Gravity Probe B
Data Analysis Challenges, Insights, and Results

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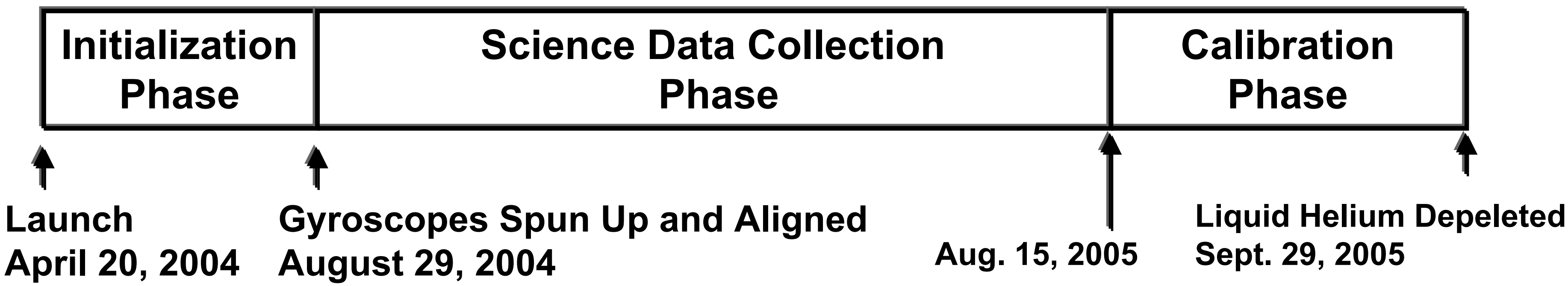
April 15, 2007

Topics

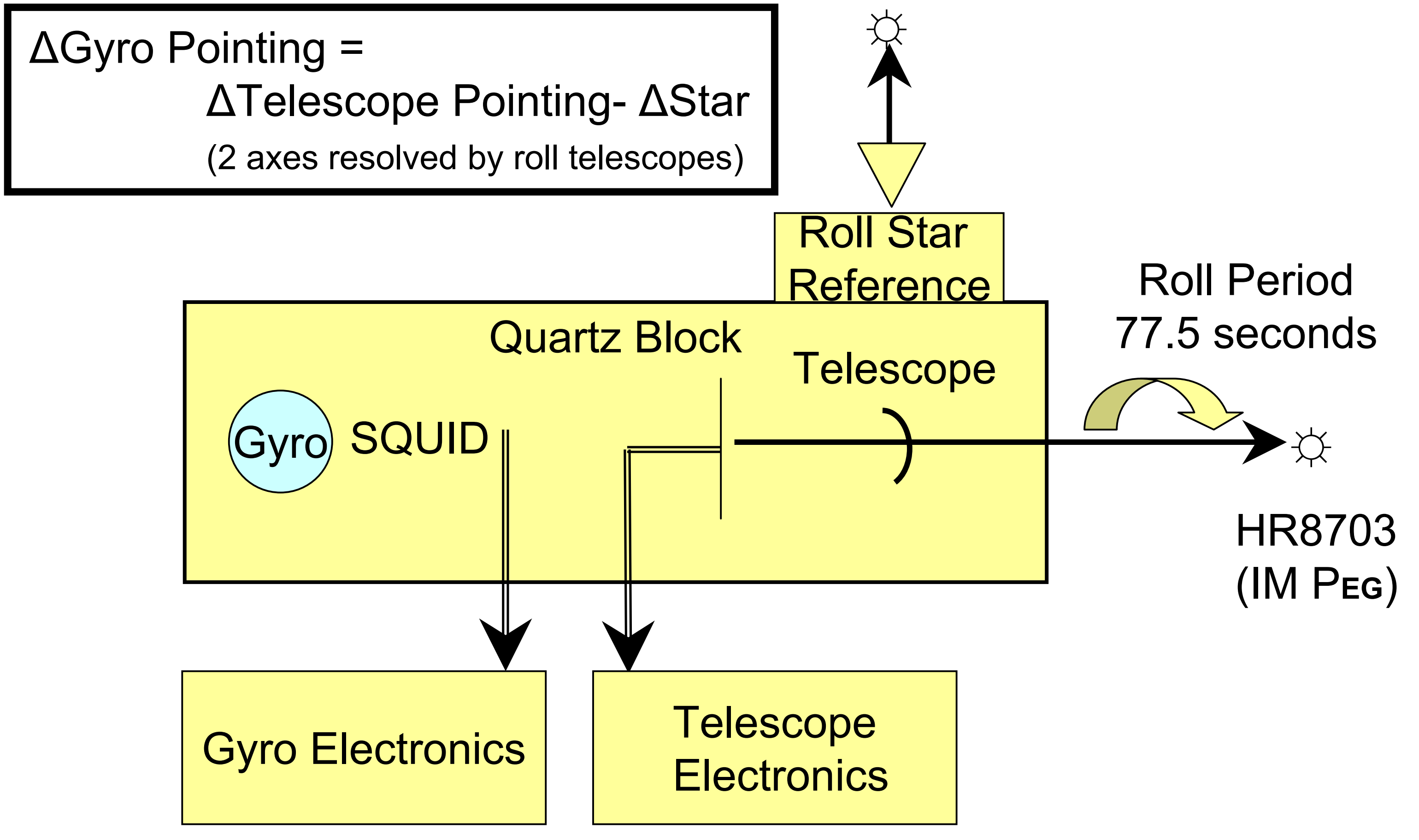
- **Data Analysis Strategy and Methods**
- **Challenges and Insights**
  - **Trapped Magnetic Flux and Slowly Changing Polhode Path**
  - **Misalignment Torque**
  - **Offsets in Gyroscope Spin Axis Orientation at Predictable Times**
- **Present Status and Future Prospects**

Data Collection Strategy and Limits on Potential Systematic Errors



- **Initialization Phase**
  - Instrument Calibration
- **Science Data Collection Phase**
  - Redundancy
  - Internal Cross-Checks
- **Calibration Phase**
  - Deliberate Enhancement of Disturbing Effects

