Progress in Two-second Filter Development

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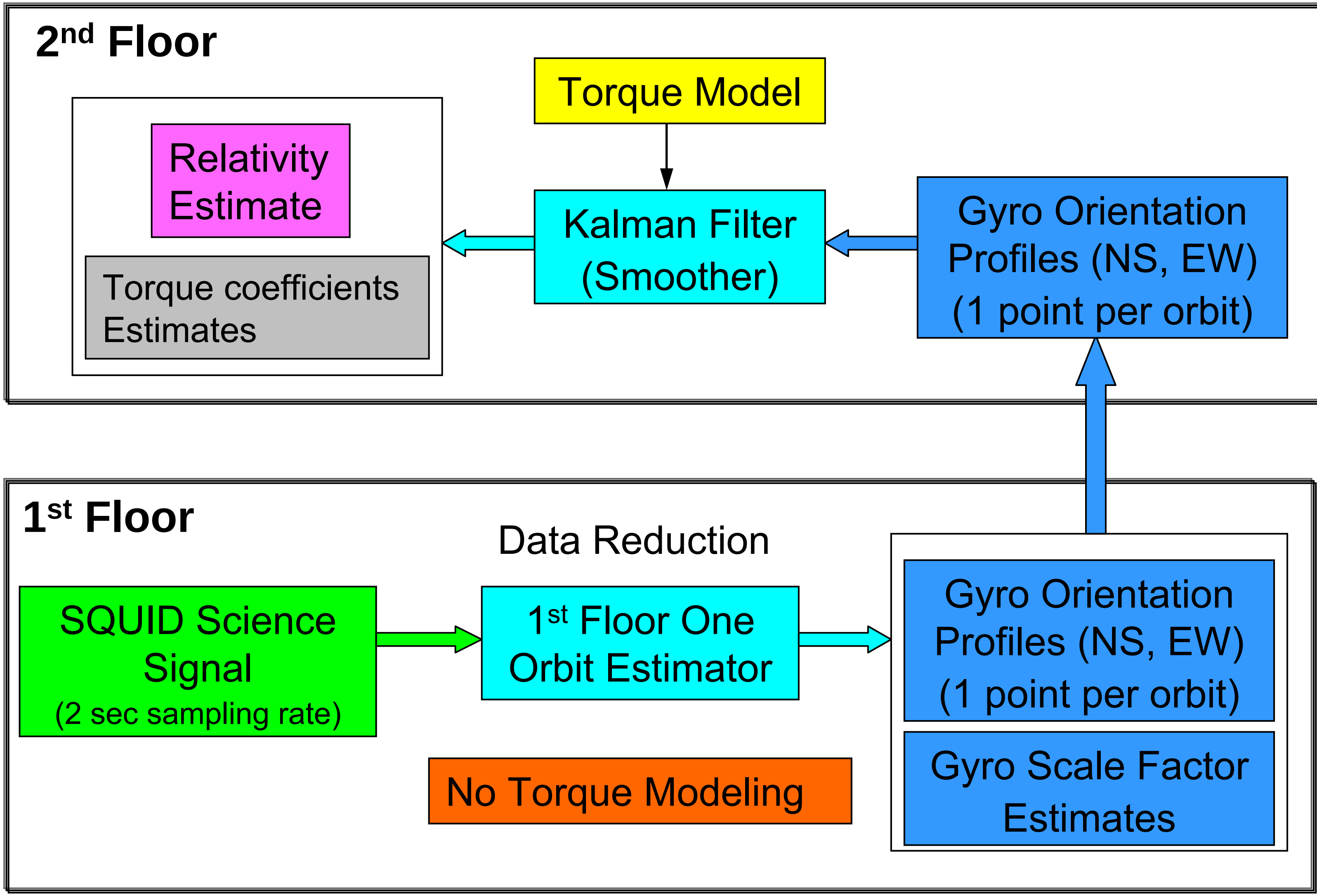
Outline

