shaw and R. Pearson (London 1809) vol. 18, p. 389.

[2] The quoted error is the 95% confidence limit on the mean of Cavendish's 23 measurements.

[3] Maxwell to Joule, quoted in History of Cavendish Laboratory (London 1910) 31.

[4] C. W. F. Everitt, James Clerk Maxwell: Physicist and Natural Philospher (New York, Scribners 1975) 118-123.

[5] R. von Eötvös, D. Pekar and E. Fekete, Ann. Phys. 68 (Leipsig, 1922)11.

[6] P. H. Roll, R. Krotkov and R. H. Dicke, Ann. Phys. 26 (New York, 1964) 442.

[7] V. B. Braginsky "Verification of the Equivalence of Gravitational and Inertial Mass" B. Bertotti, ed. in Experimental Gravitation (New York, Academic Press, 1974) 252-258.

[8] W. J. H. Moll and H. C. Burger, Phil. Mag. 50 (6th Series, 1925) 624, 626.

[9] G. Ising, Phil. Mag. 1 (7th Series, 1926) 827.

[10] R. V. Jones, J. Sci. Instrum.,38 (1961) 37.

[11] R. V. Jones and C. W. McCombie, Phil. Trans. Roy. Soc. 244 (1951) 205.

[12] P. M. S. Blackett, Phil. Trans. Roy. Soc. 245 (1952) 309.

[13] H. A. Daynes, J. Sci. Instrum. 3 (1926) 7.

[14] A. V. Hill, J. Sci. Instrum. 4 (1926) 72.

[15] R. Ruffini and A. Treves, Astrophysical Letters 13 (1973) 109.

[16] V. B. Braginsky, Physical Experiments with Test Bodies (Moscow 1970, Washington D. C. NASA translation 1972) 53f.

[17] P. W. Worden, Jr. and C. W. F. Everitt, "Tests of the Equivalence of Gravitational and Inertial Mass Based on Cryogenic Techniques" in B. Bertotti (ed) Experimental Gravitation (New York, Academic Press 1974) 393.

[18] C. W. McCombie, Rep. Prog. Phys. 16 (1953) 266.

[19] Reference [6] p. 474.

[20] G. W. Gibbons and S. Hawkins, Phys. Rev. D. 4 (1971) 2191.

[21] R. V. Pound and G. A. Rebka, Phys. Rev. Letters 3 (1959) 439; R. V. Pound and J. L. Snider, Phys. Rev. B. 140 (1965).

[22] R. H. Dicke and H. M. Goldenberg, Phys. Rev. Letters. 18 (1967) 313.

[23] H. A. Hill "Light Deflection" in R. W. Davies (ed) Proceedings of the Conference on Experimental Tests of Gravitation Theories" JPL Technical

Memorandum (Pasadena, 1970) 33-499, p. 89.

[24] W. de Sitter, M. N. Roy. Astr. Soc. 76 (1916) 699; 77(1916) 155, 481.

[25] J. A. Schouten, Proc. Amst. Acad. 21 (1919) 533.

[26] A. D. Fokker, Proc. Kon. Akad. Weten 23 (Amsterdam, 1920) 729.

[27] A. S. Eddington, Mathematical Theory of Relativity (Cambridge, 1926) 99.

[28] Letter from P. M. S. Blackett to C. W. F. Everitt, 20 November 1962 and personal conversations.

[29] L. I. Schiff, Proc. Nat. Acad. Sci. 46 (1960) 871.

[30] G. E. Pugh, WSEG Research Memorandum, No. 11, Weapons System Evaluation Group, the Pentagon, Washington 25 D. C. (12 November 1959).

[31] B. M. Barker and R. F. O'Connell, Phys. Rev. D. 2 (1970) 1428.

[32] D. C. Wilkins, Ann. Phys. 61 (New York, 1970) 277: "Precession of a Gyroscope in a Perturbed Orbit about the Earth", Stanford University Memorandum, May 1970.

[33] C. W. F. Everitt, "Review of the Significance of Gravity Gradient Torques in Different Versions of the Gyro Relativity Experiment" (Stanford University, W. W. Hansen Laboratories of Physics 1974).

[34] P. M. Selzer, W. M. Fairbank and C. W. F. Everitt, Adv. Cry. Eng. 16 (1971) 277.

[35] Staff of the Space Department of the Johns Hopkin's University Applied Physics Laboratory and Staff of the Guidance and Control Laboratory, Stanford University, Journal of Spacecraft and Rockets 2 (1974) 631.

[36] B. Cabrera, "Generating Ultra Low Magnetic Field Regions with Superconducting Shields and Their Use with a Sensitive Magnetic Charge Detector" in M. Crusius and M. Vuorio, Low Temperature Physics LT 14 (Amsterdan, North Holland Publishing Co. 1975) vol. 4 p. 270.