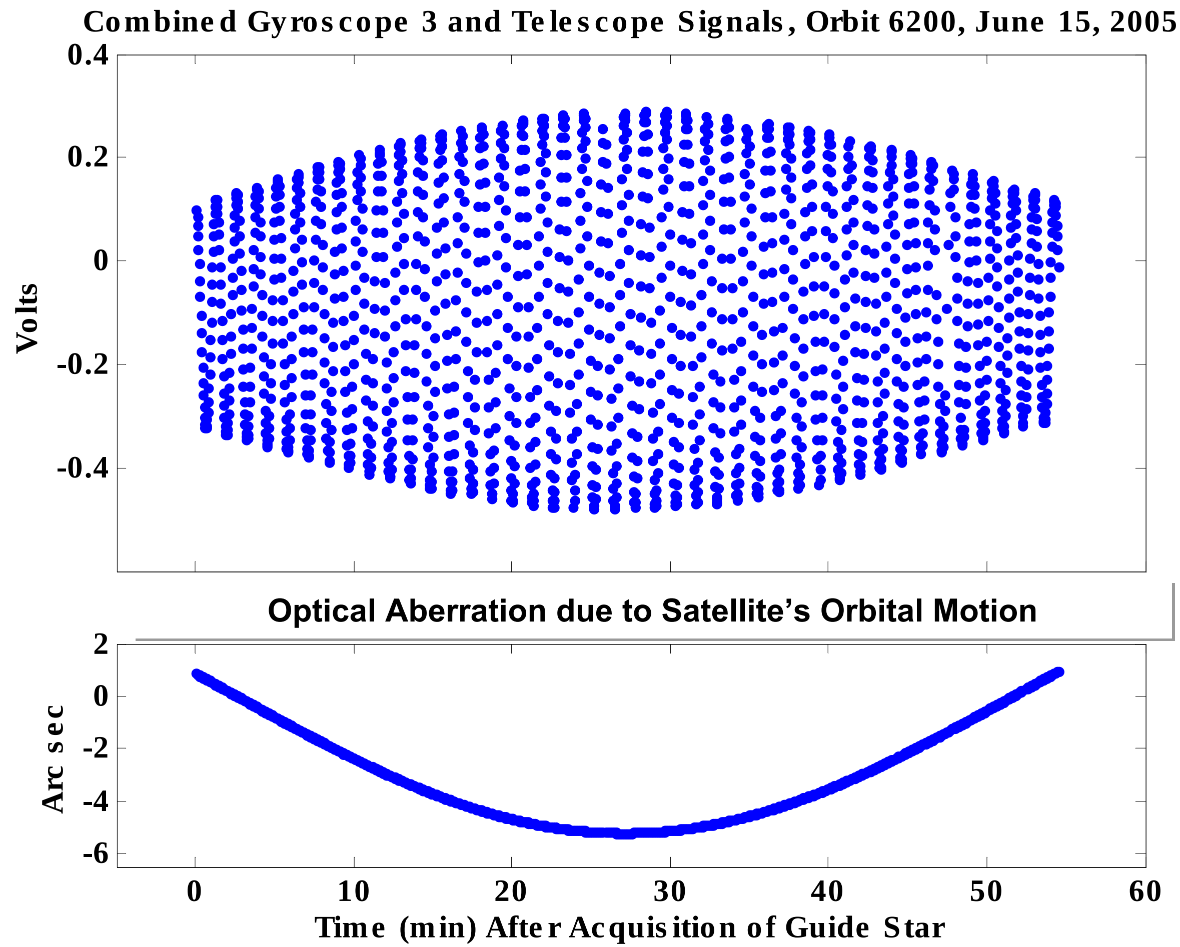
Science Data Collection: Measurement of Gyroscope Spin Axis Orientation Relative to Guide Star



Gyroscope Readout – Single Orbit

Data Analysis Procedure:

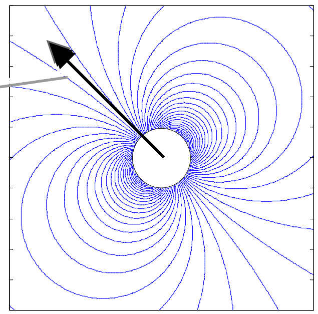
1. Find Ratios to Telescope to Gyroscope Scale Factors Using Attitude Errors
2. Combine Gyroscope and Telescope Signals
3. Calibrate Combined Signals Using Orbital Aberration



London Moment, Trapped Flux, and Polhode Motion

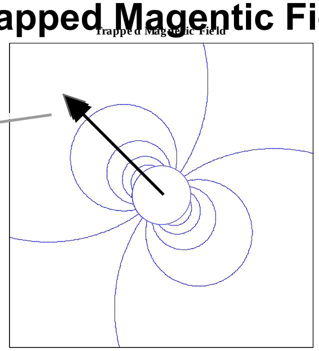
London Moment Magnetic Field

Gyroscope Spin Axis

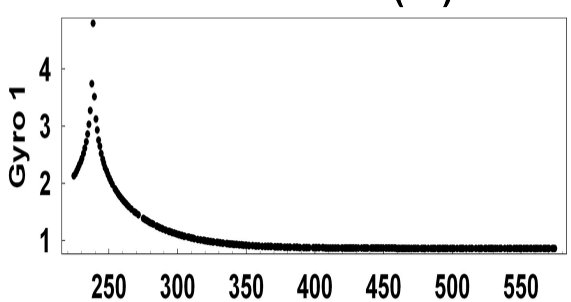


Trapped Magentic Field

Gyroscope Spin Axis



Polhode Period (hr)



Elapsed Time Since 1-Jan-2004 (days)

London Moment Readout:

- Rotating Superconductor Develops Dipole Field Aligned with Instantaneous Spin Axis
- Field at Surface of Rotor for Gyroscope Spinning at 80 Hz: 57.2 μG

Trapped Magnetic Field:

Body-Fixed

- Gyroscope 1 – 3.0 μG
- Gyroscope 2 – 1.3 μG
- Gyroscope 3 – 0.8 μG
- Gyroscope 4 – 0.2 μG

Polhode Motion and Damping:

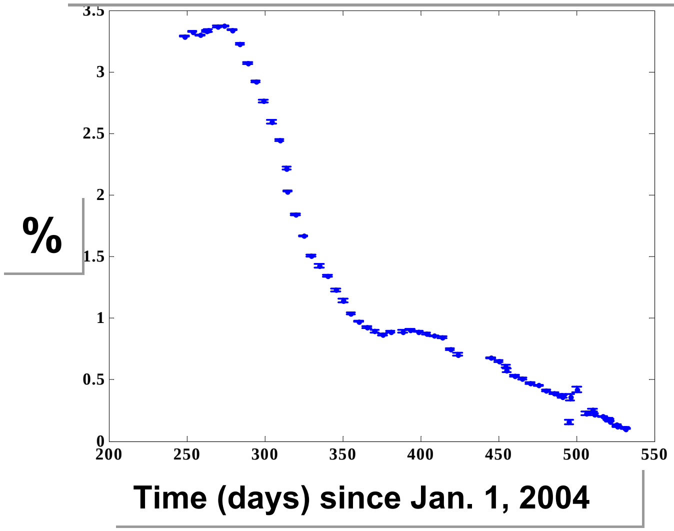
- Polhode Motion: Motion of spin axis within the body along the polhode path (Euler's Equations)
- Damping: Slow Relaxation of Polhode Path toward maximum moment of inertia