1. Two-floor data analysis:
   Initial condition for One-Floor (2-sec) Filter

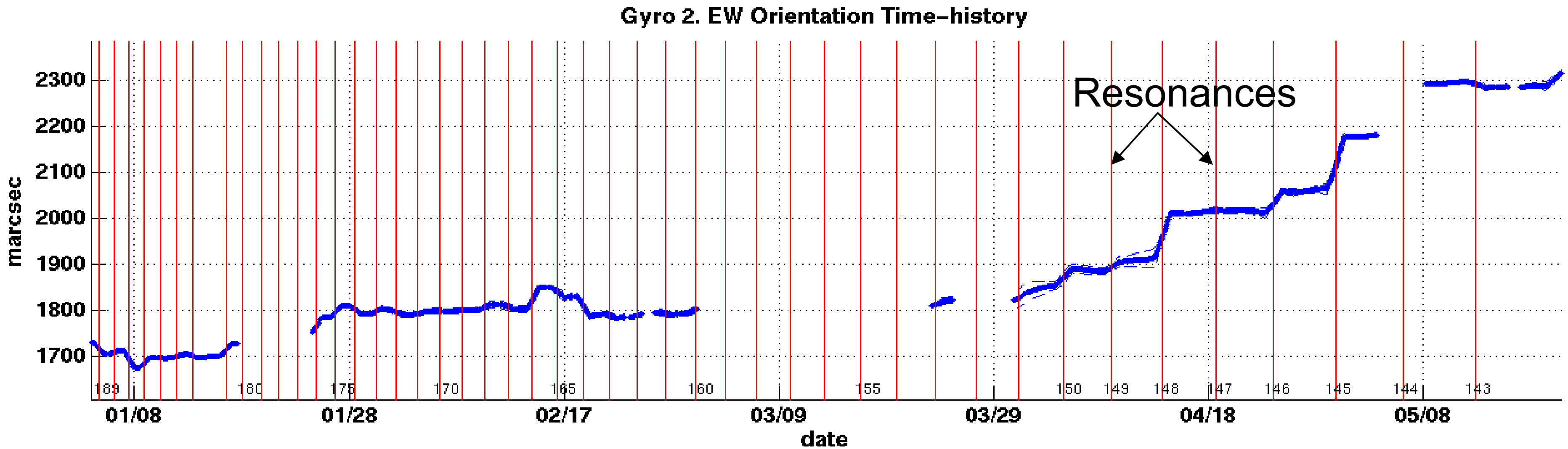
2. Progress in 2-sec Filter development

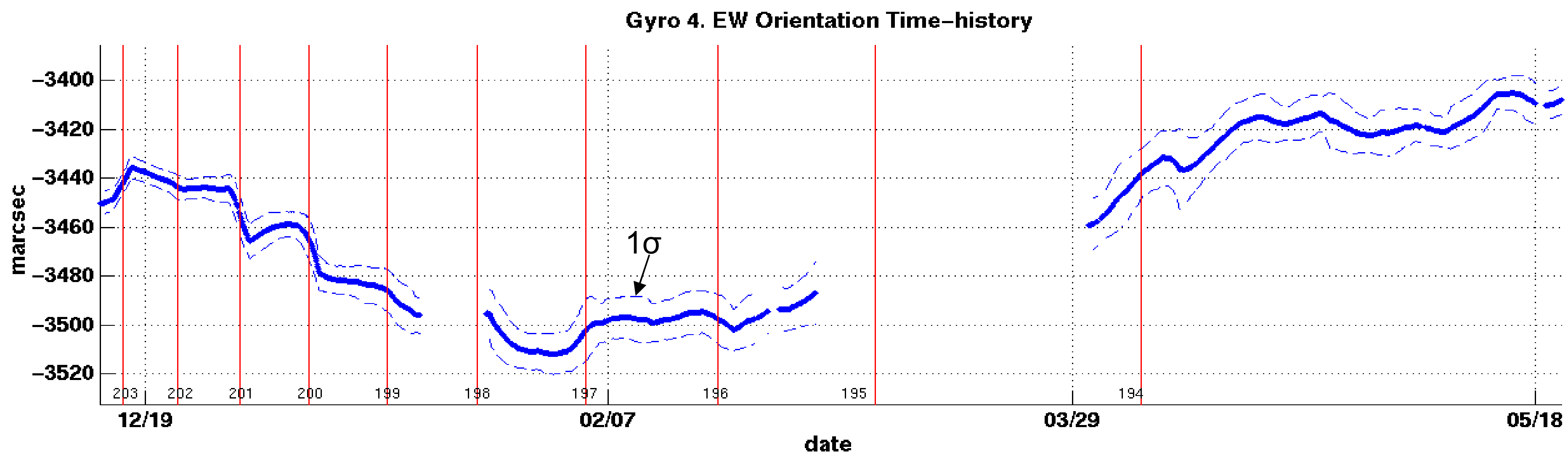
3. Next steps

Structure of Two-Floor Analysis



1st Floor Output: Gyros 2 & 4 EW Orientations





Torque Model

◯ *-Unknown (estimated) parameters*

Relativity | Misalignment torque | Roll-resonance torque

$$\frac{ds_{NS}}{dt} = r_{NS} + k(t)(\tau_{EW} - s_{EW}) + \sum_{m=0}^{Mc}\sum_{n=1}^{Nc}\varepsilon_0^n [a_{mn}^- \cos\Phi_m^-(t) - b_{mn}^+ \sin\Phi_m^-(t) + a_{mn}^+ \cos\Phi_m^+(t) - b_{mn}^- \sin\Phi_m^+(t)]$$

$$\frac{ds_{EW}}{dt} = r_{EW} - k(t)(\tau_{NS} - s_{NS}) + \sum_{m=0}^{Mc}\sum_{n=1}^{Nc}\varepsilon_0^n [b_{mn}^+ \cos\Phi_m^-(t) + a_{mn}^- \sin\Phi_m^-(t) + b_{mn}^- \cos\Phi_m^+(t) + a_{mn}^+ \sin\Phi_m^+(t)]$$

$$\Phi_m^-(t) = \Phi_r(t) - m\Phi_0(t);$$

$$\Phi_m^+(t) = \Phi_r(t) + m\Phi_0(t)$$

Resonances: $\omega_r = m\omega_p(t)$

$$k(t) = \sum_{m=0}^{M_k}[k_{1m}(\gamma_0)\cos m\Phi_0(t) + k_{2m}(\gamma_0)\sin m\Phi_0(t)]$$

$$\begin{pmatrix} k_{1m} \\ k_{2m} \end{pmatrix} = \sum_{n=0}^{N_k}\begin{pmatrix} k_{1mn} \\ k_{2mn} \end{pmatrix}\varepsilon_0^{n+1-\delta_{0m}}(t), \quad m = 0,1,\ldots,M_k$$

Polhode Phase (Φ_0), Polhode Angle (γ_0)

Trapped Flux Mapping

$\vec{\tau}(\tau_{NS}, \tau_{EW})$ - S/C pointing

$\varepsilon_0 = \tan(\gamma_0/2)$

2nd Floor Kalman Filter

"Measurements"

Gyro Orientation Profiles

State vector: $x = \left[s_{NS}(t), s_{EW}(t), r_{NS}, r_{EW}, \{k\}, \{c\} \right]$

Propagation Model:

$$x(t_{k+1}) = F(t_{k+1}, t_k)\, x(t_k)$$

Measurement Model:

$$z(t_{k+1}) = Hx(t_k) + v_k$$

Kalman Filter / Smoother

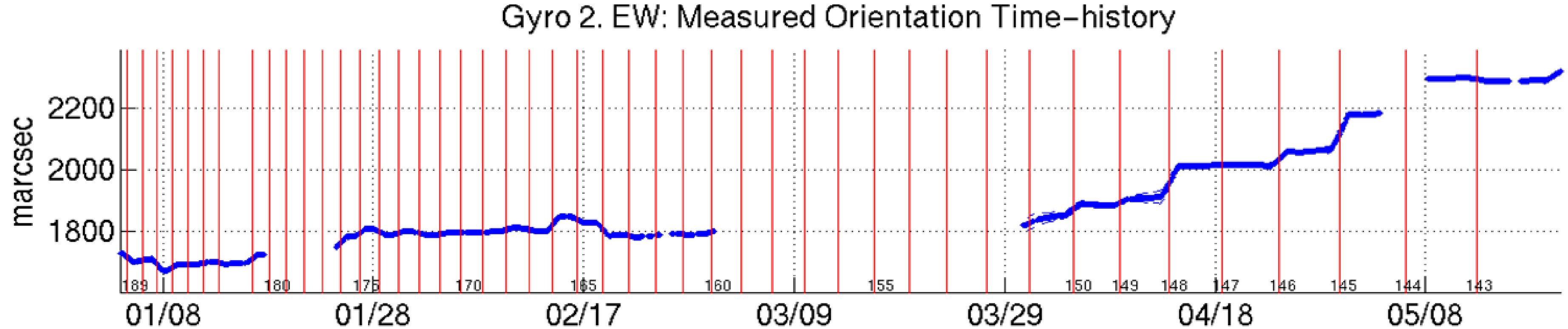
Output:

