GRAVITY PROBE B AS A GEODESY MISSION AND ITS IMPLICATIONS FOR TOPEX

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Introduction

Gravity Probe B (GP-B), a NASA Mission currently under development at Stanford University with support from Lockheed Missiles and Space Company, is designed to provide two extremely precise new tests of Einstein's general theory of relativity by means of observations on gyroscopes in Earth orbit. The experiment is to be flown in a polar orbiting drag-free satellite at an altitude between 600 and 650 km. The satellite will carry an onboard Global Positioning System (GPS) receiver for precise tracking. Launch is expected in 1995 or early 1996.

In 1987, following discussion between Smith and Everitt, we with other colleagues in the geodesy community came to the realization that Gravity Probe B could at very modest additional cost also provide new geodesy data of great importance in studies of geodynamics and also, more specifically, for improving the geoid to be applied in reducing the TOPEX (TOPographic EXperiment) altimeter data. The TOPEX altimeter has a height measurement precision of 2 cm, and oceanographic applications are limited by satellite orbit and marine

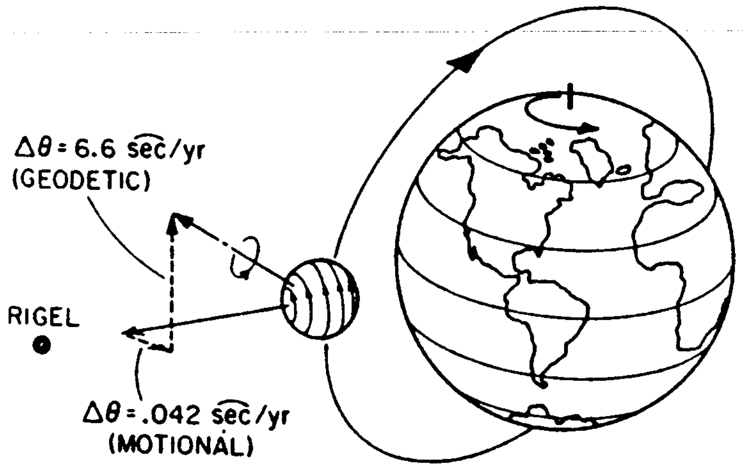

Figure 1: Relativistic Precessions of an Orbiting Gyroscope

only with recognition that an appropriately phased decision on a New Start for the Science Mission would be taken at a later time. Currently, STORE is manifested for Shuttle launch in June 1993 and a Science Mission review is scheduled for February 1991. Ground operation of FIST (First Integrated Systems Test), a full-scale prototype Science Instrument, is scheduled to commence in November 1989.

Drag-free Control as Applied to Gyroscopes and to a Geodesy Mission

The GP-B gyroscope is a sphere of fused quartz 38 mm in diameter coated with a thin layer of superconducting niobium, electrically suspended and spinning at 10,000 rpm in high vacuum. It operates at low temperature (1.8 K) and is surrounded by a superconducting magnetic shield. Readout is also magnetic, being based on observing the direction of the so-called "London moment" in the spinning superconductor. Figure 2 shows the assembled STORE/Science Mission payload, comprising four gyroscopes, a reference telescope and a "drag-free proof mass," all enclosed in a dewar vessel 3 meters long containing initially 1,580 liters of liquid helium. During the two-year lifetime of the Science Mission, this helium gradually boils away and the escaping gas is vented through a system of proportional thrusters to provide very smooth attitude and translational control of the spacecraft.

In order that Gravity Probe B reach its full potential as a relativity mission, it is extremely desirable that the spacecraft be made "drag-free." At 650 km altitude in ordinary circumstances, the accelerations acting on a spacecraft of the anticipated mass/area ratio would be on the order 5×10^{-8} g (5×10^{-9} m/sec^2), and these accelerations prove to be a significant limiting factor on gyro performance. A drag-free condition is achieved by placing near the spacecraft's center of mass a proof mass, essentially identical with one of the gyro rotors, enclosed in an evacuated spherical cavity, with reference electrodes that form part of a capacitance bridge to measure the location of the proof mass. The spacecraft is subject to drag but the proof mass, being in a shielded cavity, is not. It, therefore, allows an almost ideal geoid accuracies. Gravity Probe B appears capable of making a major contribution to reducing these uncertainties by providing a significant improvement in the knowledge of the Earth's gravity field. The net result could well be a factor of five to ten improvement in the recovery of the medium to long wavelength components of the mean dynamic topography associated with the global circulation from TOPEX and even greater improvement in the determination of the geoid itself.