Data Analysis in Presence of Trapped Magnetic Flux and Polhode Motion

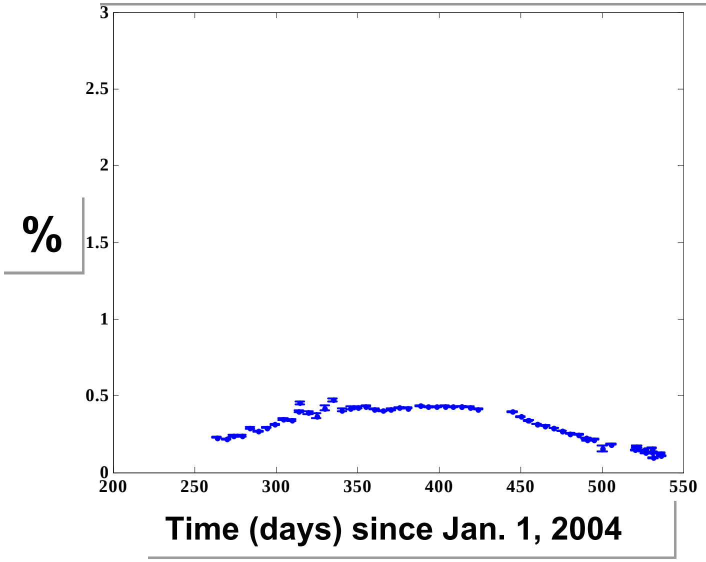
- **Impact on Data Analysis**
  1. Scale Factor is Modulated at Harmonics of Polhode Frequency
     – Magnitude of Modulation ~ 1%
  2. Amplitude and Phase of Scale Factor Modulation Slowly Changes
  3. Constant (Zero Frequency) Component Slowly Changes

- **Solution:**
  – Divide data into batches with length up to 75 orbits (5 days)
  – For Each Batch Estimate
    » Modulation Coefficients at Known Polhode Period
    » Constant Component of Scale Factor

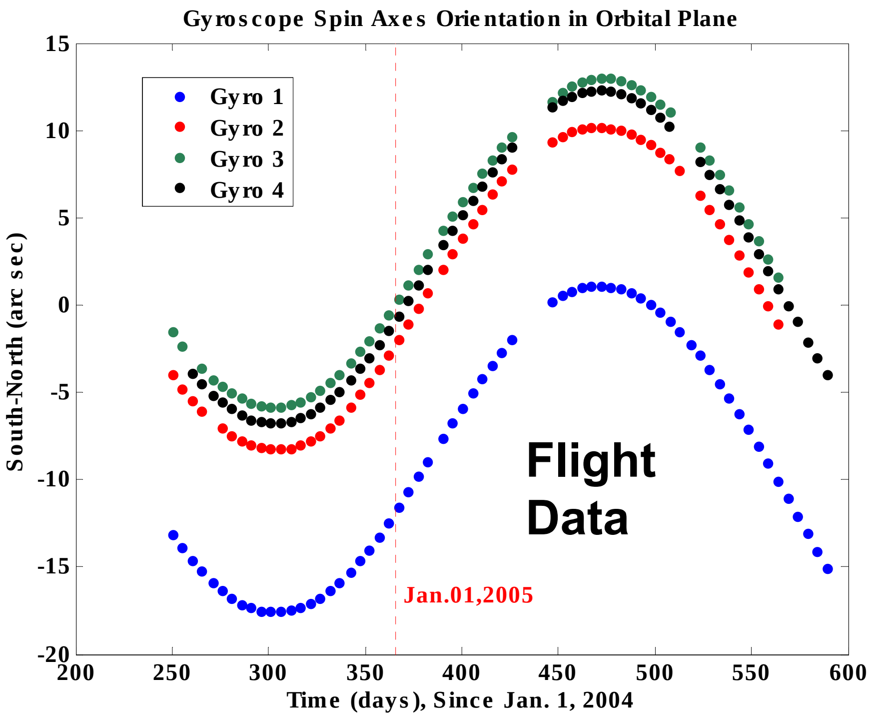
Gyro 1 Scale Factor Modulation

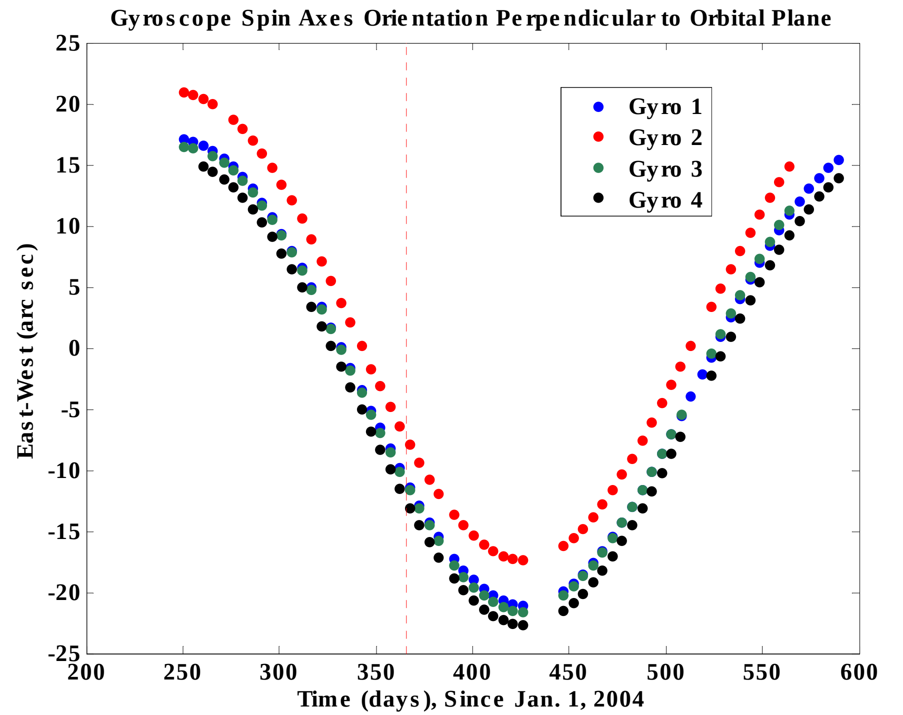


Gyro 4 Scale Factor Modulation



History of Gyroscope Spin Axis Orientation – Relative to the Apparent Position of the Star