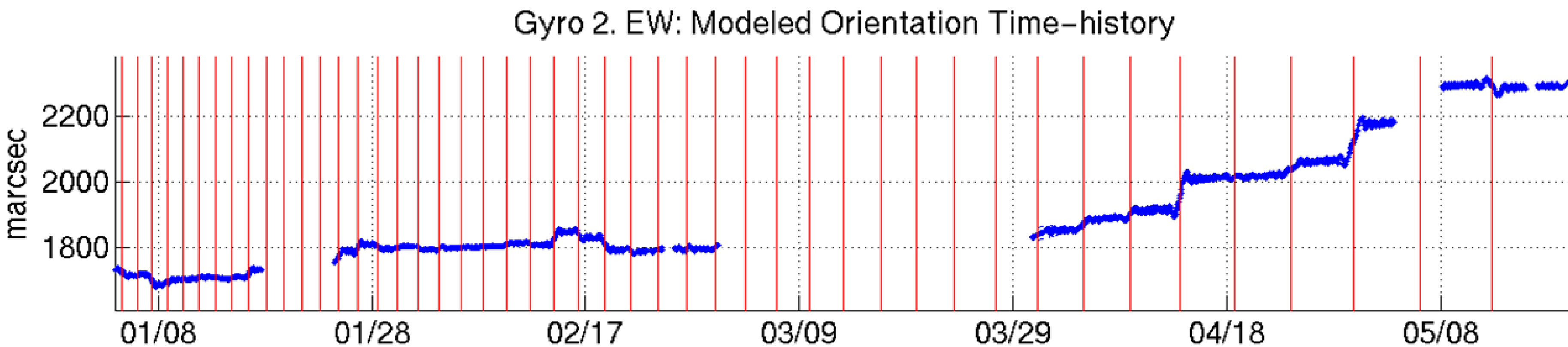
- **Torque related variables:**
  - torque coefficients
  - modeled torque contributions
- **Reconstructed "relativistic" trajectory (Orientation profile minus torque contributions)**

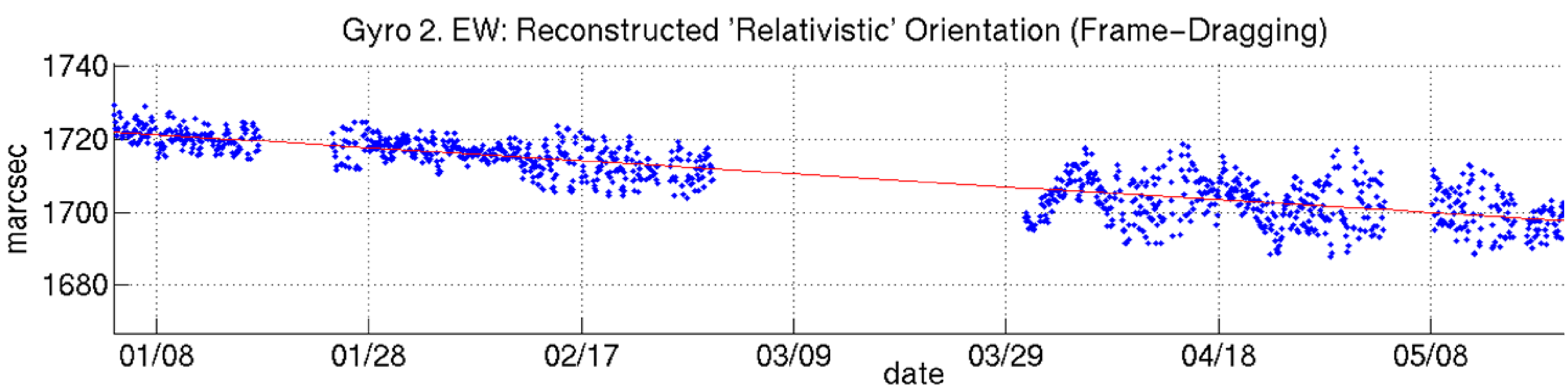
Subtraction

Relativity Estimates

Measured & Reconstructed Orientations



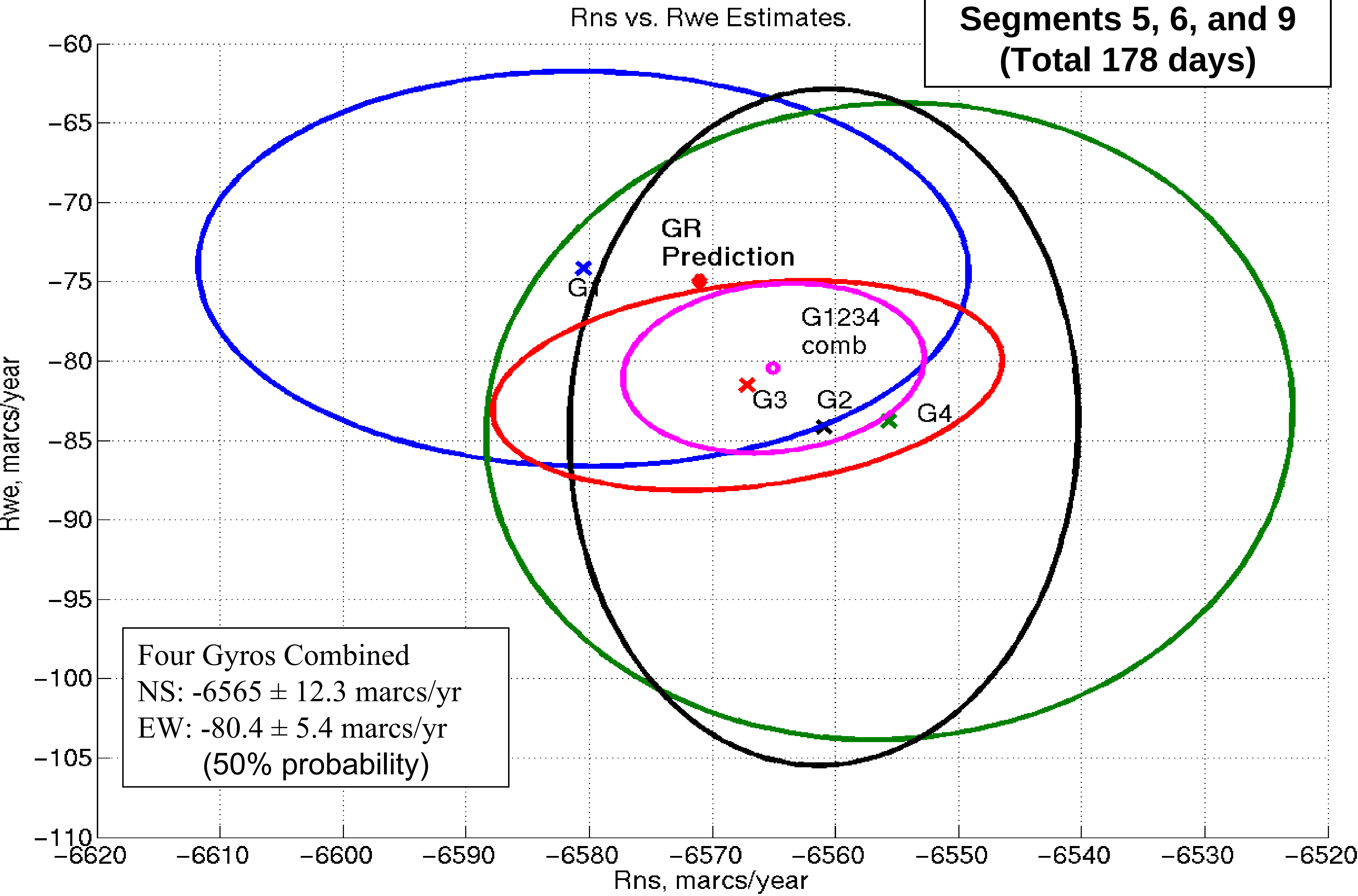




Measured & Reconstructed Orientations

Two-Floor Data Analysis Results

Segments 5, 6, and 9
(Total 178 days)



