Flight Tests of Attitude Determination Using GPS Compared Against an Inertial Navigation Unit

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ABSTRACT

Attitude determination using GPS has been applied successfully to aircraft in experiments by a number of researchers. In an effort to formally characterize its accuracy and bandwidth performance, a GPS attitude determination system was flight tested against an inertial navigation unit (INU). *Based on completely separate physical principles, this testing provides an independent means of evaluating overall performance.*

The sub-centimeter relative ranging precision offered by measurements of carrier phase serves as the foundation of attitude determination using GPS. Real-time attitude capability has been demonstrated with an accuracy better than 0.1 deg at an output rate of 10 Hz. Such performance opens the door to new applications in aviation, including heading and attitude sensing to augment traditional cockpit sensors and on-line aircraft system identification to enhance flight safety.

For the flight experiments, a King Air 200 twin turbo-prop transport aircraft (NASA 701) was outfitted with a Trimble TANS Vector attitude determination receiver. A Litton LN-93 strap-down ring laser gyro INU was operated in the main cabin as the independent reference.

For system evaluation, a number of test maneuvers were executed, including pitch angles to ±30 deg and bank angles to ±60 deg. Performance in moderate turbulence was measured. The impact of structural flexure during aircraft maneuvering was evaluated.

INTRODUCTION

Attitude determination using GPS is based on sub-centimeter precision measurements of GPS carrier phase (λ=19cm). As demonstrated by researchers from JPL,[1] Ohio,[2] and Stanford,[3] the orientation of an aircraft can be determined by measuring the relative positions of multiple GPS antennas mounted to an aircraft structure.

The purpose of the test flight presented herein is to provide a quantitative experimental basis for the kinematic accuracy performance evaluation of attitude determination using GPS. The flight test objective was to conduct maneuvers that would allow evaluation of the limit attitudes and dynamic response of the GPS attitude system.

Smooth push-over-pull-up pitch maneuvers were conducted starting with small pitch angles and increasing to ±30 deg pitch angle. Smooth roll reversals were conducted at 30, 45, and 60 deg roll angles. Steady 360 deg banked turns were conducted at 30, 45, and 60 deg roll angles. Pitch and roll doublets were also conducted to evaluate fast transient response. Attitude data from the GPS attitude system and the INU were recorded in-flight for post-test analysis.

Note that the comparison between two different forms of measurement does not explicitly yield accuracy information beyond the specification of the reference instrument. The INU attitude data are accurate to 0.05 deg rms in pitch, roll, and azimuth.[4]

As an adjunct to the flight testing presented here, a study of the impact of structural flexure was performed. Note that there is no requirement that GPS antennas used for attitude be mounted at the extremities of the aircraft. In general, attitude accuracy increases with antenna separation and so does structural flexure. To evaluate the impact of flexure on attitude determination, it was decided to mount the antennas as far apart as possible.

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Figure 1: King Air 200 Flight Test Aircraft (NASA 701)

FLIGHT TEST EXPERIMENT ARCHITECTURE

The flight experiment was conducted using the twin turbo-prop transport aircraft (King Air 200, NASA 701) shown in Figure 1.

The GPS attitude system consists of a 6 channel C/A-Code GPS receiver (Trimble TANS Vector) and four microstrip patch antennas. Figure 2 shows a plan view of the antenna installation. The antennas are mounted on the wing tips, the forward fuselage, and on the top of the vertical tail. The four antennas were mounted with their normal vectors aligned, such that they are only translated, *not rotated* (even about the antenna normal) with respect to one another. This installation eliminates any errors due to azimuthal asymmetry of the antenna pattern, because they cancel out in the phase differencing between antennas. Figure 3 shows a close-up of the left wing installation.

The attitude receiver was used as the sensor for the experiments. A 486 laptop computer provided attitude solutions and data recording for post-flight analysis. This combination is capable of providing real-time attitude solutions at an output rate of 10 Hz. However, for the test flights, the computer performed attitude solutions in real-time at a 2 Hz output rate. At the same time, it recorded raw differential phase data at 10 Hz for post-flight analysis.