The contribution which GP-B can make to the knowledge of the Earth's gravity field is referred to as the "geodesy coexperiment." It has now been studied in some detail by two groups: D. E. Smith, O. L. Colombo and E. C. Pavlis at NASA Goddard Center, and by M. Tapley and J. V. Breakwell at Stanford University. A third independent study has been performed by W. G. Melbourne at JPL. We summarize here preliminary results of these investigations and indicate directions for future work.

A second co-experiment is being considered. External forces acting on the satellite are cancelled by the drag-free control system. The control effort is therefore a measure of the external forces. When corrected for radiation pressure and other modelable disturbances [*Moe et al.*, 1976], it is a measure of atmospheric drag. The measurement can provide unprecedented spatial resolution of atmospheric drag at this altitude. Preliminary discussion with Gerald Keating of NASA Langley Research Center suggests that adding mass spectrometers would help in resolving the direction of the relative wind and would provide compositional data for correlation with observed drag variations.

The Gravity Probe B Mission

Leonard Schiff in 1960 calculated that an ideal gyroscope in orbit around a massive rotating body such as the Earth should, according to Einstein's general theory of relativity, undergo changes in spin direction of two kinds as measured with respect to the framework of the fixed stars. These two effects are illustrated in Figure 1. The larger, known as the geodetic effect, is a rotation in the orbit-plane due to the motion of the gyroscope through distorted space-time near the Earth. The smaller, or frame-dragging, effect is due to the dragging around of the local space-time reference system by the Earth itself. In a 650-km polar orbit, the geodetic effect is 6.6 arc-sec/yr and the frame-dragging effect 0.042 arc-sec/yr, the two terms being at right angles to each other as shown in the figure. A conservative error analysis establishes that determination of frame-dragging to somewhere between 0.1% and 1% and of the geodetic effect to 1 part in 10^4 or better should be feasible in Gravity Probe B.

The importance of determining these effects with extreme precision has been strongly emphasized throughout the relativity community. In 1981, the Space Science board made Gravity Probe B the number-one priority in its *Strategy for Gravitational Physics in the 1980s*.

In 1984, following a long period of basic technology development at Stanford University, the NASA Administrator approved a limited start for the Gravity Probe B flight program. To minimize technical risk, the program was divided into two phases: (1) STORE (Shuttle Test of the Relativity Experiment), an initial 7-day engineering demonstration of the total science payload onboard Shuttle, (2) Science Mission, consisting of recovery and refurbishment of the STORE instrument, integration with a spacecraft and its launch on a free-flying drag-free satellite with an expected mission lifetime of two years. The initial approval was for STORE gravitational orbit, and by applying thrust from the escaping gas to make the spacecraft chase after the ball, the effects of air drag, solar radiation pressure and other external disturbance forces are eliminated. In this way the effective transverse accelerations on the gyroscopes are reduced to 10^{-10} g or less and their performance correspondingly improved.