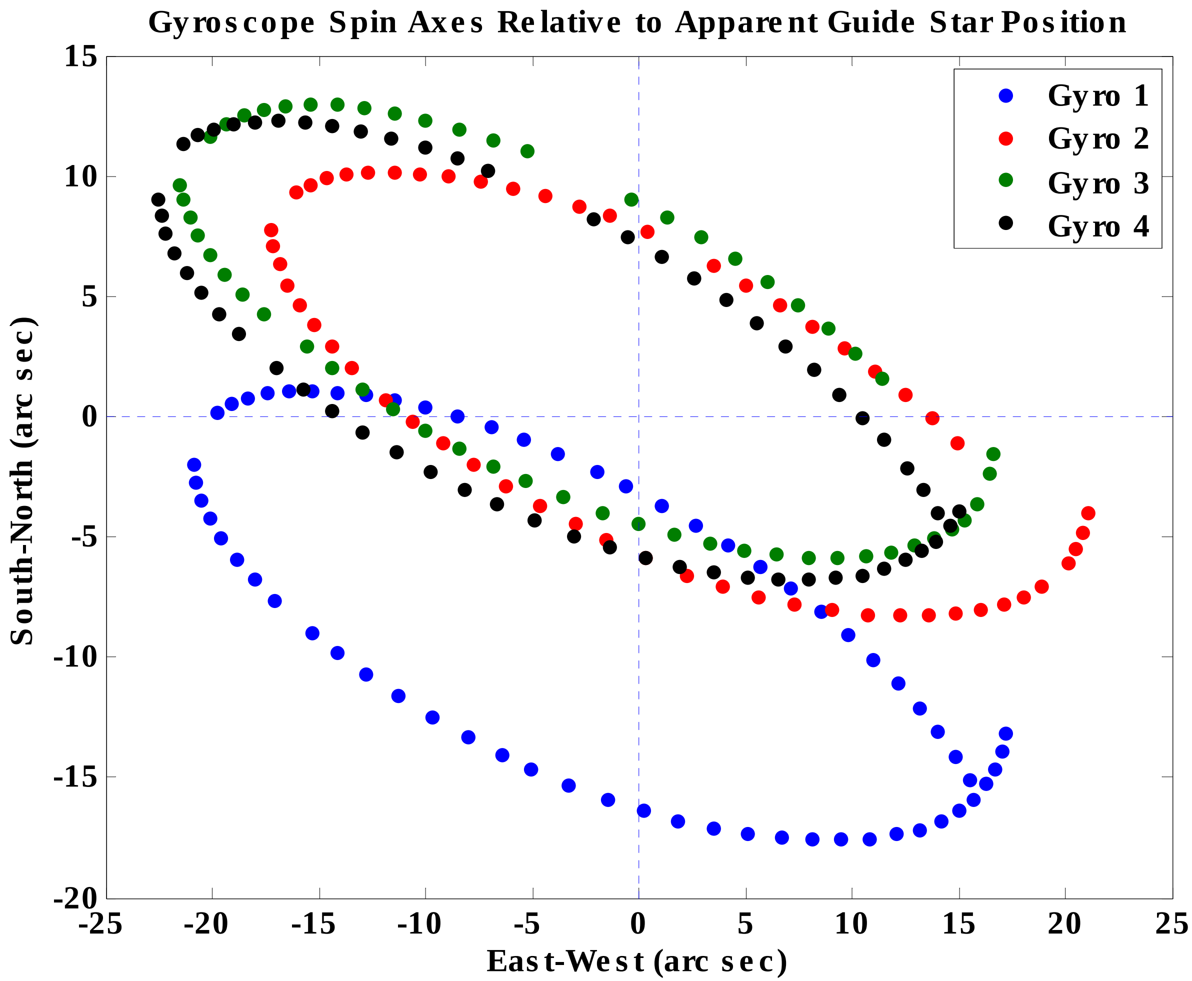


Maximum Differences Between Apparent and True Guide Star Positions

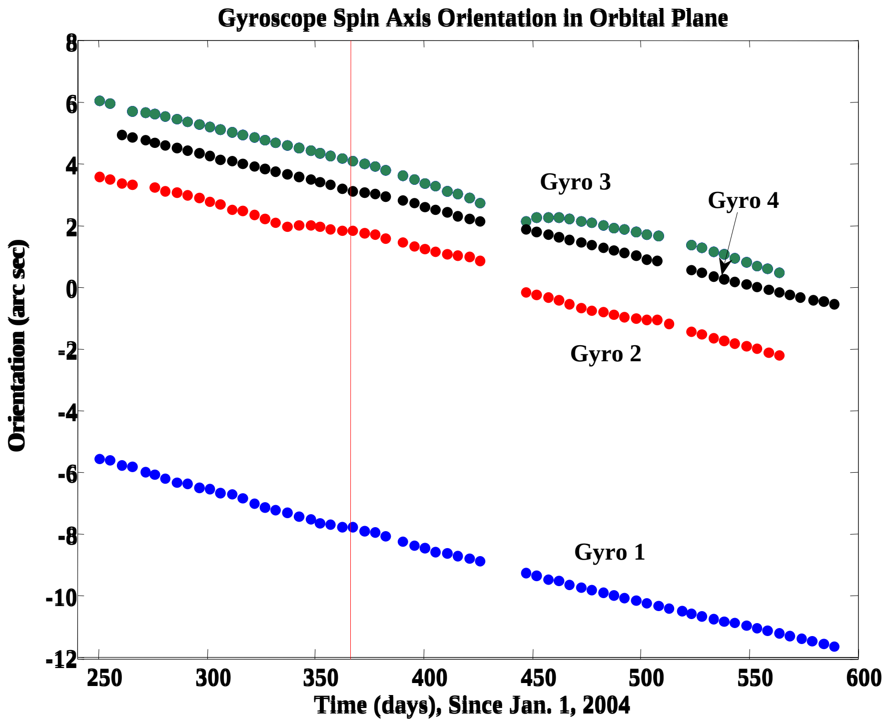
	North-South	West-East
Annual Aberration	11.0 arc sec	19.1 arc sec
Parallax	3 milli-arc sec	5 milli-arc sec
Deflection of Starlight	19 milli-arc sec	15 milli-arc sec