After the antenna installation was complete, the relative antenna positions were measured with a static "self survey" using GPS. The aircraft was parked in an area free of obstructions to minimize masking and was not disturbed during the survey period. Six and one half (6.5) hours of carrier phase measurements were recorded from

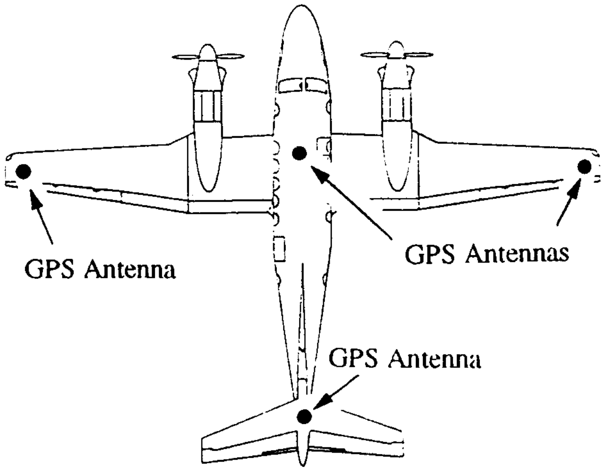

Figure 2: Plan View of GPS Antenna Installation

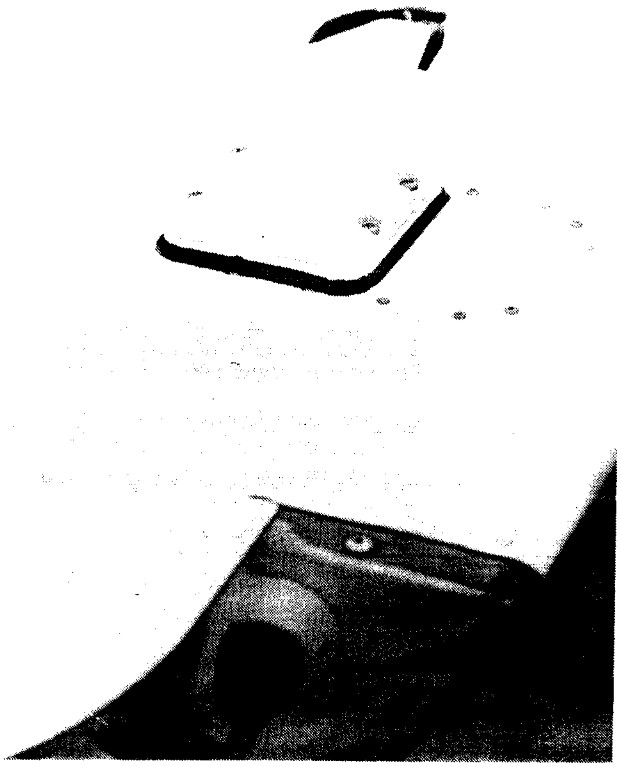

Figure 3: GPS Antenna Installation on Left Wing

the attitude receiver in order to compute a refined estimate of the relative antenna locations. The results are given in Table 1.

Table 1: Relative Antenna Vectors in Body Frame

(meters)	Baseline 1	Baseline 2	Baseline 3
x	6.887	7.702	7.080
y	-8.132	-0.124	8.087
z	-2.719	-1.807	-2.622
length	10.998	7.912	11.064

A strap-down ring laser gyro Inertial Navigation Unit (Litton LN-93) provided reference attitude measurements for comparison with the GPS attitude data. Figure 4 shows a block diagram of the entire test apparatus. The GPS attitude and inertial elements of the test setup are completely independent.

INU data were accessed at 64 Hz via the INU's 1553B data bus, and then time tagged with GPS time before being written to the host computer (68030 VME System) hard disk.

A 1 pulse-per-second (1 PPS) timing signal and its corresponding GPS time, output from a 12 channel P-Code GPS receiver (Ashtech P-12), was used to calibrate the host computer's clock to GPS time. Immediately after reading an INU record, the host computer interrogated the clock and appended GPS time to the INU record.

The GPS time tags on the INU data were the basis for comparison of GPS attitude and INU attitude measurements. It is reasonable to assume that GPS time from the two receivers is consistent to within a few hundred nanoseconds; therefore, this error was neglected.

The INU's 1553 data bus was sampled asynchronously at 100 Hz, giving rise to a 0-10 ms sampling lag (5 ms average). The host computer's time tagging latency was about 1 ms. The INU's 1553 bus latency is estimated to be about 2 ms. Therefore, the total average latency of the GPS time tag on the INU records is estimated to be 8 ms.

The GPS attitude carrier phase cycle ambiguities were resolved in real-time by using the motion-based matrix approach outlined in a previous paper.[3]

Attitude motion (from turns, either on the ground or in the air) is used to determine how many integer wavelengths lie between each pair of antennas in the direction of a given GPS satellite.

Throughout the banks and turns of normal operation, GPS satellites are frequently shaded by the aircraft structure. Once integer lock is initially obtained, it is maintained through an integer "hand-off" from satellite to satellite.

Each time a satellite becomes visible, its integers are automatically assigned based on the current attitude solution that has been computed from the other satellites in view.