Success & Limits of Two-Floor Data Analysis

- **Roll-resonance torque modeling:**
  - Reduced large part of systematic errors: previously unmodeled torque-related errors now modeled properly
  - Dramatically enhanced agreement between gyros

- **The same torque model for all 4 gyros & entire mission**

- **2nd Floor Kalman Filter is not perfect:**
  - ❖ Orientation profile time step (currently 1 point per orbit) should be << 1 roll period (Torque Model)

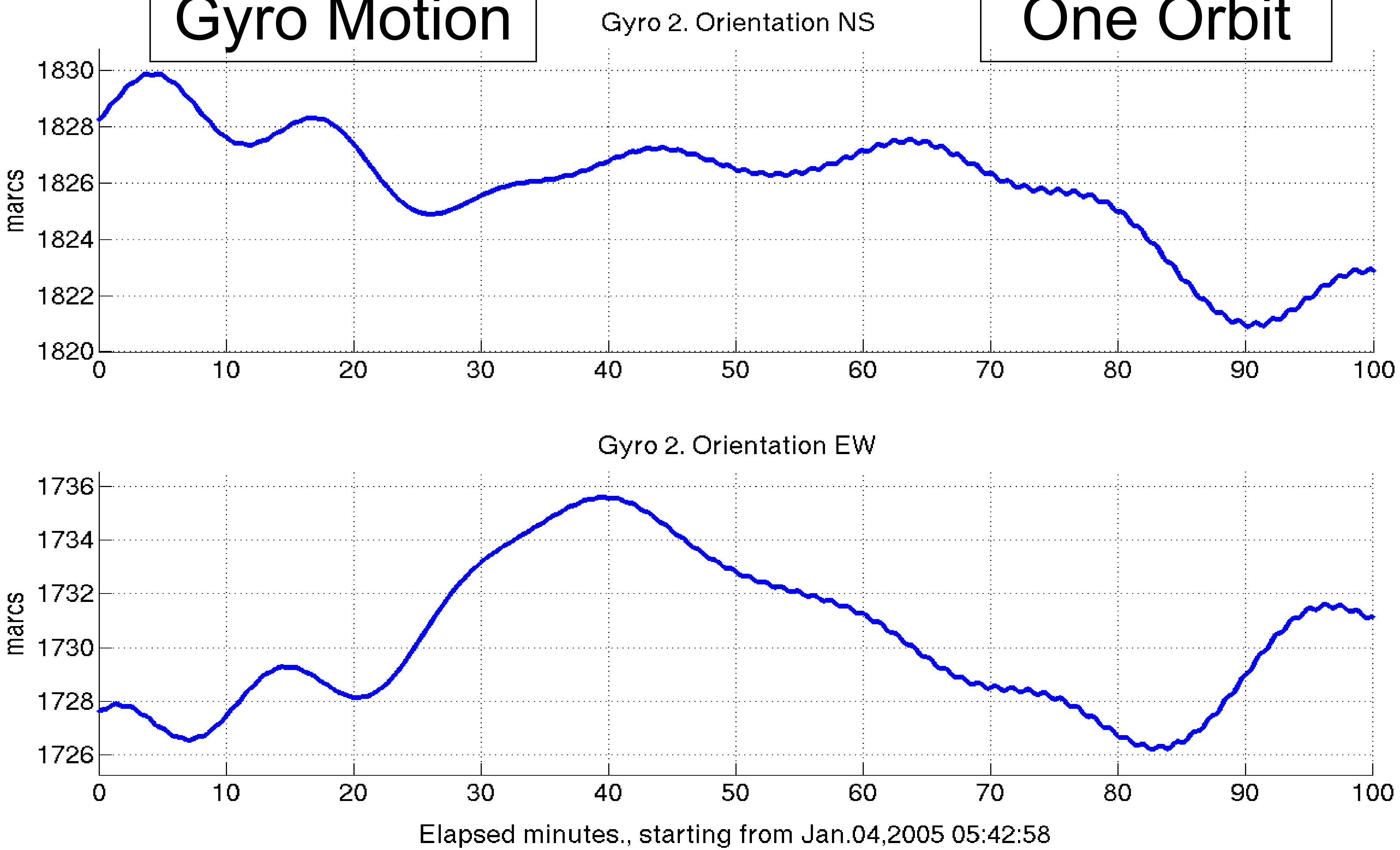
- **Final improvement of Algebraic Method: "2-sec Filter"**

Why 2- sec? → Science Signals (SQUID, Telescope,…) available at 2 sec sampling rate

