Classical Torques on Gravity Probe B Gyros

Alex Silbergleit, Mac Keiser, Yoshimi Ohshima

1. Classification of Torques

Rotor and housing; electrodes seen

– Support dependent (SD) - torques caused by suspension system operation

– Support independent (SI) - torques caused by anything else

Support Dependent Torques:

– Caused by residual accelerations (difference of squares of electrode voltages)

– Caused by preload accelerations (sum of squares of electrode voltages)

Support Independent Torques:

– Housing - fixed

– Inertially fixed

SD & SI housing-fixed – by Gyro Position:

– Nominal (gyro centered, its spin and S/C roll axes aligned)

– Misaligned (centered, spin axis not exactly along roll)

– Miscentered (spin & roll axes aligned, gyro not exactly centered)

No misaligned position housing-fixed SI torques known/detected

Small parameters: gap/rotor radius=1.7×10^{-4}; asphericity/radius~10^{-7}; miscentering/gap=3×10^{-6}, D.C.; miscentering/gap=3×10^{-5}, at roll freq.; $\Delta I/I < 4 \times 10^{-6}$; average spin-to-roll axis misalignment = 5×10^{-5};

2. Torque Theory

Systematic theory of electrostatic **Support Dependent** torques complete to lowest order in small parameters is developed

- 2 independent derivations based on: a) direct torque calculation, and b) energy conservation+ symmetry lead to the same results
- `Magic' general formula

$$\text{Torque} = (\text{position factor}) \times (\text{torque coefficient}) \times \left(V_{i+}^2 \pm V_{i-}^2\right)$$

- Position factor = 1, nominal; = NS or EW misalignment, misaligned; = miscentering/gap ratio, miscentered position torques
- Torque coefficient, K_n, depends on the rotor asphericity (deviation from perfect sphere) only. Total 15 torque coefficients, only 6 are involved in (most contributing) nominal and misaligned position torques
- V_{i+}, V_{i-} are voltages of opposite electrodes of one of 3 electrode pairs

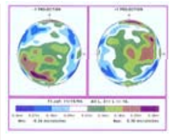
- No general theory of **SI** torques available, due to various physical origin (electrostatic, magnetic, differential damping torques, etc.)
- Expression for each **SI** torque is derived separately
- Examples of **SI** torques: direct gravity gradient torque; torque due to coupling of London moment with local shield; torque due to residual gas pressure (largest of SI torques)

3. Pre-Flight Estimates of Classical Drift

Based on torque theory, requirements, ground measurements and best parameter estimates when measurements not available

Example: torque coefficients

- Three flight-class coated rotors anodized to carefully measure their shape (up to $l=16$ in terms of spherical harmonics)
- All 15 torque coefficients computed from found rotor shape coefficients for every position of the spin axis in the body of each rotor
- Range for each torque coefficient was thus established and used in the **Support Dependent** torque estimates



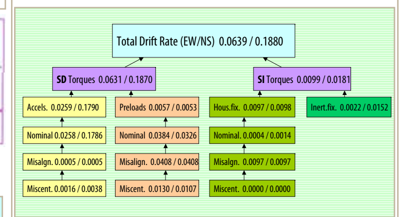
Measured surface map of flight uncoated rotor

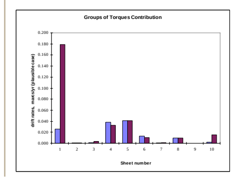
Total drift rate due to classical torques is computed for each gyro through its Error Tree, an Excel Workbook with

- >120 input parameters (mechanical, electrical, and magnetic parameters of the rotor; various S/C parameters, such as orbital ones; torque coefficients; preload accelerations, etc.)
- residual accelerations by formulas from input parameters
- >100 individual torques grouped according to their classification by formulas from input parameters and residual accelerations
- drift rates in 2 inertial directions, EW and NS

4. Pre-Flight Estimates - II

Top Sheet of Pre-Flight Error Tree for Gyro 4
(Unsupported Gyro 1; Drift Rate in *marc-sec/year*)

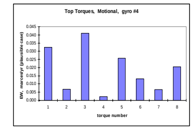




Contributions of the above groups of torques:
BLUE – EW
PINK - NS

5. Pre-Flight Estimates - III

Important result: not more than 8 torques contribute >99% of classical drift rate for **each** gyroscope **in either** of the directions (list/magnitude of torques varies slightly depending on the gyro and direction)



Top Torques, Gyro 4, EW (99.98% of drift)

1. SD, PD, MaP, D.C. preloads — 0.041 mas/yr
2. SD, PD, NP, preload difference @ roll — 0.032 mas/yr
3. SD, AD, NP, gravity grad. perp roll axis — 0.026 mas/yr
4. SD, PD, McP, D.C. prel.+ misc. at roll — 0.013 mas/yr
5. SI, HF, MaP, LM coupling to loc. shield — 0.007 mas/yr
6. SI, HF, MaP, resid. gas press.+asymm. — 0.007 mas/yr
7. SI, IF, direct gravity gradient — 0.002 mas/yr
8. SD, PD, NP, preloads+gravity gradient — 0.002 mas/yr