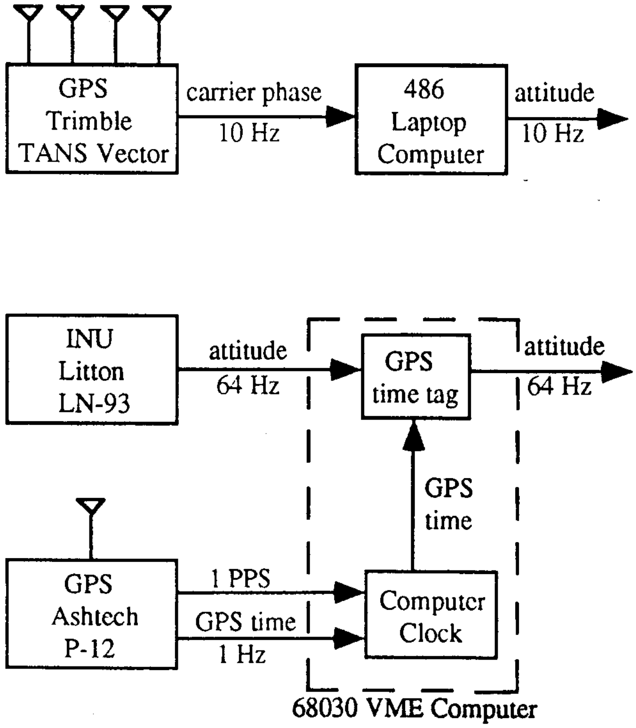

Figure 4: Block Diagram of Experimental Setup

Once the GPS receiver has performed a position fix, only two satellites are required to perform attitude fixes. This capability is based on the *common local oscillator* design of the receiver. As long as at least two satellites are in view at a given attitude, integer lock will be maintained.

Throughout all of the maneuvers performed in the test flight, the attitude receiver lost integer lock only once. The maneuver was a roll reversal from -60 to +60 deg. The integers were reaquired immediately by performing a shallow banked turn, and the identical roll reversal was repeated at a new heading without incident.

DATA ANALYSIS

Each attitude solution is comprised of a four parameter state that includes heading, pitch, roll, and instantaneous wing flex. A diagram of the aircraft body axes, GPS antenna placement, and wing flex states is shown in Figure 5.

The absolute static accuracy of attitude determination using GPS has been demonstrated previously by measurements taken against a stellar reference.[5] Therefore, it is entirely sufficient to align the GPS coordinate frame with repect to the INU using static measurements.

Static calibration data files of approximately one minute duration were collected on the tarmac immediately prior to and immediately following the test flight. The attitude alignment matrix between the GPS receiver and the INU were adjusted until the mean of the static attitude errors using both pre- and post-flight data were zero. This calibration is presented in Table 2.

Table 2: Static Calibration Results
(following application of alignment correction)

Pre-Flight (one minute mean):

(degrees)	GPS	INU	INU-GPS
Heading	-24.62	-24.66	-0.04
Pitch	1.13	1.18	0.05
Roll	0.52	0.51	-0.01

Post-Flight (one minute mean):

(degrees)	GPS	INU	INU-GPS
Heading	1.00	1.04	0.04
Pitch	1.25	1.20	-0.05
Roll	0.45	0.46	0.01

To form the relative error, INU data corresponding to the 10 Hz GPS attitude solution were computed by linear interpolation of the 64 Hz INU data with the 10 Hz GPS time tag. The differences plotted in the Results Section are the INU measurement minus the GPS attitude measurement for each axis of interest.

The latency on the INU time tags (discussed in the previous section) was observed and estimated through analysis of the roll error and roll rate data from the roll reversal maneuvers. The roll error appeared to be well-correlated with the roll rate, suggesting a timing error. From the data, the time latency was estimated to be about 7 ms—the value that was assumed for the processing of the data presented below. This is consistent with the estimate in the previous section.

Structural Flexure

The capacity of GPS to estimate structural deformation adds an additional dimension to attitude determination. Wing flexure (of almost 10 cm steady-state for the King Air) provides a direct measurement of wing loading.