[37] C. W. F. Everitt, W. M. Fairbank, W. O. Hamilton "From Quantized Flux in Superconductors to Experimens on Gravitation and Time Reversal Invariance" in John R. Klauder ed. Magic without Magic: John Archibald Wheeler (San Francisco, W. H. Freeman 1972) p. 217.

[38] J. R. Nikirk, "Fabrication of an Electronic Suspension Subsystem for a Cryogenic Electrostatically Suspended Gyroscope for the Relativity Experiment" Final Report on NASA Contract NAS8-27333 (Stanford University Center of Systems Research, January 1973).

[39] T. D. Bracken and C. W. F. Everitt, Adv. Cry. Eng. 13 (1968) 168.

[40] J. T. Anderson and C. W. F. Everitt, "A High Accuracy All-angle Gyroscope Readout Using Quantized Flux" Paper submitted to the Applied Superconductivity Conference, Stanford 1976.

[41] C. W. F. Everitt "The Gyroscope Experiment I: General Description and Analysis of Gyroscope Performance" in B. Bertotti (ed) Experimental Gravitation (New York, Academic Press 1974) p. 357, equation(29).

[42] J. A. Lipa, J. R. Nikirk, J. T. Anderson and R. R. Clappier, "A Superconducting Gyroscope for Testing General Relativity" in M. Crusius and M. Vuorio, Low Temperature Physics LT 14 14(Amsterdam, North Holland Publishing Co., 1975) 250.

[43] Reference[41], p. 341.

[44] C. W. F. Everitt, Final Report on Contract NAS 8-25705 to build and test a Precise Star Tracking Telescope (Stanford, July 1972).

[45] J. Bull and D. B. DeBra "Precise Attitude Control of the Stanford Relativity Satellite, Joint Automatic Control Conference, Ohio State University (June, 1973).

[46] D. Klinger,"Error Modeling of Precision Orientation Sensors in a Fixed Base Simulation" SUDAAR No. 481 (Ph.D. Thesis, Stanford University, 1974).

[47] R. A. Van Patten in Annual Report for the Program a Gyro Test of General Relativity and Develop Associated Control Technology" (June, 1973).

[48] J. A. Lipa, W. M. Fairbank and C. W. F. Everitt,"Gyroscope Experiment II: Development of the London Moment Gyroscope and of Cryogenic Technology for Space" in B. Bertotti (ed.) Experimental Gravitation (New York, Academic Press, 1974) 361; iden Proceedings of the Cryogenic Workshop (NASA George C. Marshall Space Flight Center, 1972) 169.

[49] R. W. P. Drever, J. Hough, R. Bland and G. W. Lessnoff, Nature 246 (1973) 340.

[50] Reference [7] page 245.

[51] R. P. Giffard "Sensitivity Limit of Resonant Gravitational Wave Detectors Using Conventional Amplifiers", Stanford University Memorandum,(December, 1975).

[52] H. J. Paik, "Low Temperature Gravitational Wave Detector" in B. Bertotti (ed.) Experimental Gravitation (New York, Academic Press, 1974) 515.

[53] J. Lense and H. Thirring, Phys. Zeits. 19 (1918) 156.

[54] Lewis F. Miller, "The Lense-Thirring Effect: A Theoretical Investigation of Possible Experiments to Measure It", Thesis submitted in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science at the Massachusetts Institute of Technology, (January 1971) 18.

[55] R. W. Davies, "A Suggested Space Mission for Measuring the Angular Momentum of the Sun" in B. Bertotti (ed.) Experimental Gravitation (New York, Academic Press, 1974) 412, Appendix II to paper discussing an experiment circulating the Earth.

[56] Reference [43] contained an error of a factor of 2 in the equation (11) corresponding to (40) here, which has been pointed out by B. M. Barker and R. F. O'Connell, Phys. Rev. D. (1976) in press.

[57] D. Schaechter, J. V. Breakwell, R. A. Van Patten and C. W. F. Everitt "A Covariance Analysis for Parameters Determined in the Relativity Mission with Two Counter-Orbiting Satellites" Astrodynamics Conference of the AIAA/AAS, San Diego, California, August, 1976) to be published.

[58] F. Graziani, J. V. Breakwell, R. A. Van Patten, C. W. F. Everitt, "Earth Tide Information from Two Counter Orbiting Polar Satellites"

International Astronautical Federation (IAF), XXVI$^{\underline{th}}$ Congress, Lisbon, Portugal, (September, 1975).

[59] R. A. Van Patten and C. W. F. Everitt, "A Possible Experiment with Two Counter-Orbiting Drag-Free Satellites to Obtain a New Test of Einstein's General Theory of Relativity, and Improved Measurements in Geodesy", Celestial Mechanics (1976) in press.

[60] Reference [43] p. 358.

[61] Reference [7] pp. 238 f.