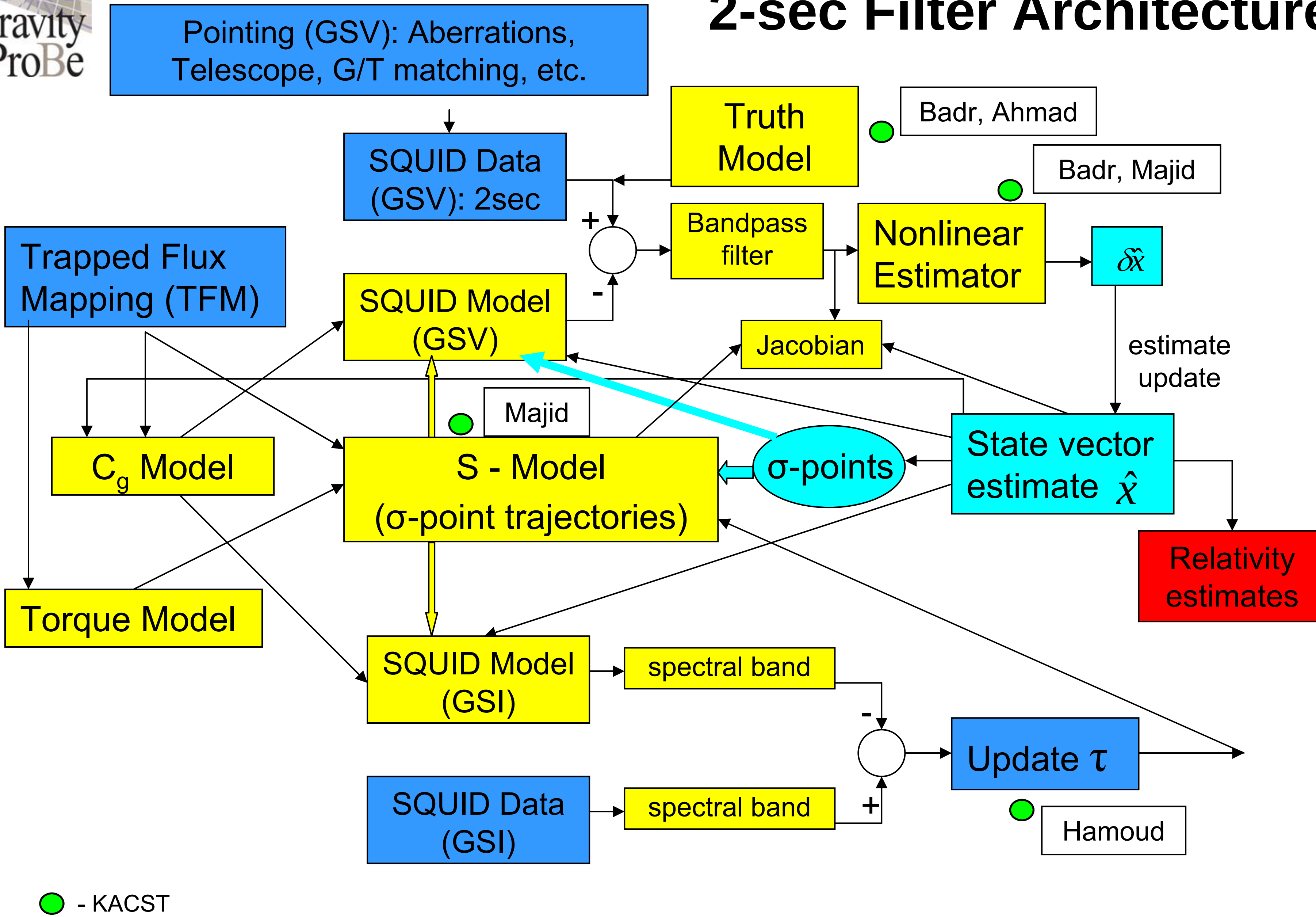
Why 2-sec Filter?