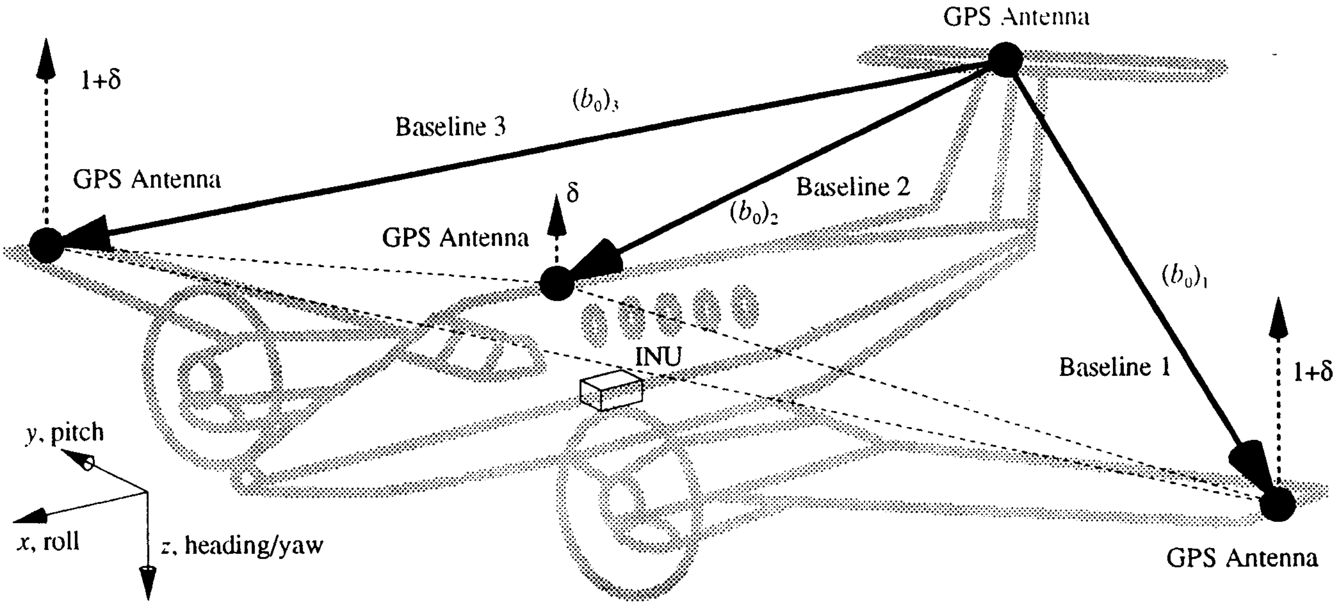

Figure 5: Antenna Baseline and Wing Flexure Definitions

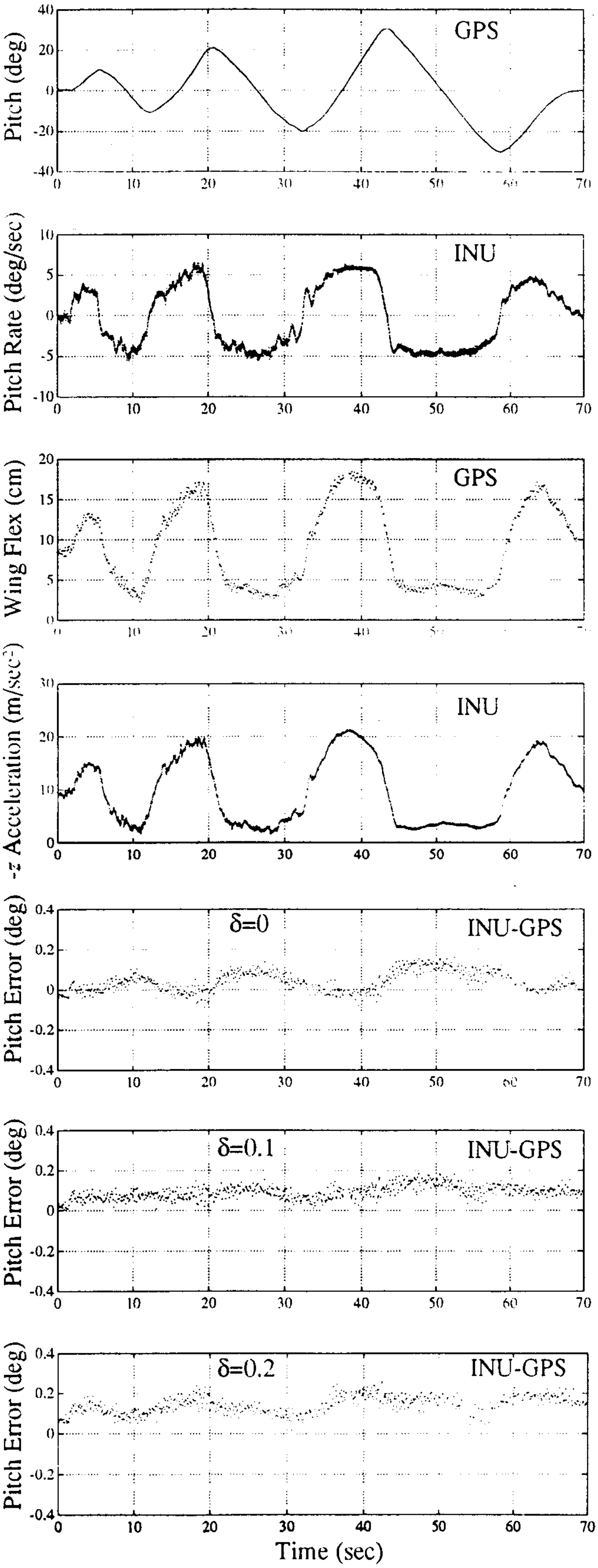

Figure 6: Pitchover Maneuvers

Attitude solutions are augmented with an estimate of the instantaneous wing flexure. The baseline vectors in the body frame of the aircraft are constrained to deform in one particular direction. A given baseline vector, b_0 (3×1), can deform in the direction of b_{flex} (3×1), by a scalar amount, f, as follows:

$$b = b_0 + b_{flex} f$$

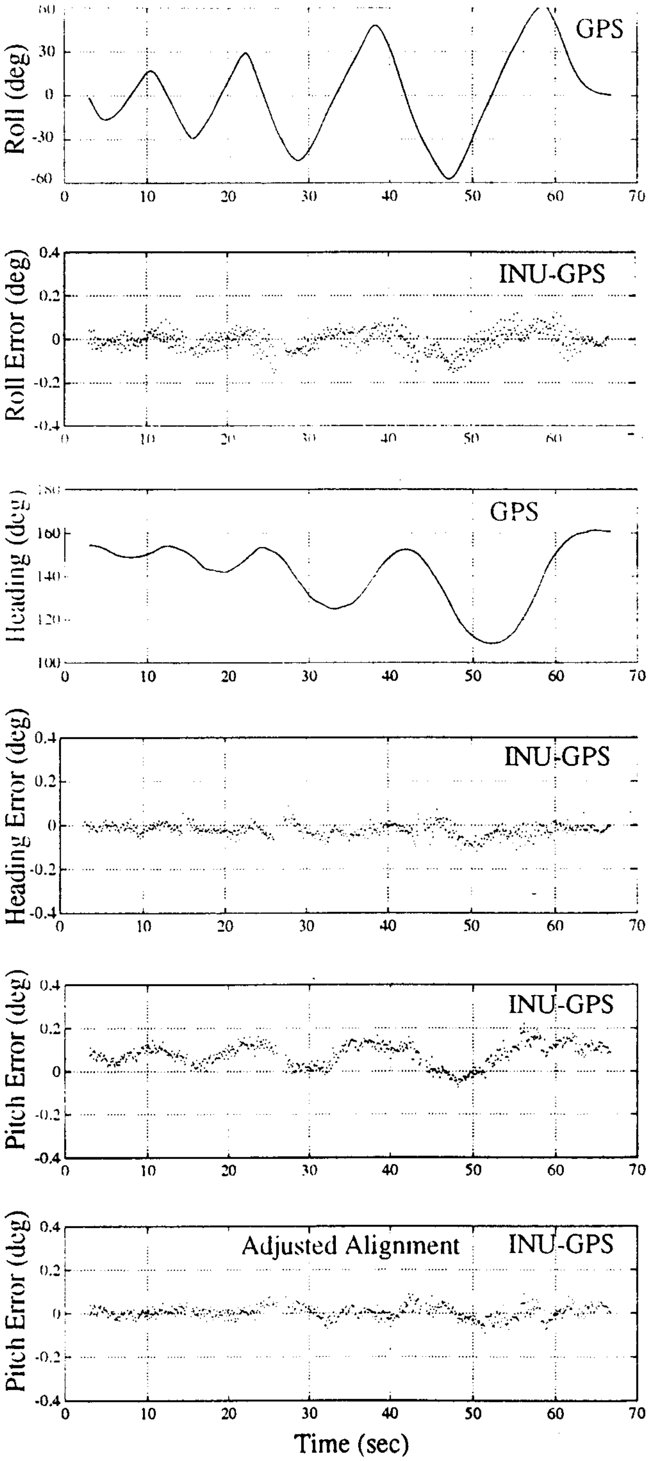

Figure 7: Roll Reversals

The entire set of m baselines is grouped together into a matrix, B, comprised as follows:

$$B = \begin{bmatrix} b_1 & b_2 & \cdots & b_m \end{bmatrix}$$

The matrix of baselines in the body frame are collectively constrained to deform in the direction indicated by the B_{flex} matrix as follows:

$$B = B_0 + B_{flex} f$$

The B_{flex} matrix is formed in an analogous manner to the B matrix. The B_{flex} matrix for the aircraft geometry shown in Figure 5 is given by

$$B_{flex} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1+\delta & \delta & 1+\delta \end{bmatrix}$$

The parameter δ specifies how much the fuselage section of the aircraft deforms during maneuvers. It could also be interpreted as a measure of how wing flexure couples into pitch attitude. For all the results presented herein (except for those in Figure 6), the value of δ was set to zero (the simplest model of wing flexure).