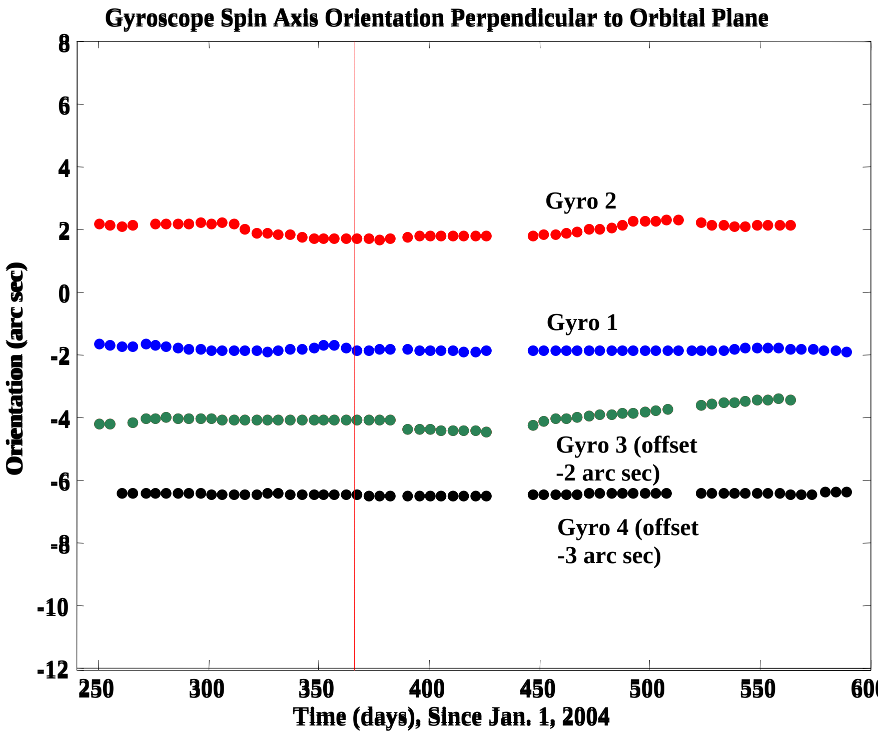
Changes in Gyroscope Spin Axis Orientation

Elliptical Path Due to Annual Aberration

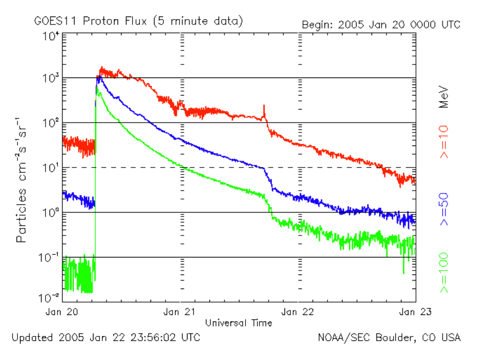


History of Gyroscope Spin Axis Orientation – Relative to the True Position of the Star





Proton Flux, Jan. 20, 2005, Measured by GOES Satellite



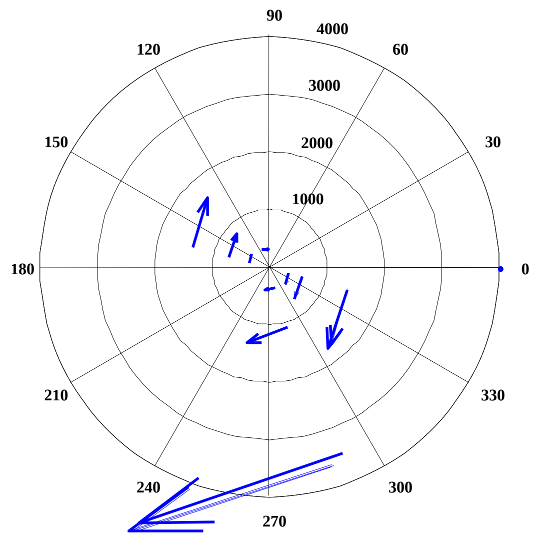
Calibration Phase – Discovery of a Misalignment Torque

- 19 Maneuvers to nearby stars or "virtual" stars
  - Maximum misalignment 7 degrees
  - Majority of maneuvers at 1 degree or less
- Maneuvers included variety of operating conditions for gyroscope suspension system and spacecraft translation control
  - Electrostatic Suspension System Operated with
    - » Voltages Applied to Electrodes Modulated at 20 Hz
    - » DC Voltages Applied to Electrodes
  - Deliberate Acceleration of the Spacecraft Along Roll Axis and Perpendicular to the Roll Axis
- Change in spacecraft roll period (77.5 sec to 120 sec)
- Calibration phase lasted from Aug. 15 through depletion of liquid helium on Sept. 29, 2005.

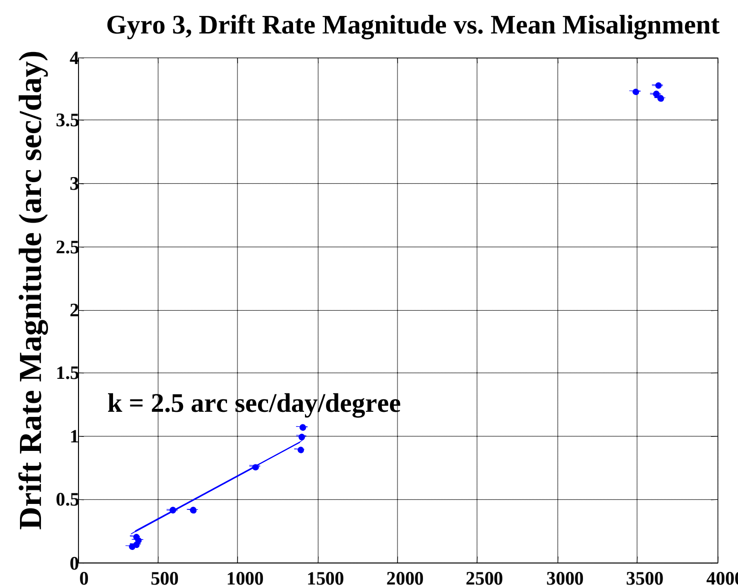
Gyro 3 Misalignment Torques – Calibration Phase

Mean Rate (marc-s/day) vs. Mean Misalignment (arc-s)

Mean North-South Misalignment



Mean West-East Misalignment



Mean Misalignment (arc sec)

For gyroscope 3

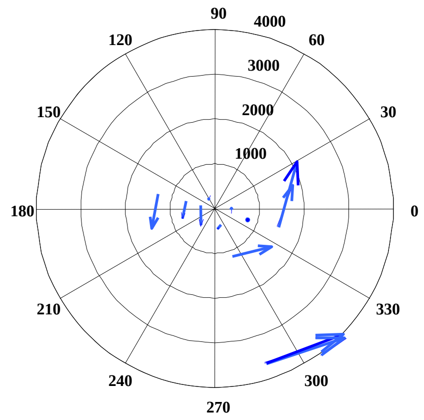
- ***drift rate is azimutal to measurement accuracy***
- ***drift rate increases linearly with the misalignment angle up to 1500 arc sec.***