GP–B EXPERIMENT MODULE

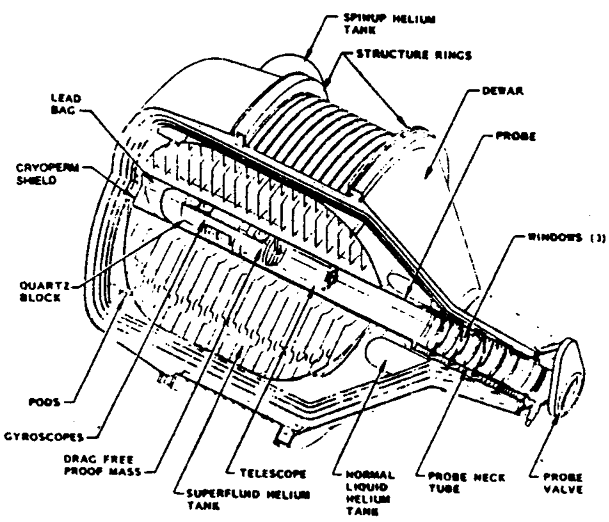

Figure 2: The Gravity Probe B Science Payload

The primary practical limit on the performance comes from the self-gravitational attraction of the spacecraft on the proof mass. In the DISCOS (DISturbance COmpensation System) controller developed by Stanford University as a spin-off from Gravity Probe B, and applied to the U.S. Navy's Triad transit navigation satellite launched in July 1972, the drag-free performance averaged to 5×10^{-12} g. This value is probably as low as one can achieve with a nonspinning spacecraft and capacitative pick-off.

For Gravity Probe B, the design goal for the drag-free system, as established by relativity, is less stringent and has been set at 10^{-10} g. This figure is the constant bias in the system. Since the spacecraft rolls about the line of sight to Rigel, which lies in the plane of the nonprecessing polar orbit, across-track, the mean acceleration is much less than that, and deviations from an ideal gravitational orbit are exceedingly small (~0.01 mm).

The inertially fixed component of bias acceleration along the line of sight to Rigel has effects of two kinds, one negligible and one not entirely so. The negligible effect is a small net displacement of the orbit center to or away from the guide star, which depends on initial conditions but is of order $2Rf/g'$, where R is the radius of the orbit, f the bias acceleration and g' the local acceleration at orbit altitude. The result is a mean displacement of 1–2 mm, which is of no concern for geodesy. The non-negligible effect is a variation in orbit eccentricity over time. Starting with a circular orbit around an ideal spherical Earth, one finds a secular decrease in perigee $R_p = 1.5 f / \bar{n}$ along a direction at right angles to the line of sight to Rigel, where $\bar{n}$ is the mean motion ($\sqrt{R/g'}$ in our earlier notation). With an f of 10^{-10} g, this would give an R_p of 50 m/yr. However, this effect is modified by the perigee advance that occurs with an eccentric orbit around the real (oblate) Earth. In a low polar orbit, the perigee advance has an 11-week period. The combination of the two effects produces a sinusoidal variation in the eccentricity of the orbit with a period of 11 weeks and an amplitude of approximately 1.7 m. This term will have to be identified and removed from the data in performing the geodesy coexperiment. That is easy to achieve.

In summary, while room exists for further study of non-constant contributions to the drag-free bias (caused, for example, by helium slosh in the dewar), the main fluctuating component is easily taken into account in reducing the data, and the more obvious terms at higher frequencies are negligible. The uncertainty in relative position between the proof mass and the GPS receiver is also negligible (<0.1 mm) owing to the great precision with which the proportional thrust system keeps the proof mass centered in its housing. Knowledge of Gravity Probe B's gravitational orbit is, therefore, only limited in practice by the accuracy of the GPS tracking data.

Tracking by GPS and Laser Ranging

Very good tracking is desired for the GP-B relativity mission, partly because of constraints in calibrating and calculating the Schiff effects and partly because of a very conservative approach to gyro performance. With GPS tracking it becomes possible to effect a very precise initial on-orbit adjustment of the orbit plane, so that its mean over the mission lifetime will remain within a very short distance (100 m) from the poles, and hence make certain disturbance torques on the gyroscopes, already small, even smaller. Details of the orbit adjustment procedure have been given by *Axelrad and Parkinson* [1989].

It is this combination of GPS tracking, very exactly polar orbit, drag-free performance and relatively low altitude that makes Gravity Probe B so interesting as a geodesy mission. By 1995, we may expect to have a worldwide terrestrial network of GPS stations available to support the TOPEX/POSEIDON Mission and for geodynamics studies (crustal motion, tides, Earth rotation). Combining these data with tracking of a relatively low satellite like Gravity Probe B provides a system of great strength both for geodesy investigations and for obtaining precise GPS ephemerides and satellite clock corrections.