The attitude solution is then given by the 3×3 transformation matrix, A, which rotates the body frame vectors, B, into the local horizontal reference frame to best match the set of n differential phase measurements, φ (n×3), from the GPS satellites. The appropriate cost function to be minimized is

$$J(A,f) = \left\| \varphi_{meas} - \varphi\left[A^T\left(B_0 + B_{flex} f\right)\right] \right\|_2^2$$

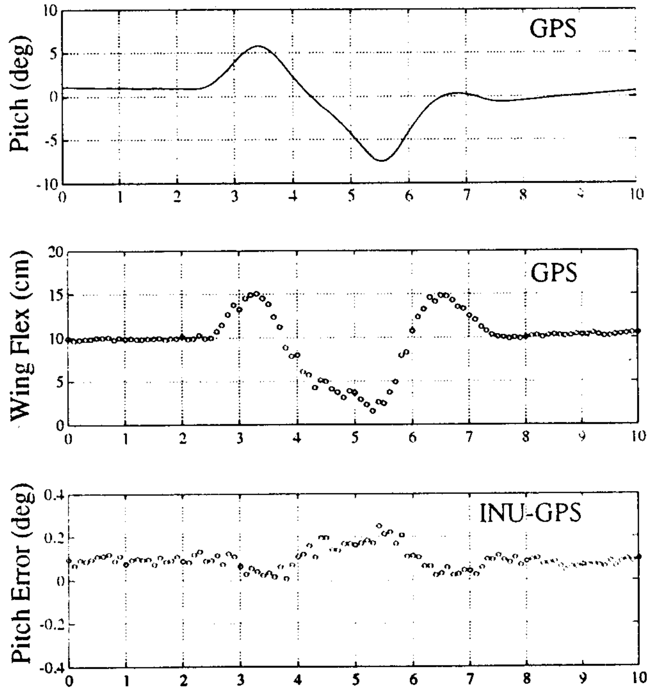

Figure 8: Pitch Doublet

RESULTS

The flight test was performed over California's Central Valley on the morning of January 12, 1993. The maneuvers from the test flight (presented herein in chronological order) are

- Static Calibration on Tarmac
- Moderate Turbulence
- Pull-up, Push-down Pitch to 30 deg
- Roll Reversals to 60 deg
- Steady Turn, 45 deg bank, 360 deg heading
- Steady Turn, 60 deg bank, 360 deg heading
- Pitch Doublet
- Straight and Level
- Roll Doublet
- Static Calibration on Tarmac

Some of these maneuvers of particular interest are described here in greater detail.

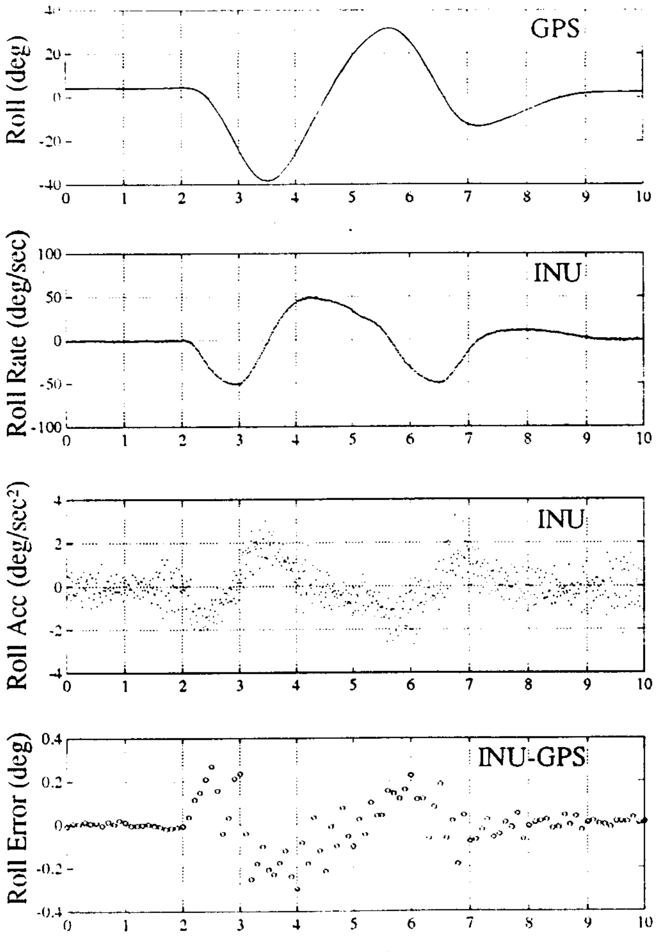

Figure 9: Roll Doublet

Pull-up, Push-down Pitchovers

Figure 6 shows the results of the pitchover maneuvers. The pilots executed pitch angles of ±10, 20 and 30 deg. The pitch rate and vertical acceleration as measured by the INU are plotted immediately following. Since pitch rate corresponds to vertical acceleration, the wing flexure varies in proportion to both acceleration and pitch rate. Note that the structure deforms by almost a full GPS L1 wavelength from peak to peak.