Legend: PD / AD = Acceleration / Preload Dependent; HF / IF = Housing- / Inertially-Fixed
NP / MaP / McP = Nominal / Misaligned / Miscentered Position

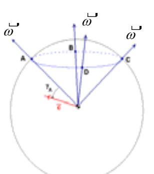
Pre-flight estimates of classical drift rate, *marc-sec/year*

	G1	G2	G3	G4
EW	0.055	0.055	0.068	0.064
NS	0.105	0.086	0.172	0.188

6. Post-Flight Estimates of Classical Drift

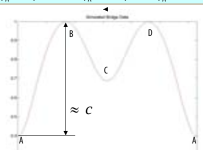
In work currently. Are obtained using on-orbit measurements and calibrations

Example 1: Flight Measurement of Rotor Mass Unbalance (MU). MU=distance between center of mass and geometrical center (fixed in body). Spin axis moves in the body along polhode path modulating gyro position signal at spin frequency. MU contributes up to 50% to all odd harmonics torque coefficients



Polhode modulation of spin harmonics amplitude

sin γ_A = very small; sin γ_A = nearly 1; sin γ_A = small again



	G1	G2	G3	G4
Pre-Flight	18.8	14.5	16.8	13.5
In-Flight	10.1	6.6	4.0	8.9

Measured MU, in *nm*

7. Post-Flight Estimates - II

Example 2: Flight Calibrations of Torque Coefficient K_3 Coefficient K_3 governs one of the top torques due to preload difference at roll frequency. In flight but **before** science this torque was used to align gyro spin axes with the direction to the Guide Star, for which the difference at roll was strongly enhanced. It was deliberately enhanced once again **after** science for precise calibration. Both times the measurement scheme was: detect the (large) drift rate, compute the torque from it, and, knowing voltages, find K_3 using `magic' formula of chart 2. Calibration values (essentially more accurate than spin alignment ones) turn out 10 (or more!) times smaller than the conservative pre-flight estimates

Measured K_3, in *pf/rad*

	G1	G2	G3	G4
Pre-Flight Estim.	-0.0682	-0.0964	-0.0872	-0.0859
Spin Axis Align., before science	-0.0061	-0.0022	-0.0184	-0.0057
Calibration, after science	-0.0008	-0.0108	-0.0183	-0.0072

After refining all the torque parameters incorporating all the relevant flight data, recalculate classical drift rate by scaling pre-flight results

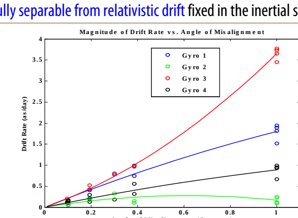
- By groups of torques, or
- By top torques

Current estimates (to be sharpened) of classical drift error in the experiment results are <1 mas/yr for gyros 1,3, 4; <2 mas/yr for gyro 2 (both directions) – ALL torques, EXCEPT just one…

8. Misalignment Torque: Patch Effect

Anomalously large drift rate detected in post-flight calibrations with spin-to-roll axis misalignment deliberately enhanced (from ~10 *as* up to several degrees). It grows with the misalignment in a non-linear way, but is linear for smaller misalignments within few tens of a degree (fit to measured drift rate below)

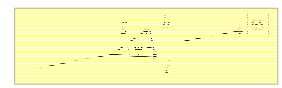
- Proportionality coefficient up to 1 – 3 *as/(deg*day)*
- Drift direction perpendicular to the misalignment
- Thus fully separable from relativistic drift fixed in the inertial space



Explanation: Patch Effect Electrostatic potential on rotor and housing surfaces may be not uniform due microcrystal structure, or dipole surface layer, etc. (patch effect). Due to patches and relative motion of rotor and housing (gyro spin, S/C roll in the inertial space), electrostatic torques on rotor are generated.

9. Patch Effect: Modeling

Complete Theory of Patch Effect Torque is developed. Potential distributions on the rotor and housing surfaces are characterized by constant coefficients of their spherical harmonics expansions in the rotor- and housing-fixed frames, respectively. By rotating the first frame to the second one, we solve the electrostatic boundary value problem in the gap, compute electrostatic energy (to lowest order in gap/radius ratio), and vary it in three independent angles (spin phase, roll phase and the misalignment) to get all the torques. The 1st torques is spin-up/down and averages out over spin period, the 2nd is also mainly spin-up/down one (small misalignment!). The 3rd is responsible for the observed drift and should be taken out of the signal

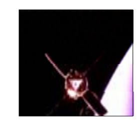


Unit vectors of spin, $\hat{s}$, and roll, $\hat{r}$, axes, misalignment vector, $\vec{\mu}$, and misalignment angle, ψ. To lowest order, $\psi = \mu$.

Gyro Spin Motion Equation (roll averaged model to linear order in μ)

$$\frac{ds_{NS}}{dt} = r_{NS} + k(t)[\tau_{EW}(t) - s_{EW}(t)]$$

$$\frac{ds_{EW}}{dt} = r_{EW} - k(t)[\tau_{NS}(t) - s_{NS}(t)]$$



W. W. Hansen Experimental Physics Laboratory • Stanford University, Stanford, CA 94305-4085 • http://einstein.stanford.edu

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