All Gyroscopes – Calibration Phase

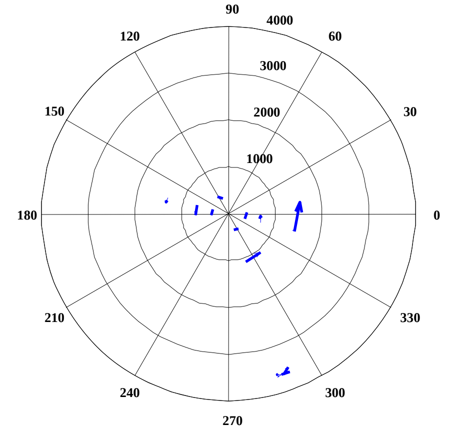
Mean Rate (marc-s/day) vs. Mean Misalignment (arc-s)

Mean North-South Misalignment

Gyroscope 1

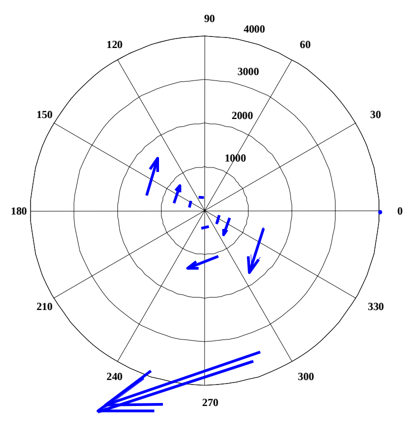


Gyroscope 2

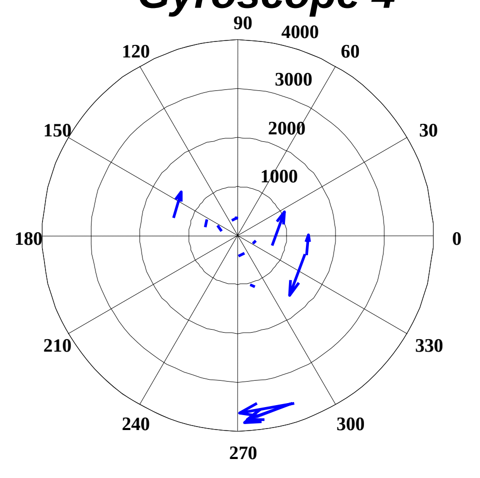


Mean North-South Misalignment

Gyroscope 3



Gyroscope 4



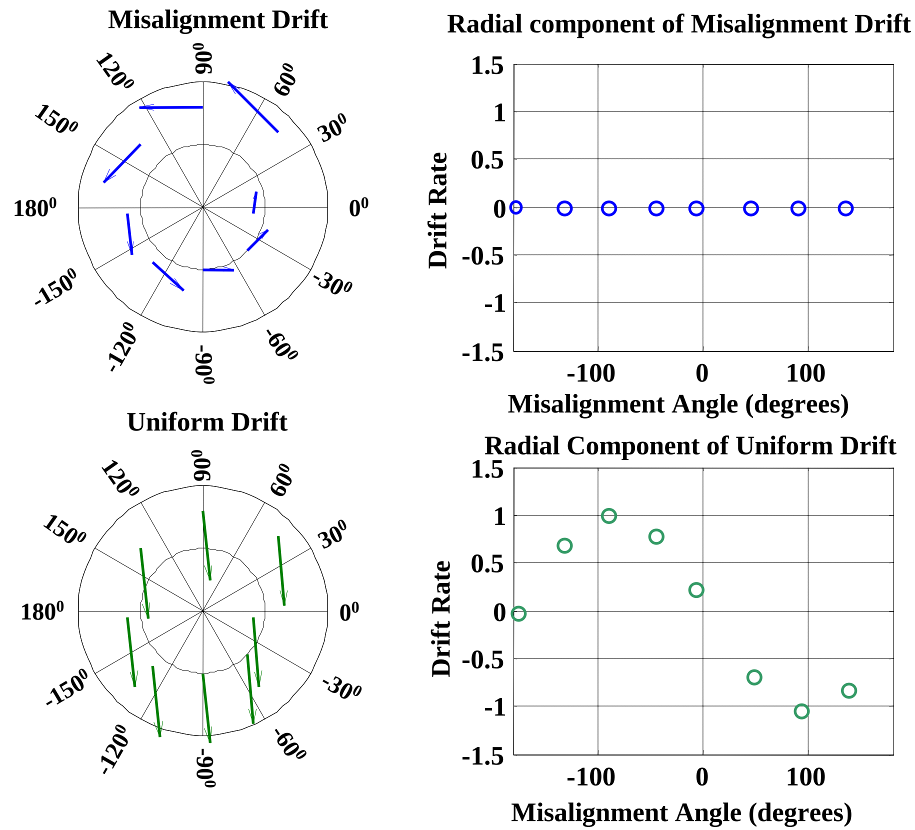
Mean West-East Misalignment Mean West-East Misalignment

Summary of Calibration Phase Results

- **Drift rate is perpendicular to misalignment**
  - Torque coefficients as large as 2.5 as/day/deg
- **Stability of misalignment torque coefficient**
  - Gyroscope 3:
    - » Gyroscope 3: Consistent calibration-phase results
    - » Offset following solar flare is consistent with magnitude of misalignment torque
    - » Sign of Torque Coefficient has changed from solar flare to calibration phase
  - Other gyroscopes:
    - » Evidence for variation in torque coefficient during calibration phase
- **Most Plausible Explanation:**
  - Interaction of patch effect fields on the gyroscope rotor with patch effect fields on the housing.