The current baseline GPS receiver for GP-B is the Monarch receiver under development by the Motorola Corporation under contract to JPL, to be flown in 1992 on the TOPEX/POSEIDON spacecraft [*Yunck et al.*, 1985; *Melbourne and Davis*, 1987; *Carson et al.*, 1988]. For enhanced reliability, TOPEX/POSEIDON will carry a dual-string version. The Monarch is a geodetic quality 12 channel dual band receiver capable of tracking 6 GPS satellites simultaneously and providing C/A, P1, P2, L1 and L2 observations from each satellite on sample periods as short as 1 sec. Here, C/A is coarse acquisition pseudorange, P is P-code pseudorange, and L1 and L2 are the L-band operating frequencies (1575 MHz and 1227 MHz). The receiver can track the reconstructed carrier for both frequencies. The carrier phase observations on each channel provide a phase-connected (i.e., no cycle dropouts) time series of cumulative carrier phase measurements with the initial measurement at acquisition being modulo 2π. Each measurement in the series is statistically independent. The random error of the Monarch receiver for 1 sec samples is expected to be less than 2 mm on the carrier phase and about 13 cm on P-code pseudorange. Systematic errors arising from the receiver are expected to be negligible in carrier phase and less than 1 cm in pseudorange. Systematic errors in the measurements will arise mainly from uncalibrated multipath variations in both phase and group delay. The spatial and temporal variability of the multipath effects will depend on the physical and electrical properties of the antenna/backplane assembly, and on possible reflecting and/or diffracting structures in the vicinity of the antenna. For TOPEX/POSEIDON the design goal is to keep the spectral power of uncalibrated multipath for periods less than 3 minutes (roughly equivalent in wavelength to the resolution limit of gravity recovery with TOPEX/POSEIDON) to less than 2 mm. For pseudorange the design goal is to achieve smoothed values over 30 minutes and longer to better than 5 cm. The smoothed pseudorange data will be used in conjunction with the carrier phase data to aid in carrier cycle ambiguity resolution and for quasi-geometric point positioning of the spacecraft.

Essential to accurate differential positioning and recovery of gravity is the concurrent tracking of co-visible GPS satellites by geodetic quality receivers embedded in a global network of ground stations. These tracking stations – for TOPEX/POSEIDON there will be roughly a dozen – provide the simultaneous observations necessary to eliminate clock errors arising from frequency standard instability in the GPS satellites and in the receivers themselves. The network also provides the terrestrial reference frame through collocation of a number of the stations at very long baseline interferometry (VLBI) and satellite laser ranging (SLR) tracking sites, which are designated as fiducial points. The tracking data streams from GP-B and from the ground network will be processed jointly to obtain global solutions for the ephemerides of the GPS satellites and GP-B, the gravity parameters, relevant geodetic and geodynamic parameters associated with ground network, and ancillary parameters such as the cycle ambiguity integers or certain receiver clock epoch parameters.

Achieving the centimeter-level positioning accuracy, which allows GP-B to recover significant gravity information for harmonics up to degree and order 60, is a consequence of three salient characteristics of the GPS-based satellite tracking system: it provides concurrent, continuous and highly accurate tracking data, it is globally distributed and therefore endowed with a high degree of observability, and it is fiducially based in a terrestrial reference frame that should have better than 1 cm accuracy by 1995.

The use of GP-B as a "flying" tracking station in combination with the Earth-bound tracking stations will strengthen the global geodetic networks, improve the GPS orbits determined from them, and contribute in the determination of Earth orientation parameters. In addition to two GPS receivers, Gravity Probe B will carry a system of corner cube reflectors for satellite laser ranging (SLR). Geodesy alone may not require these, but the intercomparison of the GPS and SLR data is of considerable importance since it will connect and unify the GPS and SLR reference frames. The availability of GPS and SLR tracking data, along with the calibrated thruster signals from the drag-free controller, also permit the aeronomy studies described earlier. Thus, Gravity Probe B advances four separate and distinct scientific enterprises, one in fundamental physics, one in aeronomy, and two in geodynamics.

To obtain the geodesy data, the data rate from Gravity Probe B will be roughly tripled from 1 kilobit/sec to 3 kilobit/sec.