To determine the impact of wing flexure on attitude sensing, the parameter δ discussed in the previous section was varied incrementally in steps of 0.1. Employing any value of δ not equal to zero is equivalent to adopting a more involved model of wing flexure which—depending on the application—may or may not be desirable.

At the normal setting of δ=0, the pitch error is biased from the INU reading by 0.04 deg with a standard deviation of 0.05 deg. However, it is apparent that there is some correlation with the wing flexure state estimate. At a setting of δ=0.1, the agreement between GPS and the INU appears to be independent of wing flexure.

Roll Reversals

Figure 7 shows the results of a series of roll reversals, executed at ±15, 30, 45, and 60 deg. The roll error has a standard deviation of 0.05 deg throughout the entire set. Heading and heading error are also shown for comparison. The controls were coordinated throughout.

The pitch error shows a distinct correlation with the roll angle. This signature is characteristic of an alignment error in heading given (in radians) by the ratio of peak-to-peak pitch disagreement to the corresponding peaks in roll angle. The following plot shows the same pitch error reprocessed, assuming alignment biases of 0.08 deg in pitch and -0.08 deg in heading. The standard deviation drops to 0.03 deg.

Pitch Doublet

The receiver response to transients on a much faster time scale was tested using doublets. In the pitch doublet, elevator control is input in one direction followed immediately by a reversal in the other direction. Figure 8 shows an example of the pitch doublet, including the wing flexure response.

Roll Doublet

Figure 9 shows an example of a roll doublet *with the maximum aileron control authority applied.* The roll excursion traverses 70 deg peak-to-peak in two seconds. The INU roll rate indicates that a maximum roll rate of 50 deg/sec is reached. Roll acceleration is formed by differencing consecutive roll rate measurements. The roll error appears to be a mirror image of the roll acceleration.

The bulk of the discrepancy between GPS and the INU is believed to originate from structural flexure about the roll axis. The ailerons, located near the wing-tips, bend the wings slightly as they accelerate the fuselage in roll. The fuselage lags behind the wings by one or two tenths of a degree in direct proportion to the angular acceleration.

In the case of symmetric bending, such as that from pitch maneuvers or steady, banked turns, wing flexure is completely observable. This is not the case for bending about the roll axis, which couples directly into roll attitude

This pointing discrepancy may or may not be important, depending on the definition of aircraft attitude in the presence of structural flexure. In cases where it is the wing location that is desired, the discrepancy would be attributed to the INU. In applications where fuselage attitude sensing is desired, this plot *places an upper bound of between 0.1 and 0.2 deg on the attitude excursions induced by flexure.* This maneuver is a worst case because *maximum aileron control authority was exercised.*

The 0-10 ms time tagging latency on the INU data introduces an apparent quasi-random error whose envelope is proportional to the attitude *rate*. (See roll error plot)

Summary

A summary of measurement errors for all the maneuvers is presented in Figure 10 and in Table 3.

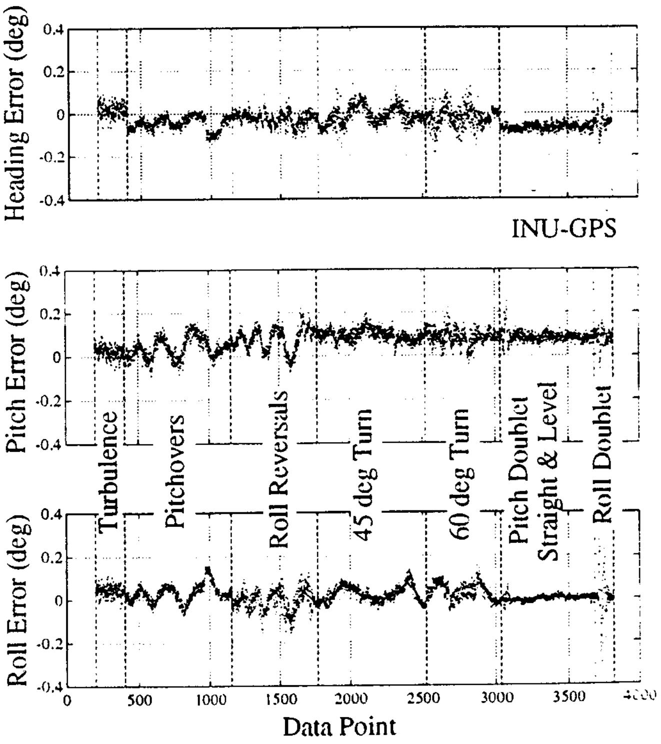

Figure 10: Summary of INU and GPS Differences

The GPS/INU differences are simply concatenated together from each segment of the flight data. In each case, the error is formed by subtracting the GPS measurement from the INU measurement.