Gyro Motion

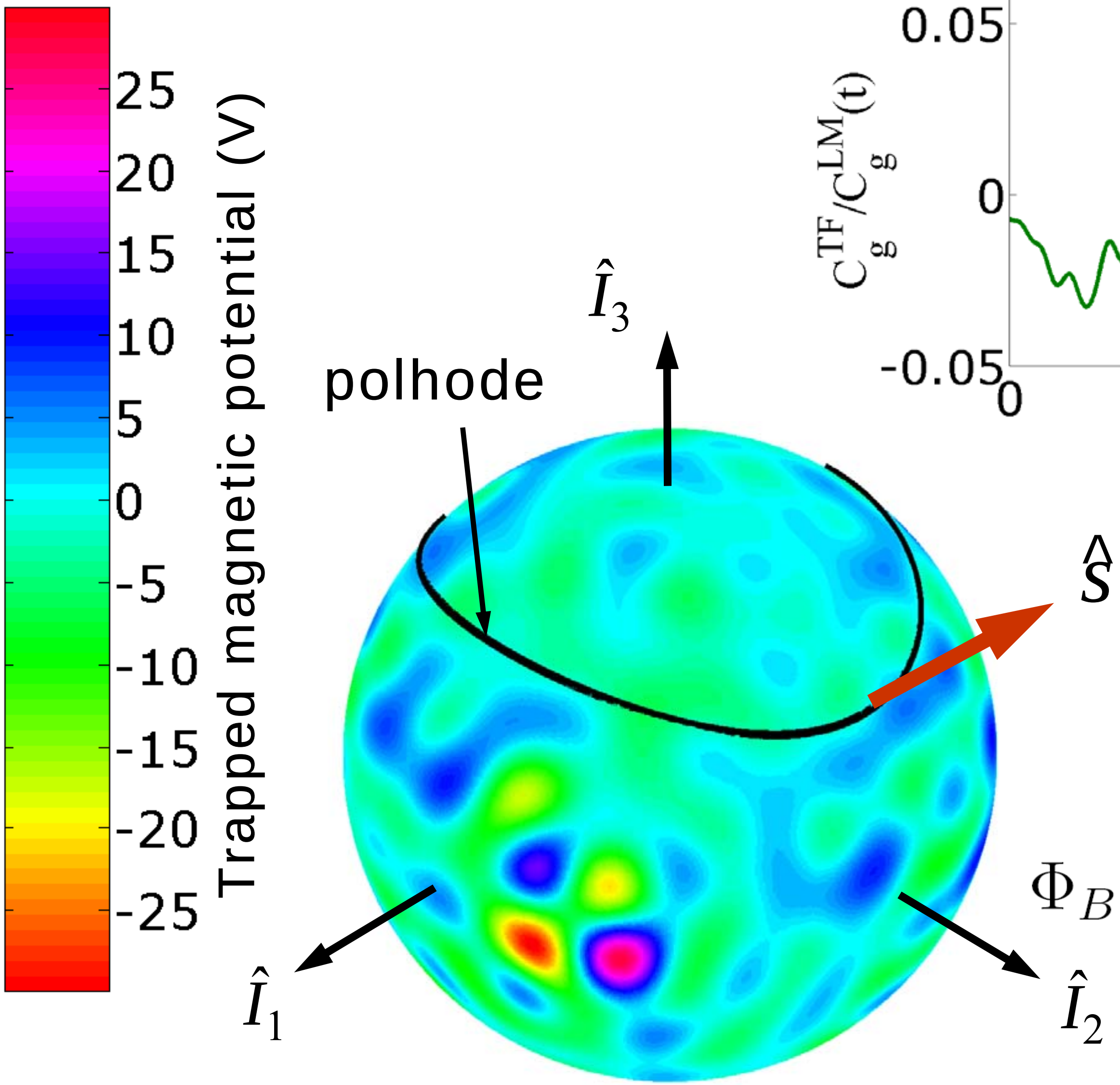
One Orbit

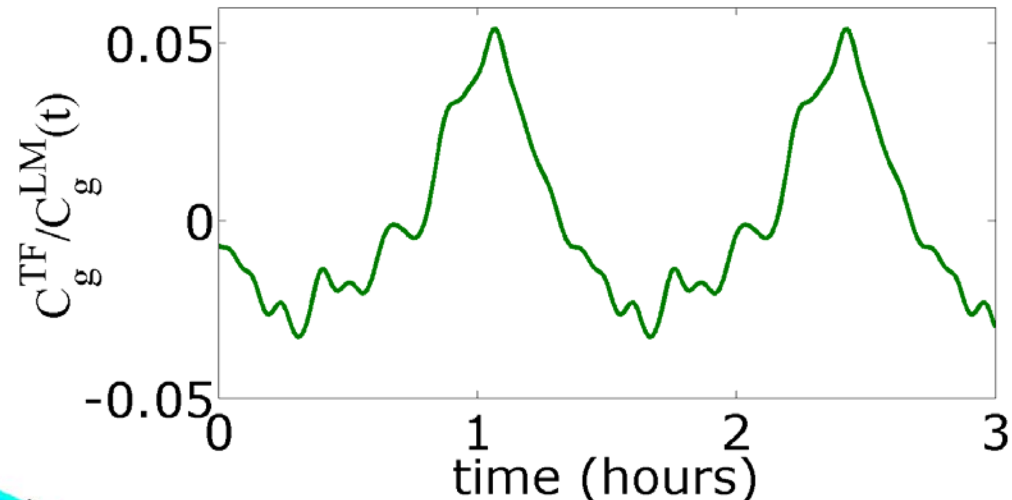


2-sec Filter Architecture



Trapped Flux & Readout Scale Factor



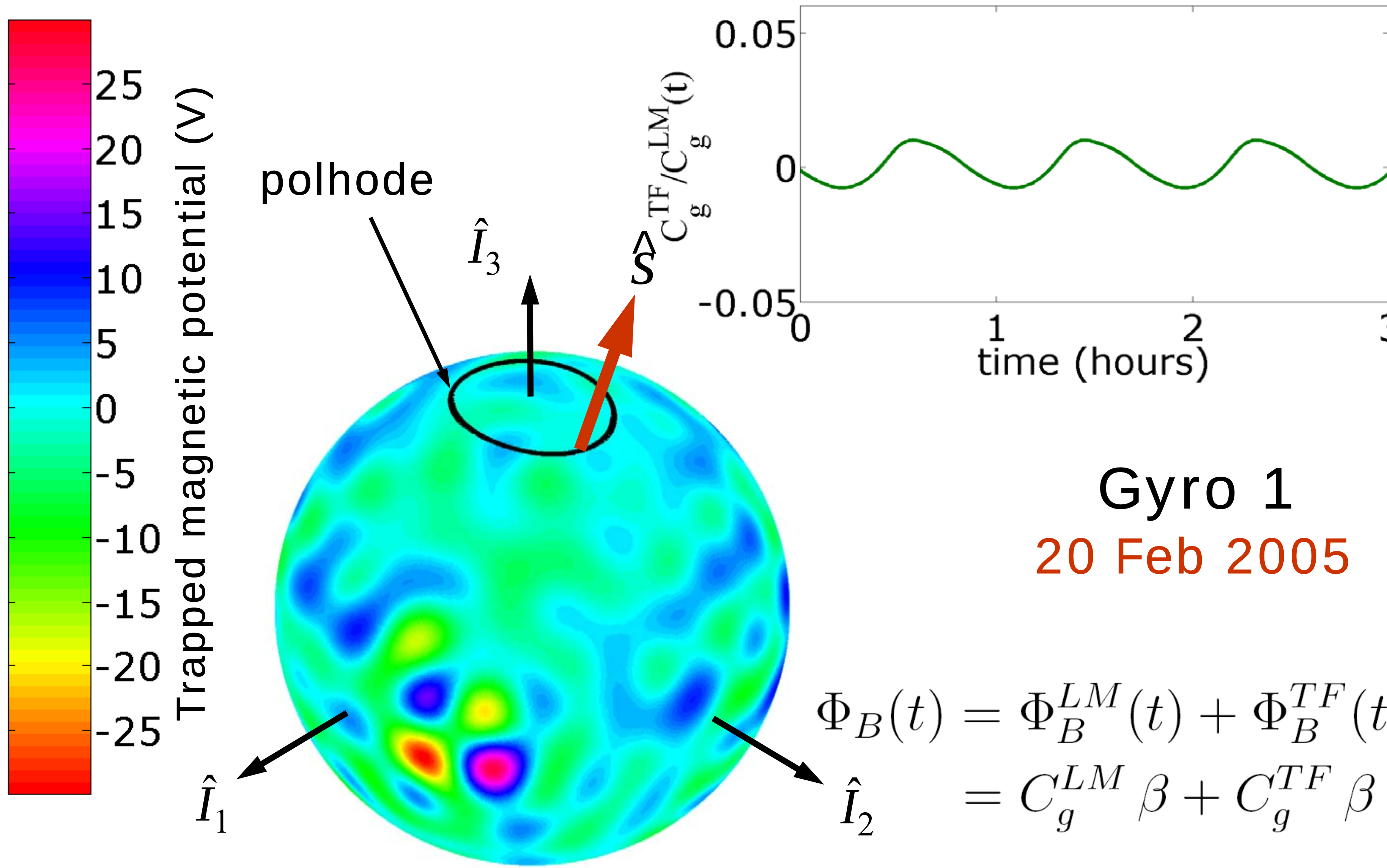


Gyro 1

4 Oct 2004

$$\begin{aligned}\Phi_B(t) &= \Phi_B^{LM}(t) + \Phi_B^{TF}(t)\\ &= C_g^{LM}\,\beta + C_g^{TF}\,\beta\end{aligned}$$