Concept of a Gravity Probe B/GPS Geodesy Experiment

Two independent studies of the Gravity Probe B geodesy mission have been conducted since 1987, one by Smith, Colombo and Pavlis at NASA Goddard Space Flight Center and the other by M. Tapley and J. V. Breakwell at Stanford. In addition, a "quick-look" study by a third independent method has been carried out by W. G. Melbourne at JPL. The results are at once encouraging and in gratifyingly good agreement with each other, as indicated in the next section.

The assumptions underlying these studies are all essentially the same and are as follows:

- The GP-B spacecraft is drag-free; its orbit is polar, circular, 600 km in altitude, and repeats monthly.
- The mission lasts for two years, during which an average of 7 GPS satellites from the full constellation of 24 are tracked simultaneously, at all times, from GP-B. The sampling rate is once per second, and there are four simultaneous measurements per GPS satellite: pseudorange and carrier phase, in both the L1 and the L2 bands.
- The carrier cycle ambiguity integers on L1 and L2 are estimated from averaging the appropriate linear combinations of the L1, L2, P1 and P2 measurements over some tens of minutes (up to a maximum of half a revolution of GP-B, or 40 minutes).
- The estimated ambiguity integers are added to the corresponding carrier phase measurements, resulting in full L1 and L2 ranges, which are then combined to correct the effect of ionospheric refraction. The corrected ranges have larger measurement uncertainties than the original L1 and L2 measurements by a factor of 3.5.
- These corrected ranges are subtracted from ranges to the same GPS satellites measured simultaneously from ground receivers. This differencing eliminates the GPS spacecraft clock errors. The resulting single differences are subtracted from each other to form double differences to suppress both the clock errors of the ground receivers and of the receiver on GP-B. At the same time, this double-differencing doubles the measurement noise. Finally, double differences, involving four or more GPS satellites altogether, could be combined to estimate geometrically the instantaneous position of GP-B. The uncertainty of the ranges, propagated into those of the x, y, z coordinates (here aligned with the across, along and radial orbital coordinates), is the uncertainty of the corrected ranges multiplied by the Geometric Dilution of Precision (GDOP) per coordinate. This quantity depends on the GPS/GP-B geometry at measurement time. Errors in the final data will be affected by the original measurement errors (unresolved biases, noise), and also by the errors in the ephemerides of GPS.
- The data set is used to estimate corrections to the orbits of Gravity Probe B and all GPS satellites involved, simultaneously with the potential coefficients of the gravity field up to a high degree and order. This is accomplished by a least squares technique, using a mathematical description of the data and orbits which are linearized about *a priori* values of the orbit and gravity field parameters.

Results of Initial Simulations

Within the assumptions just stated, with measurement errors conservatively treated as 1 cm rms white noise in the reconstructed carrier phase (after correcting for ionospheric refraction) and residual biases in the one-way GPS/GP-B ranges treated as 10-cm process noise with a triangular covariance function and a 15-minute correlation length, two different estimation procedures were applied by the Stanford and Goddard groups in the initial simulation of the geodesy mission completed in 1988.

The Goddard approach consisted in applying analytical perturbation theory to set up the full normal matrix for the gravity field least squares estimation procedure [*Smith et al.*, 1988]. An approximate analytical theory based on the linearized dynamics of a circular orbit, was used to derive a mathematical model for the gravitational perturbations of the coordinates of GP-B. The theory is based on the trigonometric expansions of the gravitational potential and accelerations along the orbit, given as functions of time. These expansions become true Fourier series when the orbit repeats precisely, their fundamental frequency being that of this repeat, and the orbital perturbations in x, y and z are also Fourier series with the same frequencies. This model leads to a normal matrix that is very sparse [*Colombo*, 1984]. Moreover, the non-zero elements can be computed analytically, and with a proper arrangement of the unknowns, the matrix becomes block-diagonal. All this permits very efficient calculation of the inverse, and thus of the variances and covariances of the estimated coefficients. Best and worst case estimates were made to provide upper and lower bounds on the errors of the coefficients. The worst case assumes the loss of all information with the same frequency content as that of the orbit errors (multiples of once per revolution of GP-B plus/minus multiples of one cycle per twelve hours, which is the orbital frequency of GPS, spread out further over plus/minus one cycle per day, supposing that one-day orbits are estimated). It also assumes residual 10-cm standard deviations for the 15-minute residual range biases. The best case assumes that all biases have been resolved and there are no GPS orbit errors.