Since the plots are shown in flight chronological order, long-term bias trends are also observable. The first segment of data was taken during what the NASA test pilots characterized as "moderate turbulence".

The standard deviation of the disagreement between the INU and GPS was always less than or equal to 0.05 deg—exactly the specification on the INU readout.

Table 3: GPS/INU Comparison Results

Pitch Maneuvers (±10, 20 and 30 deg):

(degrees)	mean	sigma	max
Heading	-0.04	0.03	0.15
Pitch	0.04	0.05	0.16
Roll	0.03	0.04	0.16

Moderate Turbulence:

(degrees)	mean	sigma	max
Heading	0.02	0.03	0.09
Pitch	0.03	0.03	0.10
Roll	0.05	0.02	0.11

Roll Reversals (±15, 30, 45, and 60 deg):

(degrees)	mean	sigma	max
Heading	-0.02	0.03	0.12
Pitch	0.08	0.05	0.22
Roll	-0.01	0.05	0.15

Steady 360 deg Turn (45 deg bank):

(degrees)	mean	sigma	max
Heading	-0.01	0.04	0.15
Pitch	0.10	0.03	0.20
Roll	0.02	0.04	0.14

Steady 360 deg Turn (60 deg bank):

(degrees)	mean	sigma	max
Heading	-0.02	0.05	0.18
Pitch	0.09	0.03	0.23
Roll	0.03	0.04	0.16

Straight and Level, Calm Air:

(degrees)	mean	sigma	max
Heading	-0.06	0.02	0.11
Pitch	0.08	0.02	0.13
Roll	0.00	0.01	0.03

Slowly drifting biases of a few hundredths of a degree appear in the data. The total differences rarely exceeded a tenth of a degree. As mentioned in the above discussion on the roll reversal maneuvers, it appears likely that there is still some residual alignment error between the GPS and INU body reference frames.

SUGGESTIONS FOR FUTURE RESEARCH

The comparison between GPS and INU attitude could probably be improved by a factor of two by carrying out three improvements in the experiment.

- Static Alignment

Performing a longer static measurement before and after the test flight would improve the knowledge of the relative alignment between the INU and GPS reference frames.

- Independent Flexure Measurement

Measurement of fuselage flexure by independent means would clarify the modeling of structural deformation during maneuvers. A simple HeNe laser mounted on the INU bulkhead could sight along the fuselage to a target at the rear of the aircraft.

- INU Time Tagging

Reducing the time tagging latency on the INU data would eliminate the quasi-random element in the comparison data whose envelope is proportional to attitude rates. (See Roll Error in Figure 9.)

CONCLUSIONS

As a commercial passenger transport, the King Air did not appear to possess sufficient aerodynamic control authority to break GPS carrier tracking loop lock with dynamics alone. Nevertheless, the aircraft provides an excellent testing platform on par with flight dynamics rarely encountered in even the most demanding regimes of commercial passenger and general aviation.

Across a wide and strenuous range of large-angle dynamic maneuvers (up to ±60 deg of roll and ±30 deg of pitch), GPS attitude determination performed accurately—even while subject to interruptions and masking of individual GPS satellites due to shading during the course of maneuver execution.

The disagreement between the INU and GPS attitude seldom exceeded a tenth of a degree—even in the presence of structural deformation on the order of a GPS wavelength (19 cm). The standard deviation of the difference measurements did not exceed 0.05 deg for any of the maneuvers. The specification on the INU attitude accuracy is 0.05 deg rms.

Worst case structural deformation about the roll axis due to exercising maximum aileron control authority places an upper bound of between 0.1 and 0.2 deg of roll attitude as the very worst case. Nominal aircraft operation is unlikely to come anywhere near this figure.

APPLICATIONS

The capability of using GPS for attitude determination on aircraft and other vehicles opens up many new applications.

For aircraft operation, GPS offers pilots an artificial horizon and directional indicator that is invulnerable to drift or magnetic variation. The sensor could serve either as primary or backup to the existing complement of sensors carried on aircraft. GPS attitude determination is also likely to play a role in the precision landing of aircraft using GPS, integrity monitoring of GPS, and real-time detection of wind shear.

For aircraft instrumentation, GPS can provide attitude data for remote sensing experiments, flight testing, and aircraft system identification to enhance flight safety.[3]

Applications also extend to space, marine, and land vehicles as a heading and/or attitude sensor or as a means of pointing instrument platforms.

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