GRAVITY PROBE B SCIENCE DATA ANALYSIS: FILTERING STRATEGY

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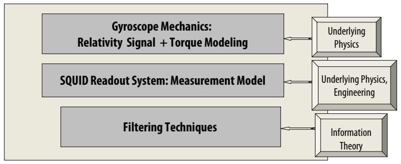
Filtering Approach: Background

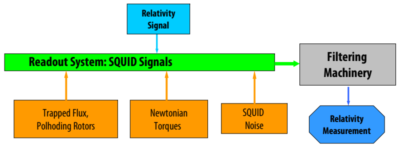


Gyroscope Mechanics: Relativity Signal + Torque Modeling ← Underlying Physics

SQUID Readout System: Measurement Model ← Underlying Physics, Engineering

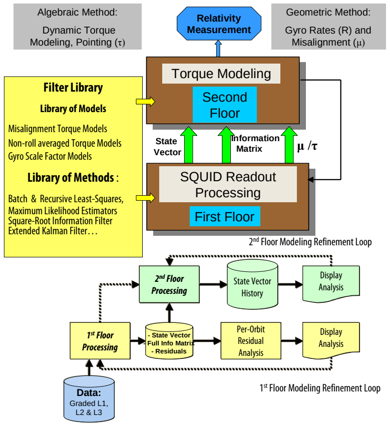
Filtering Techniques ← Information Theory

Challenges of GP-B Data Analysis



Gyro Rates change continuously ↔ Torque Models
Gyro Scale Factor varies continuously ↔ Scale Factor Models
Simultaneous estimation of relativity, torques, scale factor
Noisy nonlinear measurements ↔ Estimation Algorithms

'Two-Floor' Processing



Algebraic Method: Dynamic Torque Modeling, Pointing (τ)

Geometric Method: Gyro Rates (R) and Misalignment (μ)

Filter Library

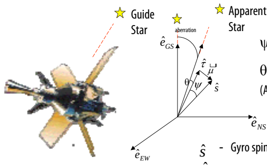
Library of Models

- Misalignment Torque Models
- Non-roll averaged Torque Models
- Gyro Scale Factor Models

Library of Methods:

- Batch & Recursive Least-Squares,
- Maximum Likelihood Estimators
- Square-Root Information Filter
- Extended Kalman Filter…

Data Analysis Geometry



ψ – misalignment angle

θ – pointing error (Attitude Control System)

$\hat{s}$ - Gyro spin axis orientation

$\hat{\tau}$ - Vehicle roll axis orientation

$\vec{\mu} = \hat{\tau} - \hat{s}$ - Gyro misalignment vector

Inertial reference frame

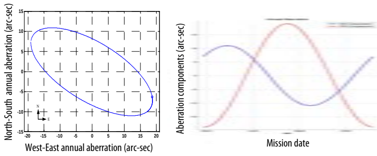
$\{\hat{e}_{NS}, \hat{e}_{EW}, \hat{e}_{GS}\}$

($\hat{e}_{GS}, \hat{e}_{NS}$ in orbital plane)

$\hat{s}(s_{NS}, s_{EW}, s_{GS})$ $\hat{\tau}(\tau_{NS}, \tau_{EW}, \tau_{GS})$

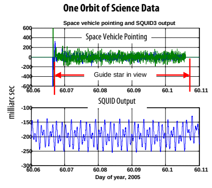
Gyro Orientation Time-history: $s_{NS}(t), s_{EW}(t)$

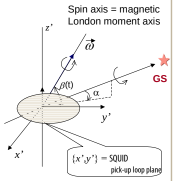
Annual Aberration: dominant component of $\hat{\tau}$



SQUID Readout Signal

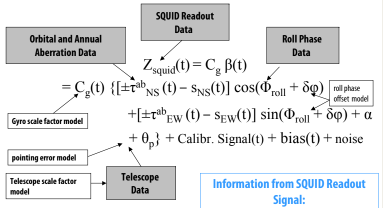
One Orbit of Science Data





Repeat every 97 minutes for a year....

Measurement Model



SQUID Readout Data

Orbital and Annual Aberration Data

Roll Phase Data

$$Z_{squid}(t) = C_g\,\beta(t)$$
$$= C_g(t)\,\{[\pm\tau^{ab}_{NS}(t) - s_{NS}(t)]\cos(\Phi_{roll}+\delta\varphi)$$
$$+[\pm\tau^{ab}_{EW}(t) - s_{EW}(t)]\sin(\Phi_{roll}+\delta\varphi) + \alpha$$
$$+\,\theta_p\} + \text{Calibr. Signal}(t) + \text{bias}(t) + \text{noise}$$

roll phase offset model

Gyro scale factor model

pointing error model

Telescope scale factor model

Telescope Data

Information from SQUID Readout Signal:
Gyro orientation $s_{NS}(t)$ and $s_{EW}(t)$

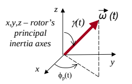
Gyro Scale Factor Polhode Variations

First Floor

Second Floor Connections

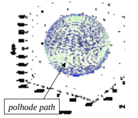
$$C_g = C_{g0}\left\{1 + \sum_{n=0}^{N}\left[a_n(\gamma)\cos\left(n\Phi_p(t)\right) + b_n(\gamma)\sin\left(n\Phi_p(t)\right)\right]\right\},$$

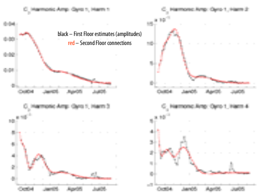
$$a_n(\gamma) = \sum_{k=0}^{K} a_{nk}\,\varepsilon^{n+2k},\quad b_n(\gamma) = \sum_{k=0}^{K} b_{nk}\,\varepsilon^{n+2k},\quad \varepsilon(t) = \tan\left(\frac{\gamma(t)}{2}\right)$$