In the Stanford approach [*Tapley and Breakwell*, 1988], a two-dimensional Fourier transform (i.e., the Flat Earth approximation [*Breakwell*, 1979]) was used to determine the improvements in the geopotential with the GEM-T1 covariance matrix adopted as *a priori* information.

In the Flat Earth approximation, one takes the Fourier transform of the geopotential spectrum on the surface of the Earth, and treats that as existing on an infinite plane rather than on a sphere. The equation to describe the upward continuation of the geopotential is thus transformed from a convolution integral (Poisson's equation) to a simple multiplication. The effect of this field on the motion of a satellite in low orbit then has a very simple mathematical expression as a function of frequency and altitude. An error model (based upon some *a priori* knowledge) for estimates of the geopotential spectrum reduces under these assumptions to a single integral in the frequency domain, carried out for each wavelength in the spectrum. Breakwell and M. Tapley carried out these integrals for several cases similar to GP-B, including variations in mission length, altitude, and accuracy and frequency of measurements. The baseline case, identical to that of the Goddard approach, appears below.

Figure 3 illustrates the expected errors in the normalized potential coefficients accuracy as a function of the coefficient degree, n, as obtained by the two methods. It will be seen that the

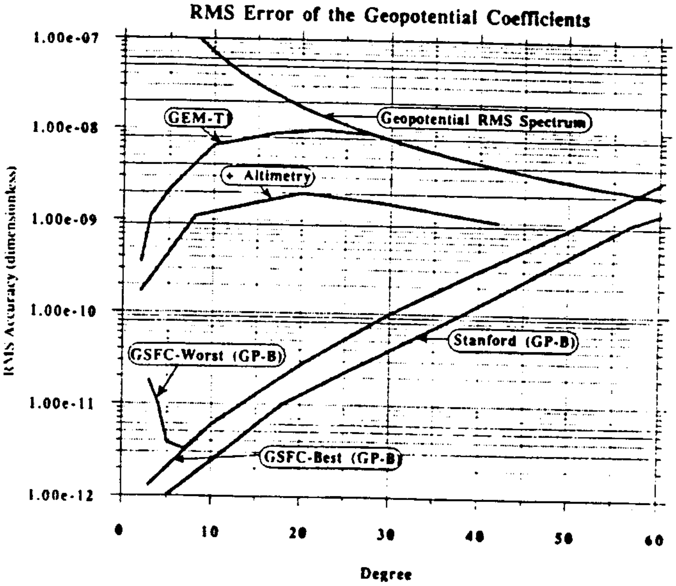

Figure 3: Estimated Recovery of Dimensionless, Normalized Spherical Harmonic Potential Coefficients

two curves agree to within a factor of 3 overall, with the Stanford result being the lower. In the Goddard curve, the best and worst cases are very close to each other except at the very low degrees, with unresolved biases and orbit errors (both low degree effects) being the main reason for the low degree divergences. Also shown on the diagram are the GEM-T1 curve and an estimate of the improvement that might be effected to GEM-T1 by combining conventional tracking with altimetry.

Thus, these preliminary estimates indicate that the Gravity Probe B data is likely to surpass anything now in existence by two orders of magnitude through degree and order 20 and be of substantial value even to orders as high as degree and order 40. The curve of the geopotential rms spectrum obtained via Kaula's rule implies that the mission would (according to these estimates) run out of resolving capability around degree and order 65.