Trapped Flux & Readout Scale Factor



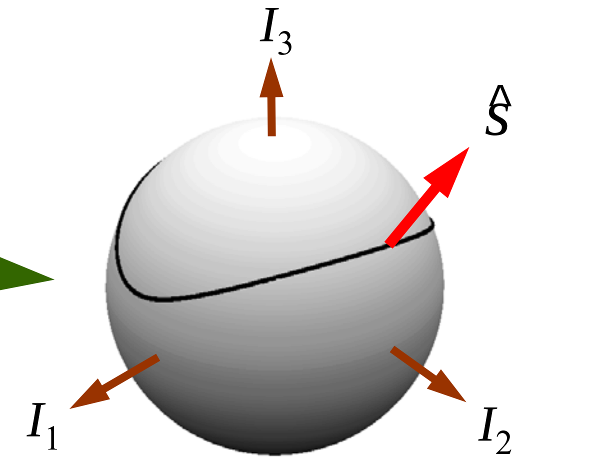
Gyro 1

20 Feb 2005

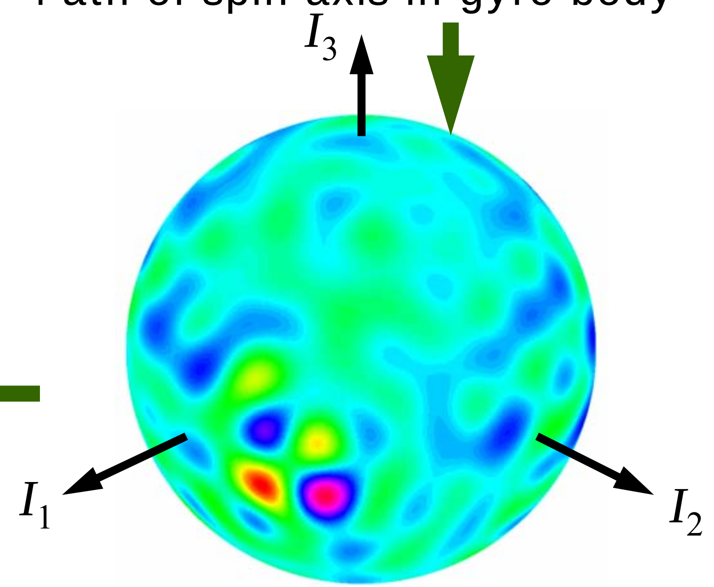
$$\Phi_B(t) = \Phi_B^{LM}(t) + \Phi_B^{TF}(t)$$
$$= C_g^{LM}\,\beta + C_g^{TF}\,\beta$$

Products of Trapped Flux Mapping

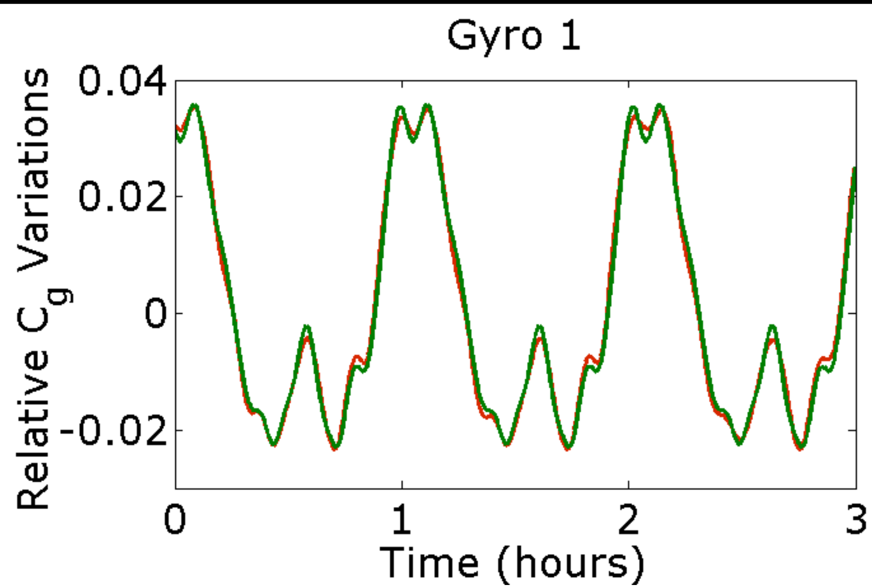
Parameter	Error
Angular velocity, ω	10 nHz ~ 10^{-10}
Polhode phase, ϕ_p	~ 1°
Rotor orientation	~ 2°
Trapped magnetic potential	~ 1%
Gyroscope scale factor, C_g	~ 10^{-4}



Path of spin axis in gyro body



Trapped magnetic potential



2-sec Filter Estimation Algorithm

- **Estimation problem:**

State vector

$$z_k = h_k(x, u_k) + v_k \qquad k = 1, 2, \ldots N \;\; (\sim 10^7)$$

SQUID Signal

SQUID Signal Model (NONLINEAR function of x)

SQUID noise

- **Conventional method (widely used): Extended Kalman Filter (EKF) / Iterative EKF**
  - **Linearization at current estimate $\rightarrow$ linear Kalman Filter for each iteration**
- **Jacobian computed via Sigma-Point Technology**
- **2-sec Filter capitalizes on ~ 3 years of experience**

KACST Contribution #1

- **Badr Alsuwaidan**

  Activity: Truth model development & implementation

  Motivation: Debugging and testing of complex filter is IMPOSSIBLE without "Truth Model"

  - ✓ **Badr developed software to generate "Truth Model SQUID signal"**
  - ✓ **Badr & Vladimir Solomonik integrated "Truth Model" with 2-sec Filter**
  - ✓ **"Truth Model" now 1 of 2 main Filter modes (together with "Flight Data" mode)**
  - ✓ **"Truth Model" allows:**
    - Filter debugging
    - Analyze filter convergence
    - Tuning of filter parameters
    - Sensitivity analysis preparation

KACST Contribution #2

❖ **Majid Almeshari**

Activity: Parallelization of 2-sec Filter

Motivation: 2-sec Filter computation time (1 processor) = few days.
parallel processing: few days → few hours

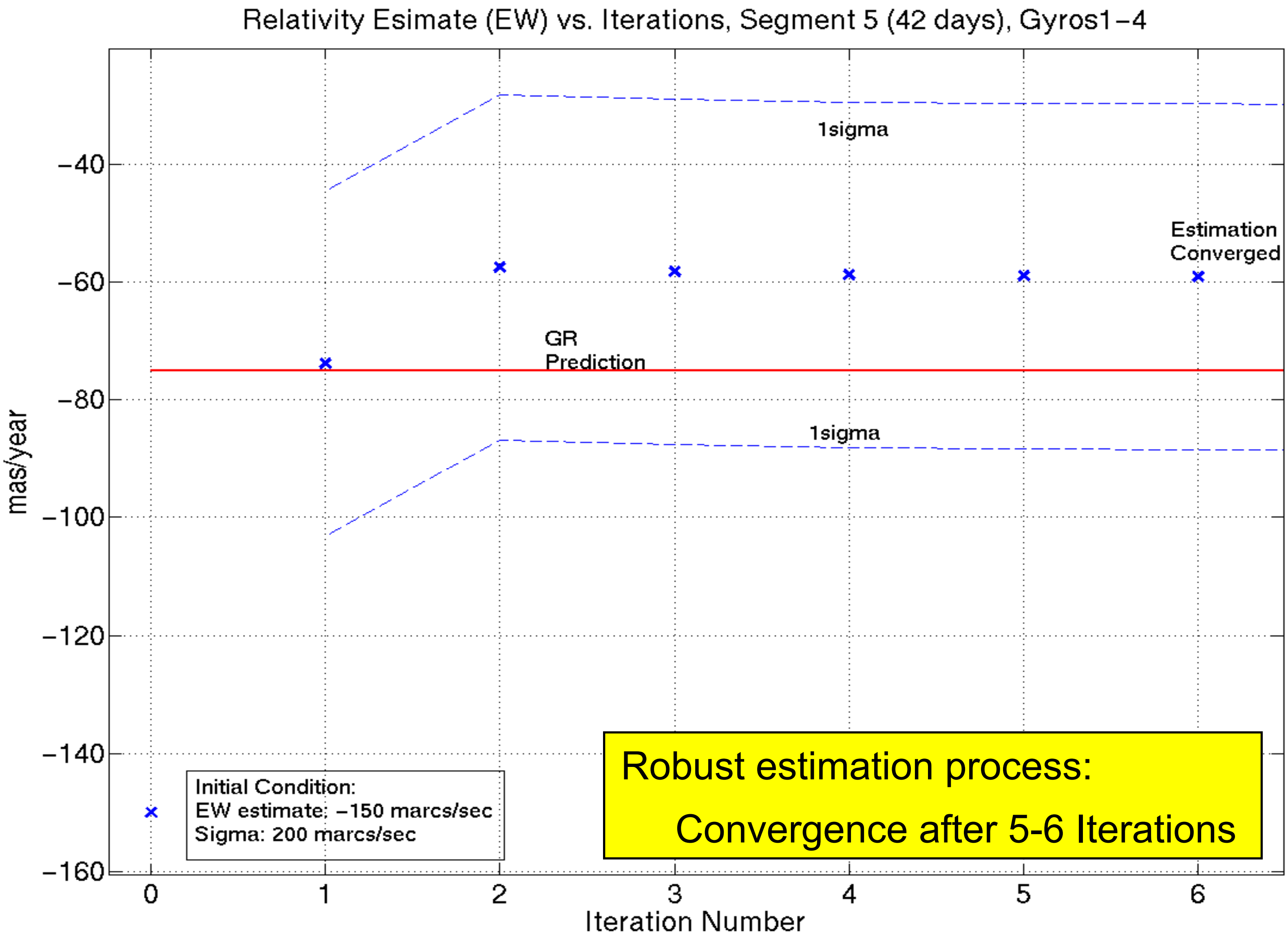
- ✓ **Parallel 2-sec Filter demonstrated on 44 processor (64 bit) cluster**
- ✓ **Extensive timings of serial code complete: allows efficient parallelization**
  - ➢ **~ 80% of computation time is Jacobian → initial parallelization effort**
- ✓ **Majid's work revealed that comm. time is largest bottleneck**
  - ➢ **Restructuring underway to minimize intra-processor communication time**