$\Phi_p(t)$ – Polhode phase; $\gamma(t)$ - Polhode angle

$\{a_{nk}, b_{nk}\}$ - Estimated parameters (constant through mission)





Precise determination of polhode phase and polhode angle is based on measurements of polhode period and identification of rotor asymmetry parameters (work in progress)

Best Expected Scale Factor Modeling – Trapped Flux Mapping (work in progress)

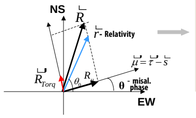
Torque Modeling

Misalignment Torque

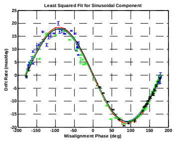
$$\vec{R}(t) = \frac{d\vec{s}}{dt} = \vec{r} + \vec{R}_{Torque}(t)$$

Relativity

Geometric Approach $\vec{R}_{Torq}(t) \perp \vec{\mu}(t).$



$$R_\mu = \vec{R}\cdot\hat{\mu} = \vec{r}\cdot\hat{\mu}$$
$$= r\cos\left(\theta_0 - \theta(t)\right).$$
$$r_{NS} = r\sin\theta_0$$
$$r_{EW} = r\cos\theta_0.$$



Algebraic Approach $\vec{R}_{Torq}(t) = (\hat{\tau}\times\hat{s})\,k(t)$

$$\frac{ds_{NS}}{dt} = r_{NS} + k(t)[\tau_{EW}(t) - s_{EW}(t)]$$
$$\frac{ds_{EW}}{dt} = r_{EW} - k(t)[\tau_{NS}(t) - s_{NS}(t)]$$

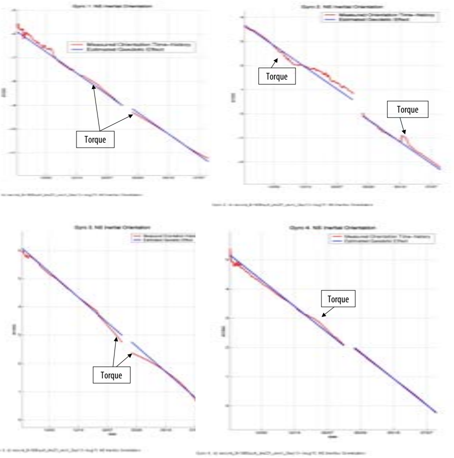
First Floor output – batch-based gyro orientation estimates $\{\hat{s}^1_{NS}, \hat{s}^1_{EW}\}$ fitted to piece-wise constant torque coefficient model (analytic solution of differential equations of motion)

$$s_{NS}(t) = s_{NS}(t_0) + [r_{NS} - k s_{EW}(t_0)](t-t_0) - k r_{EW}\frac{(t-t_0)^2}{2} + k\int_{t_0}^{t}\tau_{EW}(t')dt'.$$
$$s_{EW}(t) = s_{EW}(t_0) + [r_{EW} + k s_{NS}(t_0)](t-t_0) + k r_{NS}\frac{(t-t_0)^2}{2} - k\int_{t_0}^{t}\tau_{NS}(t')dt'.$$
$$k(t-t_0) << 1$$

Second Floor Torque Model

Iterative Nonlinear Fit: Maximum Likelihood Estimator (MLE)

Gyro Motion: Geodetic Measurement)



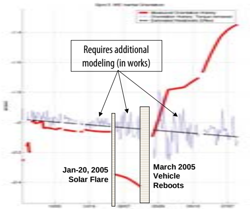
Current Estimates ("Glimpses")

-6595 ± 10 milliarcsec/year

-6604 ± 7 milliarcsec/year

Gyro Motion: Frame-Dragging Effect

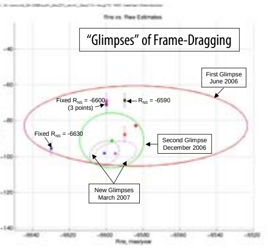
Torque Modeling Example : Motion of Gyroscope 3



RED – measured (estimated) gyro orientation time-history

BLUE –the same after subtracting estimated effect of misalignment torque

DASHED LINE – estimated relativistic motion

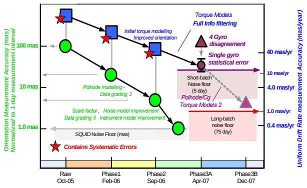


Why "glimpses" so far?

Robust estimation of frame-dragging effect requires more precise modeling of C_g polhode variations and Newtonian torques

(work in progress)

The Way Forward



Core remaining data analysis issues:

- Full understanding of the physics of classical torque sources.
- Elimination of polhode effects on the SQUID scale factor (C_g)
- Systematic error sources identification to continue (most, though not all, instrumentation issues understood)

Progress requires insight derived from careful analysis

Abstract

Nonlinear filtering provides one component of the data analysis strategy to determine the relativistic precession of GP-B science gyroscopes. The filtering methodology is based on: 1) models of the gyroscope motion, 2) models of the science readout signal, 3) filtering techniques. A "two-floor" data analysis process has been developed. The first floor focuses on modeling of the readout system: gyroscopes' scale factor polhode variations, telescope signals, matching of the gyroscope and telescope scale factors/bias, and SQUID calibration signal modeling. Nonlinear parameter estimation is performed for a set of independent batches that generates state vector and information matrices for each batch. The second floor separates the relativistic precession from the torque-induced motion of the science gyroscopes. Batch-based estimates from the first-floor filter are treated as "measurements" of the second floor state vector and connected through the torque model and other constraints. Estimates of relativistic precession and its covariance are obtained from the "second-floor" filters. Supporting validation tools such as spectral and statistical analyses of the filter residuals were developed to interface with the filter outputs for multiple sensitivity analyses

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