The JPL analysis was based on an earlier TOPEX/POSEIDON analysis [*Melbourne and Tapley*, 1983; *Yunck et al.*, 1985] which was adjusted to the GP-B orbit altitude and mission duration. Here, a circular coplanar analysis was performed in which the Earth was treated as an infinite cylinder with an invariant potential field in the z-direction. In this case a Fourier expansion is applied in the solution to Laplace's equation. A standard Hill variable approach was used to obtain the perturbations to a circular orbit arising from this series. Six GPS satellites were placed in infinitely distant orbits; three satellites with 120 deg mutual separations were revolving about the Earth at 0.5 radians/hour and three were counter-rotating at the same rate. The GP-B satellite was placed at a radial distance of 1.1 Earth radius and revolving at 4 radians/hour. Six ground stations were assumed, also with 60 deg spacing. This geometry provides rough equivalence in the flatland model to the actual three-dimensional geometry expected to hold for GP-B. The basic observable was carrier phase with the ambiguities resolved. White noise clocks were assumed for the GPS satellites and linear clocks were assumed for the GPS receivers. With the geometry as described and using 90 deg horizon masks, the coefficients of the information matrix were analytically developed for the orbit and timing parameters and for the Fourier coefficients. The density of the measurements was assumed to be continuous and uniform so that the orthogonality properties of the Fourier coefficients could be invoked in inverting the information matrix. The vacuum equivalent measurement accuracy was 1 cm at 1 sec; for GP-B the mission was assumed to last two years which is equivalent to about 160 complete orbits in this two-dimensional model.

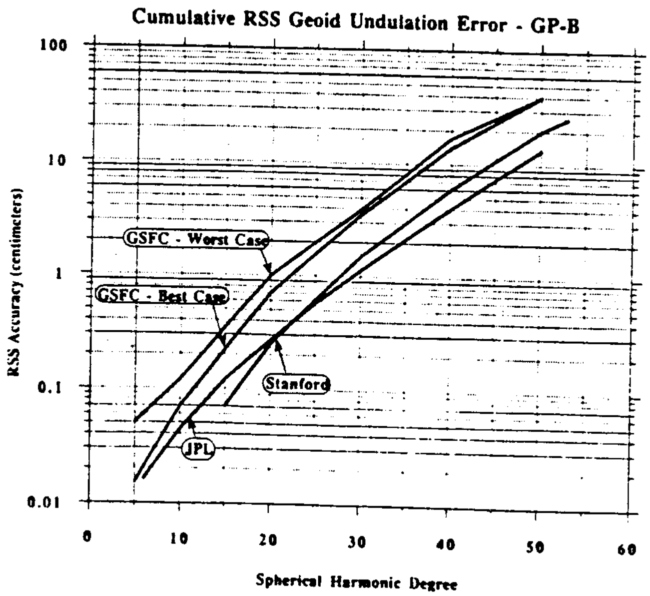

Figure 4: Estimates of Cumulative Geoid Undulation Error

Figure 4 shows the translation of the coefficient errors into rss cumulative geoid errors. The geoid error grows about one order of magnitude for each factor of 10 in the degree and order of the coefficients, reaching 10 cm at degree and order 40 for the Goddard simulation. The predicted cumulative geoid undulation error for the JPL analysis is also shown on Figure 4. No systematic errors were considered; hence, these covariance results should be considered as limiting accuracies and appropriate caution should be exercised. These results may be scaled upward by adopting more conservative assumptions for the basic measurement accuracy and about the degree to which the measurement errors may be considered as a random process.

Work in Progress and Future Directions

A detailed simulation of the recovery of geopotential coefficients from GPS/GP-B tracking data is now underway at NASA Goddard Space Flight Center. The simulation involves the same software used to produce the Goddard Earth Models (GEMs): Geodyn-II and Solve-II, based on numerical orbit integration. The purpose is to obtain the variance-covariance matrix of the Bayesian least squares adjustment of the gravitational coefficients up to degree and order 65 plus the orbits of GPS and GP-B in two-day arcs, ground-based GPS receivers coordinates, double difference ambiguities and polar motion parameters for a total of about 6500 unknowns. Ten days of 40 sec normal point data have been simulated and are currently being analyzed, this being the minimum amount of data needed to resolve a full 65 degree and order field.

At Stanford, work is proceeding on several fronts to support the geodesy experiment. A computer simulation to study the precision with which the orbit of GP-B can be placed at 90 degrees inclination is under development by P. Axelrad. Analytical studies by Breakwell and M. Tapley of the unification of SLR and GPS reference frames by comparing the two sets of measurements are also in progress. M. Tapley is investigating an experiment to make an independent measurement of the frame-dragging effect through observations of the GP-B orbit. Cooperation with Goddard and the University of Texas will generate computer simulations of the GP-B mission to elucidate the relationships of tides to the gravity field solutions.