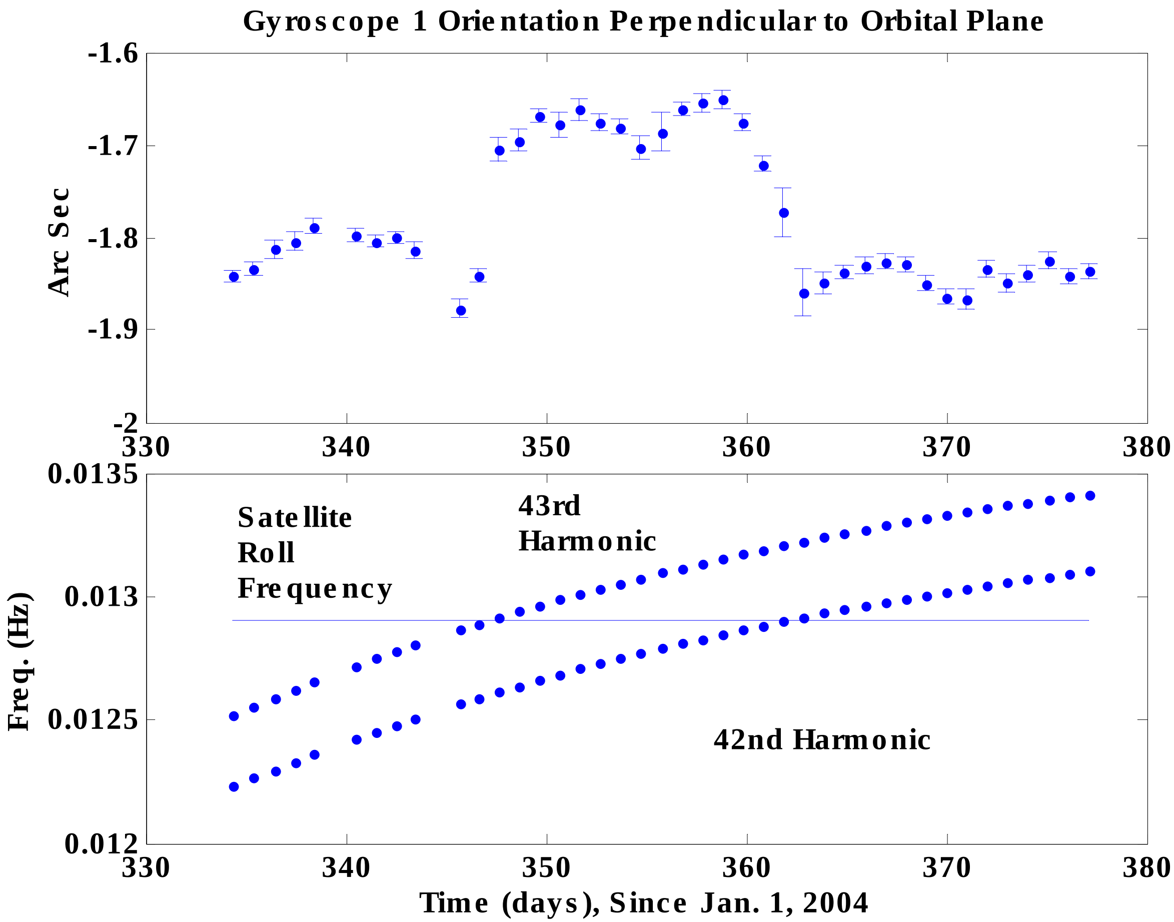
Data Analysis in the Presence of Misalignment Torques

Data simulated for illustration purposes



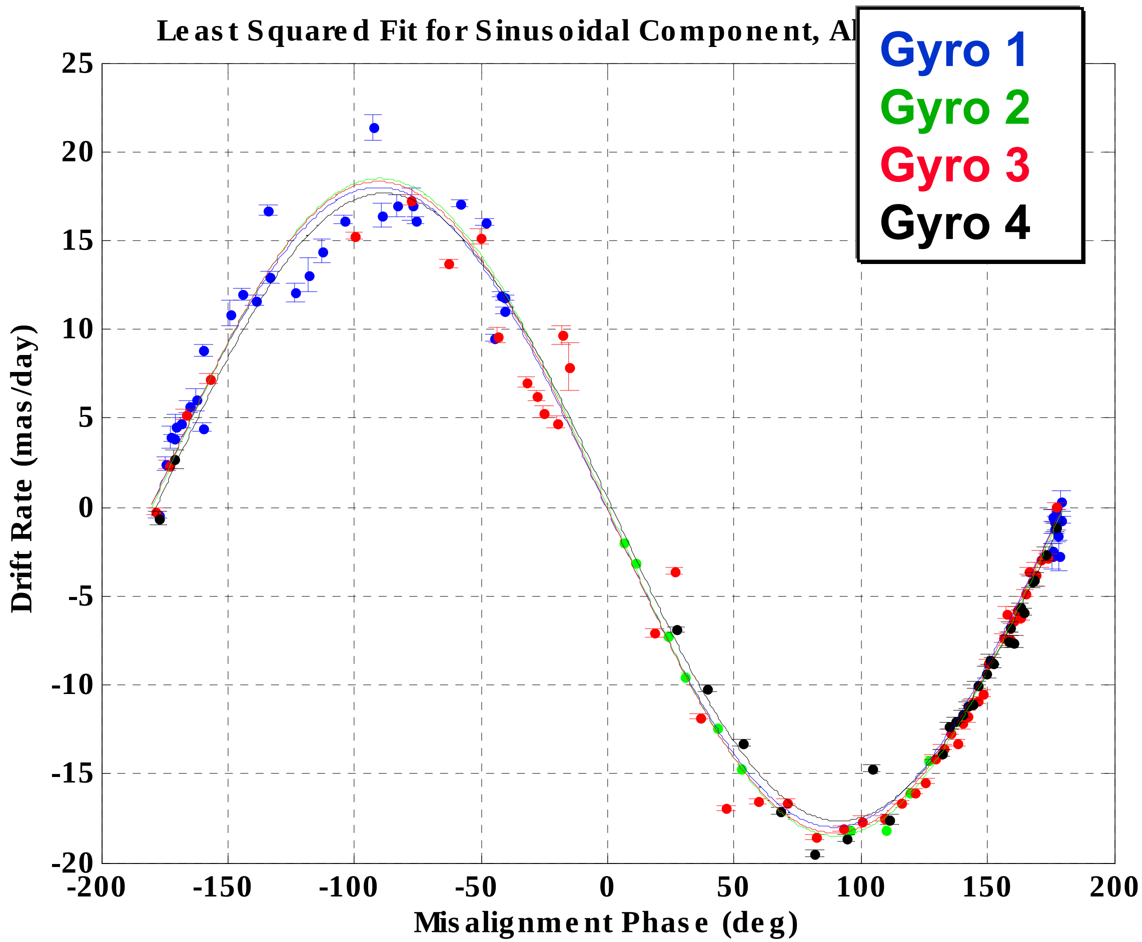
- ***Radial Component of Drift Rate Contains NO Contribution from Misalignment Drift***
- ***Magnitude and Direction of Uniform (Relativistic) Drift Rate May Be Determined From Variation of Radial Component with Misalignment Phase***

An Additional Observation: Offsets in Gyroscope Spin Axis Orientation



Data is Not Used in Analysis When an Harmonic of Polhode Frequency Coincides with Satellite Roll Frequency

Relativistic Drift Rate – All 4 Gyroscopes



Consistency between gyroscopes and data runs better than 100 mas/year.