Schedule: Parallel 2-sec Filter as primary analysis by October

Testing of 2-sec Filter

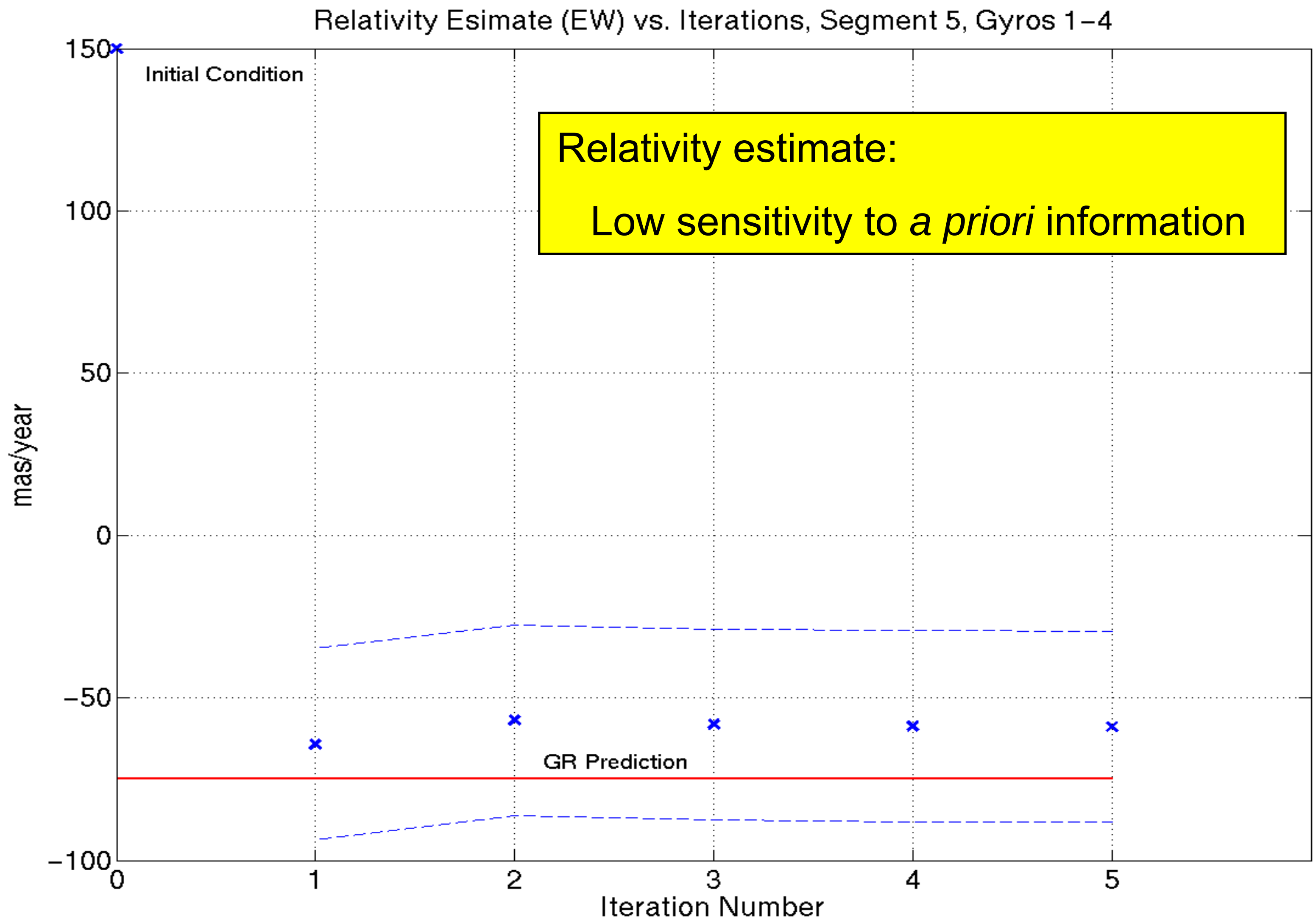
- **Testing performed on a single-processor (8 GB RAM)**
- **Constraint allows only Segment 5 (~40 day analysis)**
- **Parallel processing necessary for complete (10 Segment) analysis** (see Majid's presentation)
- **Segment 5, 1 gyro not enough data to estimate relativity parameters**
- **Segment 5, with 4 gyros allows convergence**

Testing Results (Flight Data): Segment 5



Testing Results (Flight Data): Segment 5

Sensitivity to initial conditions



Near Term Steps

- **Extend analysis to segments 5, 6 & 9 (Aug ’09)**
  - Requires proper integration of segments
- **Implement parallel processing (Oct ’09)**
  - Analysis with all segments requires efficient parallel processing
- **Continue testing & fine-tuning of filter (Dec ’09)**