Applications to Satellite Altimeter Mission

The oceanographic and ice topography applications of satellite altimeter height measurements from future satellite altimeter missions, which include Geosat-II, the European Remote Sensing satellite (ERS-1), TOPEX/POSEIDON and the Earth Observing System (Eos), are dependent on the computation of accurate orbits for the satellite. For many applications, an accurate marine geoid is required as well. The most demanding of the requirements comes from attempts to use satellite altimeter measurements to measure the general ocean circulation. The determination of the large-scale ocean circulation is arguably one of the more important results future satellite missions will provide. If the height of the ocean surface relative to the geoid can be measured, the absolute geostrophic surface current velocity at a given location can be inferred, and with the addition of appropriate hydrographic measurements, the variation of the geostrophic current velocity with depth can be computed [*Wunsch and Gaposchkin*, 1980]. It follows then that errors in the gravity field affect the determination of the sea surface topography, ζ, in two ways: 1) ζ is measured with respect to the geoid, an equipotential surface defined by the gravity field model, and 2) errors in the gravity field are the primary limitation in the accuracy of the satellite height computation.

It has been shown that for TOPEX/POSEIDON, the primary limitation on the orbit accuracy is the error in the geopotential model for the Earth's gravity field. *Tapley and Rosborough* [1985] demonstrate that the gravity model error will produce an orbit error component that is geographically correlated, and this component cannot be removed without actually improving the Earth's gravity geopotential model.

While the TOPEX Project geopotential model effort is expected to achieve a geopotential improvement which will satisfy the radial orbit error requirements, the effort is not likely to satisfy the requirements for an accurate marine geoid. The use of the satellite altimeter data in a simultaneous solution for the ocean surface topography and the coefficients of the gravity field has the potential for achieving a significant improvement in the gravity field.

Figure 5 shows the role geoid error plays in the recovery of the ocean surface topography. The power of the sea surface topography decreases with degree, while the error in the geoid increases with degree. For current gravity field solutions, the ocean surface topography may be recovered to degree 6, or to degree 10 if altimetry is used directly. If the error in the

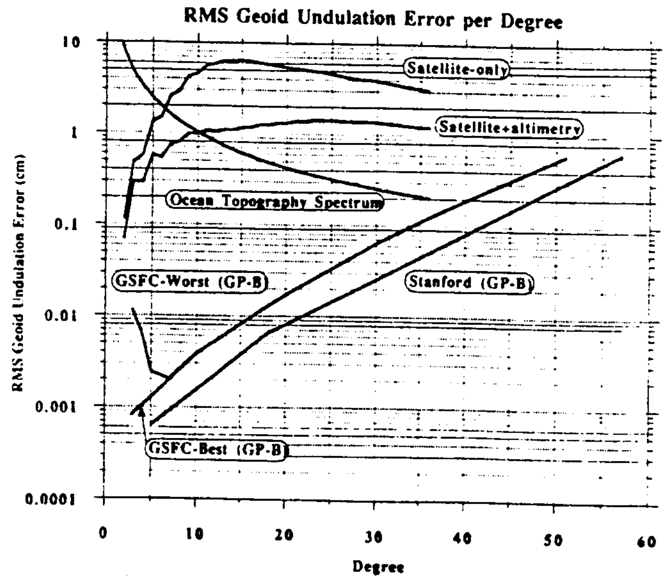

Figure 5: Estimated Geoid Errors per Degree

geopotential recovered by the GP-B coexperiment is commensurate with the accuracy shown in Figure 5, it will be possible to determine the ocean topography to degree 36 or higher.

From the previous discussion, it is evident that the requirement for improvement in the geopotential model is central to the successful estimation of the features of the general ocean circulation from a mission such as TOPEX/POSEIDON. The Gravity Probe B geodetic coexperiment can provide a gravity field improvement which will enhance significantly the scientific results from the TOPEX/POSEIDON Mission.

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