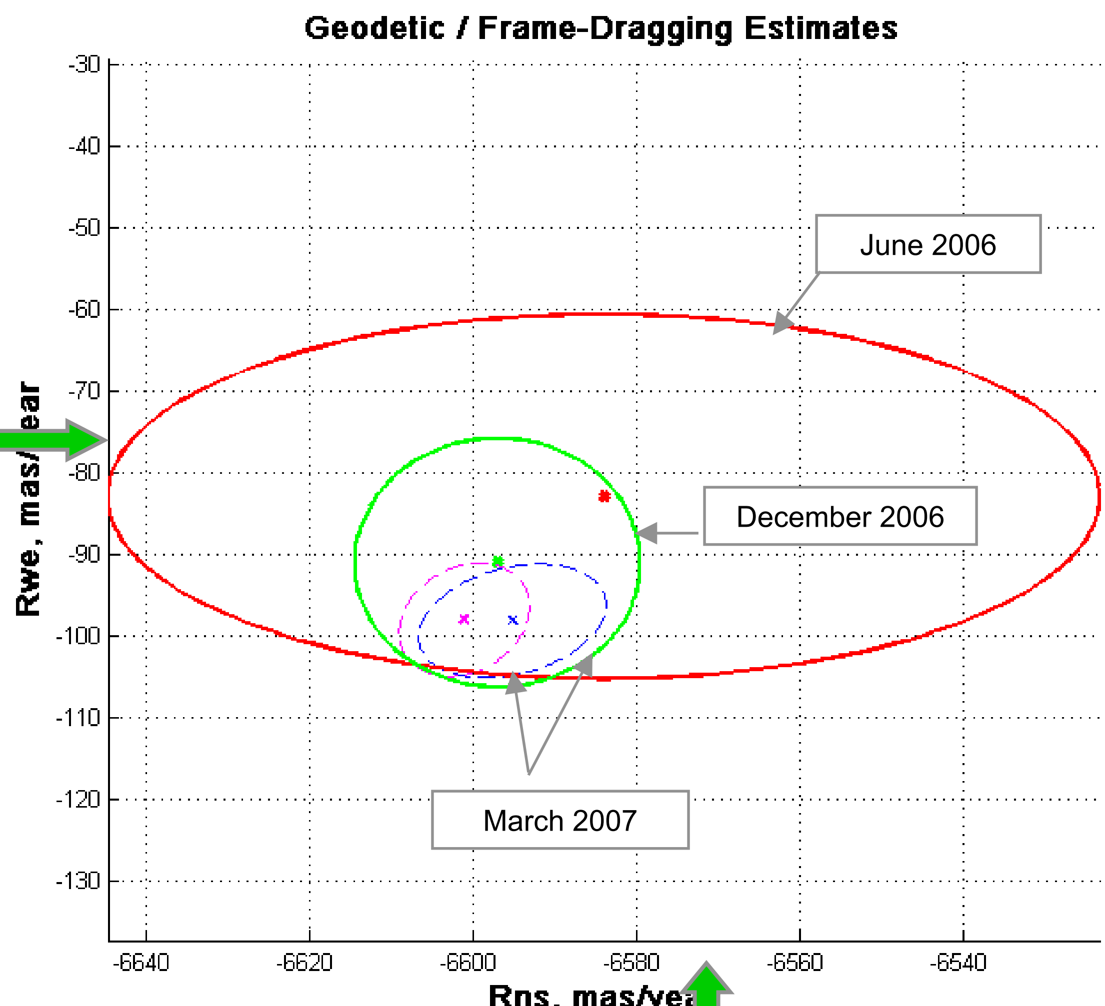
A complementary data analysis approach: Algebraic Method

- **Method:**
  - Determine Gyroscope Spin Axis Orientation from Short-Term Analysis
  - Simultaneously Estimate Relativistic Drift Rate and Misalignment Torque Coefficient Using Known History of Satellite Roll Axis Orientation

	Geometric Method	Algebraic Method
Short Term Analysis	• 10 to 75 orbits • Modulation of Scale Factor at Polhode Frequency	•1 to 5 orbits • Modulation of Scale Factor at Polhode Frequency
Information for Long Term Analysis	Gyroscope Drift Rate	Gyroscope Spin Axis Inertial Orientation
Additional Information Needed	Misalignment between satellite roll axis and gyroscope spin axis	Inertial Orientation of Satellite Roll Axis
Long Term Analysis	Relativistic Drift Rate and Misalignment Torque	• Relativistic Drift Rate Misalignment Torque •Polhode Variations of Scale Factor •Continuous Dynamic Estimation of Gyroscope Orientation

Two Complementary Data Analysis Methods Provide Valuable Cross-Checks and Additional Insight

Preliminary Results: Algebraic Method



Frame-Dragging from GR

Geodetic from GR

Error Ellipses Show Statistical Error Only

Initial Geodetic Effect Results

	'Geometric'	'Algebraic'			
	Full Year	1 158 days	2 85 days	3 82 days	4 41 days
1σ statistical error only	-6638 ± 11	-6584 ± 52	-6597 ± 20	-6595 ± 10	-6604 ± 7

Separate gyro, ~ 5-day batches

Combined gyro processing, continuous filtering

Progress in modeling with algebraic approach evident

SQUID noise limit

Net expected

-6571 ± 1 *

[marc-s/yr]

-6638 ± 97

Residual gyro-to-gyro inconsistencies due to incomplete modeling ~ 100 marc-s/yr

* Earth -6606, solar geodetic +7, proper motion +28 ± 1 → net expected -6571 ± 1

Summary

- **Solutions to Difficulties in Data Analysis Have Been Found**
  1. **Trapped Flux and Changing Polhode Path**

     Solution: Estimate Modulation and Constant Components of Scale Factor for Batches up to 75 orbits long.
  2. **Misalignment Torques**

     Two Complementary Solutions: Geometric and Algebraic Methods
  3. **Offsets in Gyroscope Positions When Harmonics of Polhode Frequency Coincide with Satellite Roll Frequency**

     Solution: Do not use data near resonances
- **All Four Gyroscopes are Consistent with One Another and with Drift Rate Predicted by General Relativity to better than 100 mas/yr.**
- **Formal Statistical Error is Significantly Smaller than Typical Inconsistencies Between Gyroscopes or Between Data Runs**

Methods for Improving Consistency of Results

- **Trapped Magnetic Flux and Polhode Motion**
  1. Include temporal variation of scale factor modulation
  2. Use known polhode path
  3. Trapped Flux Mapping
  4. Crosschecks;
     - » Annual Aberration
     - » Flux Slipping

- **Misalignment Torques**
  1. Provide for slowly changing torque coefficient and misalignment
  2. Tighten Limits on Misalignment Torques due to Incomplete Roll Averaging

- **Offsets in Gyroscope Spin Axis Orientation When Harmonics of Polhode Coincide with Roll Frequency**
  1. Compare data with physical model
  2. Include additional terms in data analysis or modify data selection criteria

Acknowledgements